## Supporting information

Let  $0 < \alpha < 1$  be a significance level. We illustrate here that false-positives are uncontrolled in the test used in [1] and under control of  $\alpha$  in the correct test described in Section 2.3. To do so, consider the following framework:

- i/ Create a synthetic set  $\mathcal{X}$  by drawing its *n* iid elements from  $\mathcal{N}(0, 1)$ , the Gaussian distribution with zero mean and variance 1. *n* is set to  $10^3$ .
- ii/ Generate  $N = 5 \times 10^4$  independent realisations of such set  $\mathcal{X}$ . All N sets thus have a true zero mean by construction.
- iii/ Test each set: obtain a *p*-value per set and, given the significance level  $\alpha$ , a rejection decision per set.

We then consider both the probability of type I error estimated by  $\hat{p}_{\rm I} = R/N$ , where R counts the number of rejected sets  $\mathcal{X}$  (for the given  $\alpha$ ), as well as the  $\alpha$ -quantile  $p_{\alpha}^*$  of the N p-values obtained: the value under which there are  $\alpha N$  p-values. If the test used in iii/ is correct, both  $\hat{p}_{\rm I}$  and  $p_{\alpha}^*$  should be very close to  $\alpha$ .

Figure 13 compares results obtained with the test published in [1] and the one described in Section 2.3. For both tests, at q = 0 and as expected, both indicators  $\hat{p}_{\rm I}$  and  $p^*_{\alpha}$  are at  $\alpha$ , as it should be. However, as q increases, the test from [1] deviates from  $\alpha$  quite significantly. For instance, for q = 0.2, the 0.01-quantile of the computed p-values is  $3.5 \times 10^{-4}$ , almost two orders of magnitude lower than what it should be!

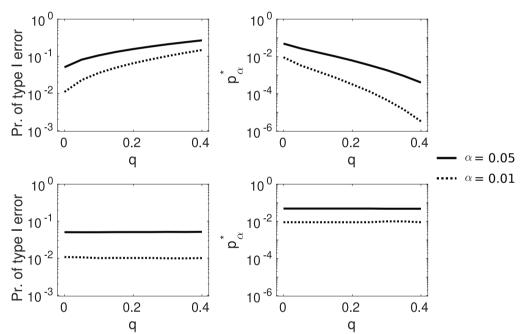


Fig 13. Results obtained on artificial data of zero mean (see the Supporting information for details). Top line: results of the test published in [1]. Bottom line: results of the correct test detailed in Section 2.3. Left: the estimated type I error  $\hat{p}_{I}$  as a function of q (the trimming intensity), for two different values of  $\alpha$ . Right: the  $\alpha$ -quantiles of the *p*-values versus q, for two different values of  $\alpha$ . Number of bootstrap samples used for both tests: 2000.

On the contrary, in the test used in this paper, and for all values of q, both  $\hat{p}_{\rm I}$  and  $p^*_{\alpha}$  are equal to what is expected from a well-controlled test, namely  $\alpha$ .