**Supplementary Information: Relationship between the correlation coefficient and R2**

The explained variance, under the assumption that the observations were normalized to have zero mean is:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

The correlation coefficient under is:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Writing as a function of ρ produces

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Based on the following the property of the unbiased estimator of the variance:

the relation between and becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

The assumption of using responses that are centered (normalized to have zero mean ) reflects the interest of encoding models in describing the variations in the brain response around their mean value as a function of the differences between the features of the presented stimulus. In the more general scenario where is non-centered the term becomes into in the previous formula.

**Supplementary Information: Derivation of the analytical noise ceiling**

The definition of noise ceiling for a vector of responses and vector of responses of true brain responses (not directly observed from the data) is (Eq14):

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

This definition refers to the covariation across the components of the “noise free response” and the estimated response (noise contaminated). Note that each component of the can be written as: , where the variability of each component of around the expected value due to experimental noise. The variance of derived is , (derived from one of the methods presented in this article. e.g GLS) where the index *ii* refers to *i* the diagonal entries of the corresponding covariance matrix. Substituting into the NC definition produce:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

Noticing that, (for unbiased estimators of ) the expected value of the estimation error each is 0, and that and are independent. The expected value of the noise ceiling becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

The estimator of is obtained by substituting and by its corresponding estimators. The estimator of is . Consider that if and and are independent, then: . The estimator of is:

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

With: . Substituting these estimators into the noise ceiling formula we obtain that the estimator of the noise ceiling is:

|  |  |  |
| --- | --- | --- |
|  |  | (9) |