## Supporting information

Let $0<\alpha<1$ be a significance level. We illustrate here that false-positives are uncontrolled in the test used in [1] and under control of $\alpha$ in the correct test described in Section 2.3. To do so, consider the following framework:
i/ Create a synthetic set $\mathcal{X}$ by drawing its $n$ iid elements from $\mathcal{N}(0,1)$, the Gaussian distribution with zero mean and variance 1. $n$ is set to $10^{3}$.
ii/ Generate $N=5 \times 10^{4}$ independent realisations of such set $\mathcal{X}$. All $N$ sets thus have a true zero mean by construction.
iii/ Test each set: obtain a $p$-value per set and, given the significance level $\alpha$, a rejection decision per set.

We then consider both the probability of type I error estimated by $\hat{p}_{\mathrm{I}}=R / N$, where $R$ counts the number of rejected sets $\mathcal{X}$ (for the given $\alpha$ ), as well as the $\alpha$-quantile $p_{\alpha}^{*}$ of the $N p$-values obtained: the value under which there are $\alpha N p$-values. If the test used in iii/ is correct, both $\hat{p}_{\mathrm{I}}$ and $p_{\alpha}^{*}$ should be very close to $\alpha$.

Figure 13 compares results obtained with the test published in [1] and the one described in Section 2.3. For both tests, at $q=0$ and as expected, both indicators $\hat{p}_{I}$ and $p_{\alpha}^{*}$ are at $\alpha$, as it should be. However, as $q$ increases, the test from [1] deviates from $\alpha$ quite significantly. For instance, for $q=0.2$, the 0.01 -quantile of the computed $p$-values is $3.5 \times 10^{-4}$, almost two orders of magnitude lower than what it should be!


Fig 13. Results obtained on artificial data of zero mean (see the Supporting information for details). Top line: results of the test published in [1]. Bottom line: results of the correct test detailed in Section 2.3. Left: the estimated type I error $\hat{p}_{\mathrm{I}}$ as a function of $q$ (the trimming intensity), for two different values of $\alpha$. Right: the $\alpha$-quantiles of the $p$-values versus $q$, for two different values of $\alpha$. Number of bootstrap samples used for both tests: 2000.

On the contrary, in the test used in this paper, and for all values of $q$, both $\hat{p}_{\mathrm{I}}$ and $p_{\alpha}^{*}$ are equal to what is expected from a well-controlled test, namely $\alpha$.

