**Supplementary Information: Relationship between the correlation coefficient and R2**

The explained variance, under the assumption that the observations were normalized to have zero mean $\left(\overbar{\hat{β}}=\frac{1}{n}\sum\_{i}^{}\hat{β}\_{i}=0\right)$ is:

|  |  |  |
| --- | --- | --- |
|  | $$R^{2}=1-\frac{\sum\_{i}^{n}\left(\hat{β}\_{i}-β\_{i}^{\*}\right)^{2}}{\sum\_{i}^{n}\hat{β}\_{i}^{2}}=-\frac{\sum\_{i}^{n}β\_{i}^{\*}^{2}}{\sum\_{i}^{n}\hat{β}\_{i}^{2}}+2\frac{\sum\_{i}^{n}\hat{β}\_{i}β\_{i}^{\*}}{\sum\_{i}^{n}\hat{β}\_{i}^{2}}$$ | (1) |

The correlation coefficient under $\overbar{\hat{β}}=0$ is:

|  |  |  |
| --- | --- | --- |
|  | $$ρ=\frac{\sum\_{i}^{n}\hat{β}\_{i}\left(β\_{i}^{\*}-\overbar{β}^{\*}\right)}{\sqrt{\left(\sum\_{i}^{n}\hat{β}\_{i}^{2}\right)\sum\_{i}^{n}\left(β\_{i}^{\*}-\overbar{β}^{\*}\right)^{2}}}$$ |  (2) |

Writing $R^{2}$as a function of ρ produces

|  |  |  |
| --- | --- | --- |
|  | $$R^{2}=-\frac{\sum\_{i}^{n}β\_{i}^{\*}^{2}}{\sum\_{i}^{n}\hat{β}\_{i}^{2}}+2ρ\sqrt{\frac{\sum\_{i}^{n}\left(β\_{i}^{\*}-\overbar{β}^{\*}\right)^{2}}{\sum\_{i}^{n}\hat{β}\_{i}^{2}}}$$ | (3) |

Based on the following the property of the unbiased estimator of the variance:

$\sum\_{i}^{}β\_{i}^{2}=\left(n-1\right)\hat{σ}\_{β}^{2}+n\overbar{ β}^{2}$ the relation between $R^{2}$ and $ρ$ becomes:

|  |  |  |
| --- | --- | --- |
|  | $$R^{2}=2ρ\frac{\hat{σ}\_{β^{\*}}}{\hat{σ}\_{\hat{β}}}-\frac{\hat{σ}\_{β^{\*}}^{2}}{\hat{σ}\_{\hat{β}}^{2}}-\frac{n \overbar{β}^{\*}^{2}}{(n-1)\hat{σ}\_{\hat{β}}^{2}}$$ | (4) |

The assumption of using responses that are centered (normalized to have zero mean $\left(\frac{1}{n}\sum\_{i}^{}\hat{β\_{i}}=0\right)$ ) reflects the interest of encoding models in describing the variations in the brain response around their mean value as a function of the differences between the features of the presented stimulus. In the more general scenario where $\hat{β}$ is non-centered the term $\frac{n \overbar{β}^{\*}^{2}}{(n-1)\hat{σ}\_{\hat{β}}^{2}}$ becomes into $\frac{n\left(\overbar{β}^{\*} - \overbar{\hat{β}}\right)^{2}}{(n-1)\hat{σ}\_{\hat{β}}^{2}}$ in the previous formula.

**Supplementary Information: Derivation of the analytical noise ceiling**

The definition of noise ceiling for a vector of responses $\hat{β} $and vector of responses of true brain responses $β$ (not directly observed from the data) is (Eq14):

|  |  |  |
| --- | --- | --- |
|  | $$ρ\_{NC}=Ε\left(ρ\right)\_{β^{\*}=β}$$$$ρ\_{NC}=\frac{ \frac{1}{(n-1)}\sum\_{i}^{n}\left(\hat{β}\_{i}-\overbar{\hat{β}}\right)\left(β\_{i}-\overbar{β}\right)}{\sqrt{σ\_{\hat{β}}^{2}σ\_{β}^{2}}}$$ |  (5) |

This definition refers to the covariation across the components $i$ of the “noise free response” $β$ and the estimated response $\hat{β}$ (noise contaminated). Note that each component of the $\hat{β}\_{i} $can be written as: $\hat{β}\_{i} =β\_{i}+ε\_{i}$, where $ε\_{i}$ the variability of each component of $\hat{β}$ around the expected value due to experimental noise. The variance of $ε\_{i}$ derived is $\hat{σ}\_{ε\_{i}}^{2}=\hat{V}\_{ii}$, (derived from one of the methods presented in this article. e.g GLS) where the index *ii* refers to *i* the diagonal entries of the corresponding covariance matrix. Substituting $\hat{β}\_{i} =β\_{i}+ε\_{i}$ into the NC definition produce:

|  |  |  |
| --- | --- | --- |
|  | $$ρ\_{NC}=\frac{\frac{1}{(n-1)}\sum\_{i}^{n}\left(β\_{i}-\overbar{β}+ε\_{i}-\overbar{ε}\right)\left(β\_{i}-\overbar{β}\right)}{\sqrt{σ\_{\hat{β}}^{2}σ\_{β}^{2}}}$$ |  (6) |

Noticing that, (for unbiased estimators of $β$) the expected value of the estimation error each $ε\_{i}$ is 0, and that $ε\_{i} $and $β\_{i}$ are independent. The expected value of the noise ceiling becomes:

|  |  |  |
| --- | --- | --- |
|  | $$ρ\_{NC}=\frac{\frac{1}{(n-1)}\sum\_{i=1}^{n}\left(β\_{i}-\overbar{β}\right)^{2}}{\sqrt{σ\_{\hat{β}}^{2}σ\_{β}^{2}}}=\frac{σ\_{β}}{σ\_{\hat{β}}}$$ | (7) |

 The estimator of $ρ\_{NC}$ is obtained by substituting $σ\_{β}$ and $σ\_{\hat{β}}$ by its corresponding estimators. The estimator of $σ\_{\hat{β}}$ is $\hat{σ}\_{\hat{\left(β\right)}}=\frac{1}{n-1}\sum\_{i}^{}\left(\hat{β}\_{i}-\overbar{\hat{β}}\right)^{2}$ . Consider that if $β\_{i}=β\_{i}+ε\_{i}$ and $β\_{i}$ and $ε\_{i}$ are independent, then: $σ\_{\hat{β}}^{2}=σ\_{β}^{2}+σ\_{ε}^{2}$. The estimator of $σ\_{β}^{2}$ is:

|  |  |  |
| --- | --- | --- |
|  | $$\hat{σ}\_{β}^{2}=\hat{σ}\_{\hat{β}}^{2}-\hat{σ}\_{ε}^{2}$$ | (8) |

With: $\hat{σ}\_{ε}^{2}=\frac{1}{n}\sum\_{i=1}^{n}\left(\hat{V}\_{ii}\right)$. Substituting these estimators into the noise ceiling formula we obtain that the estimator of the noise ceiling is:

|  |  |  |
| --- | --- | --- |
|  | $$\hat{ρ}\_{NC}=\frac{\sqrt{\hat{σ}\_{\hat{β}}^{2}-\frac{1}{n}\sum\_{i=1}^{n}\hat{V}\_{ii}}}{\hat{σ}\_{\hat{β}}}$$ | (9) |