

Suppressors of Selection – Supporting Information – S1 Text

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The fixation probability is a rational function

Given a connected undirected graph $G = (V, E)$, the Markov chain associated to the invasion process is as follows: The set of states \mathcal{S} of the chain is the power set of the vertex set V , where each set $S \in \mathcal{S}$ contains the nodes occupied by mutant individuals. The transition probability between two states $S, S' \in \mathcal{S}$ is

$$P_{S,S'} = \begin{cases} \frac{r \sum_{i \in S} w_{ij}}{w_S} & \text{if } S' \setminus S = \{j\}, \\ \frac{\sum_{i \in V \setminus S} w_{ij}}{w_S} & \text{if } S \setminus S' = \{j\}, \\ \frac{r \sum_{i,j \in S} w_{ij} + \sum_{i,j \in V \setminus S} w_{ij}}{w_S} & \text{if } S = S', \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $r > 0$ is the relative fitness for the invader mutant, d_i is the degree of the node i , and $w_{ij} = 1/d_i$ if $(i, j) \in E$ and $w_{ij} = 0$ otherwise. Notice that all the outgoing probabilities $P_{S,S'}$ have the same denominator, the *total reproductive weight of S*

$$w_S = r \sum_{i \in S} \sum_{j \in V} w_{ij} + \sum_{i \in V \setminus S} \sum_{j \in V} w_{ij} = r|S| + N - |S|.$$

Let $\Phi_S(r)$ be the probability of all population will become mutant at some point starting with mutants in the vertices belonging to S , that is, the probability of reaching the absorbing state $V \in \mathcal{S}$ starting from $S \in \mathcal{S}$. Hence, the *average fixation probability* is

$$\Phi(r) = \frac{1}{N} \sum_{i \in V} \Phi_{\{i\}}(r).$$

To compute this quantity it is enough to solve the system of linear equations

$$\Phi_S(r) = \sum_{S' \in \mathcal{S}} P_{S,S'} \Phi_{S'}(r), \quad (2)$$

with boundary conditions $\Phi_\emptyset(r) = 0$ and $\Phi_V(r) = 1$. If $\mathbf{P} = (P_{S,S'})$ is the transition matrix, Eq 2 can be written as

$$\mathbf{P} \cdot \Phi = \mathbf{P} \cdot \begin{pmatrix} 0 \\ \Psi \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \mathbf{b} & \mathbf{Q} & \mathbf{c} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \Psi \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \Psi \\ 1 \end{pmatrix} = \Phi, \quad (3)$$

where $\Phi = (0, \Psi, 1)$ is the vector with coordinates $\Phi_S(r)$, $(1, \mathbf{b}, 0)$ is the vector with coordinates $P_{S,\emptyset}$, and $(0, \mathbf{c}, 1)$ is the vector with coordinates $P_{S,V}$. It can be rewritten as

$$(\mathbf{I} - \mathbf{Q}) \cdot \Psi = \mathbf{c}, \quad (4)$$

where \mathbf{I} is the identity matrix. It is clear that the entries of $\mathbf{I} - \mathbf{Q}$ and the coordinates of \mathbf{c} belongs to $\mathbb{Q}[r]$. Therefore, Cramer's rule implies that the entries of the fundamental matrix $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$ and the coordinates of the solution Ψ of the linear system Eq 4 are rational functions on r with rational coefficients. Then,

Lemma 1. *The average fixation probability $\Phi(r)$ is a rational function on r with rational coefficients.* \square

In order to bound the degree of the numerator and denominator of $\Phi(r)$, we must simplify Eq 4 as follows. If we multiply each equation by the reproductive weight w_S of the corresponding state S , we obtain the simpler equivalent linear system

$$[\mathbf{T}(\mathbf{I} - \mathbf{Q})] \cdot \Psi = \mathbf{T} \cdot \mathbf{c},$$

where \mathbf{T} is the diagonal matrix with entries w_S associated to the states $S \in \mathcal{S}$ different from \emptyset and V . Obviously, all the coordinates of the matrix $\mathbf{T}(\mathbf{I} - \mathbf{Q})$ and the vector $\mathbf{T} \cdot \mathbf{c}$ are functions of the form $ar + b$ with a and $b \in \mathbb{Q}$. By Cramer's rule, the fixation probability of any state S is

$$\Phi_S(r) = \frac{\det[\mathbf{T}(\mathbf{I} - \mathbf{Q})]_S}{\det[\mathbf{T}(\mathbf{I} - \mathbf{Q})]}$$

where $[\mathbf{T}(\mathbf{I} - \mathbf{Q})]_S$ is the matrix obtained from $\mathbf{T}(\mathbf{I} - \mathbf{Q})$ by replacing the column associated to S with the vector $\mathbf{T} \cdot \mathbf{c}$. If the size of both matrices is $d \times d$, since each entry is a polynomial of degree at most 1, $\Phi_S(r)$ is the quotient of two polynomial with rational coefficients of degree at most d . Hence,

Lemma 2. *The average fixation probability $\Phi(r)$ is a rational function with both numerator and denominator polynomials of degree equal to the number of states minus two.*

Remark 3. A priori, the degree of the average fixation probability function is $2^N - 2$ where N is the size of the graph. However, using the symmetries of the graph it is possible to reduce the number of possible states, and hence the degree of $\Phi(r)$. For example, the average fixation probability function for the complete graph K_N has degree, at most, $N - 1$. For the complete bipartite graphs $K_{n,m}$ it would have degree $\leq (n - 1)(m - 1) - 2$ (see Refs. 1–3 for the concrete expressions). As we explained in the paper, for the ℓ -graphs, the degree $d = \frac{N(N+1)}{2} - 2$.

On the other hand, since the average fixation probability tends to 1 as $r \rightarrow \infty$, both numerator and denominator have the same lead coefficient, so it can be assumed to be 1.

The SageMath program

The essential tool in order to prove that ℓ_6 , ℓ_8 and ℓ_{10} are suppressor of selection for any fitness value $r > 1$ is a SageMath program that symbolically

- computes the exact fixation probability $\Phi(r)$ for the graphs ℓ_6 , ℓ_8 and ℓ_{10} when $r \in \{1, \dots, d + 1, 1/2, \dots, 1/d\}$,
- solves the linear system

$$\sum_{i=0}^d a_i r^i = \Phi(r) \left(\sum_{i=0}^d b_i r^i \right),$$

where $\Phi'(r) = \sum_{i=0}^d a_i r^i$ and $\Phi''(r) = \sum_{i=0}^d b_i r^i$ are the numerator and the denominator of the rational function $\Phi(r)$. The symmetries of the ℓ -graphs allow to reduce the degree to $d = \frac{N(N+1)}{2} - 2 \ll 2^N - 2$. The code is available at <https://bitbucket.org/snippets/alvarolozano/7jnka/>. Next, we give the exact values of $\Phi = \Phi'/\Phi''$ and $\Delta = \Delta'/\Delta''$ for the graphs ℓ_6 , ℓ_8 and ℓ_{10} .

Fixations functions for ℓ_6 , ℓ_8 and ℓ_{10}

Graph ℓ_6

$$\Phi'(r) = r^{17} + \frac{20010803101}{2356885440} r^{16} + \frac{12872954057}{368903808} r^{15} + \frac{146529682470259}{1590897672000} r^{14} + \frac{20006411662050293}{114544632384000} r^{13} + \frac{57705463723529081}{229089264768000} r^{12} + \frac{9771555843017869}{34363389715200} r^{11} + \frac{1025122250624863}{4042751731200} r^{10} + \frac{60836752923313811}{343633897152000} r^9 + \frac{9233242322315201}{98181113472000} r^8 + \frac{1542392404026107}{42954237144000} r^7 + \frac{3153322564729}{357951976200} r^6 + \frac{496442293}{474304896} r^5$$

$$\Phi''(r) = r^{17} + \frac{22367688541}{2356885440} r^{16} + \frac{1883112090533}{42423937920} r^{15} + \frac{579196292648299}{4242393792000} r^{14} + \frac{23779172425823219}{76363088256000} r^{13} + \frac{17230349794290811}{30545235302400} r^{12} + \frac{55592085032472109}{65454075648000} r^{11} + \frac{71798191286820983}{65454075648000} r^{10} + \frac{37885437915832519}{30545235302400} r^9 + \frac{37885437915832519}{71798191286820983} r^8 + \frac{71798191286820983}{55592085032472109} r^7 + \frac{55592085032472109}{17230349794290811} r^6 + \frac{17230349794290811}{30545235302400} r^5 + \frac{30545235302400}{65454075648000} r^4 + \frac{65454075648000}{65454075648000} r^3 + \frac{1883112090533}{42423937920} r^2 + \frac{22367688541}{2356885440} r + 1$$

$$\Delta'(r) = r^5(r-1) \left(-\frac{1}{408} r^{13} - \frac{17959403797}{509087255040} r^{12} - \frac{2269495014623}{9163570590720} r^{11} - \frac{165933379973713}{152726176512000} r^{10} - \frac{4365681088426001}{1374535588608000} r^9 - \frac{4487811876686051}{687267794304000} r^8 - \frac{6774104367192077}{687267794304000} r^7 - \frac{673669214287879}{59762416896000} r^6 - \frac{13704670736178649}{1374535588608000} r^5 - \frac{2361661570426121}{343633897152000} r^4 - \frac{2306212053017441}{687267794304000} r^3 - \frac{122207986609099}{85908474288000} r^2 - \frac{299174081287}{818175945600} r - \frac{22137397}{474304896} \right)$$

$$\Delta''(r) = r^{21} + \frac{22367688541}{2356885440} r^{20} + \frac{1925536028453}{42423937920} r^{19} + \frac{619458132022099}{4242393792000} r^{18} + \frac{27245137277038619}{76363088256000} r^{17} + \frac{15493177389178517}{21818025216000} r^{16} + \frac{552157240360000477}{458178529536000} r^{15} + \frac{411797892764062669}{229089264768000} r^{14} + \frac{550050599259865931}{229089264768000} r^{13} + \frac{1329324154659596831}{458178529536000} r^{12} + \frac{54074574184168127}{16969575168000} r^{11} + \frac{54074574184168127}{16969575168000} r^{10} + \frac{1329324154659596831}{458178529536000} r^9 + \frac{458178529536000}{550050599259865931} r^8 + \frac{411797892764062669}{229089264768000} r^7 + \frac{552157240360000477}{458178529536000} r^6 + \frac{15493177389178517}{21818025216000} r^5 + \frac{229089264768000}{27245137277038619} r^4 + \frac{619458132022099}{4242393792000} r^3 + \frac{1925536028453}{42423937920} r^2 + \frac{22367688541}{2356885440} r + 1$$

Graph ℓ_8

$$\Phi'(r) = r^{31} + \frac{134280899640541}{8309364148800} r^{30} + \frac{51436353602527862867}{399248328621542400} r^{29} + \frac{261395239481852545538087881}{387350728428620436480000} r^{28} + \frac{58380787959652779590779931453}{22369504566752830206720000} r^{27} + \frac{273244285509514020032806460287}{34414622410388969548800000} r^{26} + \frac{109546794936308370917902477449001}{5541864357182636644761600000} r^{25} + \frac{103820319576966457199577599649}{25100123467406200012800000} r^{24} + \frac{92197306752696283843238979311641}{1243784941702528405340160000} r^{23} + \frac{11740712284825831069430912101427861}{101806100783799547251916800000} r^{22} + \frac{1032937952392948232127781620508111}{6568135534438680467865600000} r^{21} + \frac{2080310449462551016365104268144504161}{10995058884650351103207014400000} r^{20} + \frac{753897025982731385467385013243433}{3739815947159983368437760000} r^{19} + \frac{24635466779770270651137435076943581}{1251563161394232439985295422113577} r^{18} + \frac{1251563161394232439985295422113577}{7853613489035965073719296000} r^{17} + \frac{129353633937062954155376640000}{3477901185411691168774411224802753} r^{16} + \frac{5510437720885200677122578161372417}{72335913714804941468467200000} r^{15} + \frac{470903112909175542318799571869746773}{29556609904974062105395200000} r^{14} + \frac{596923826377497575290462451325719}{28934365485921976587386880000} r^{13} + \frac{7066876349108602065069037131219649}{15230658652580143385602731564041827} r^{12} + \frac{15230658652580143385602731564041827}{845773760357719315631308800000} r^{11} + \frac{845773760357719315631308800000}{141930027424171970518414844712131} r^{10} + \frac{911745036811073646792134165773}{6544677907529970894766080000} r^9 + \frac{141930027424171970518414844712131}{196340337225899126842982400000} r^8 + \frac{11363885694605603}{354149237420452970496000} r^7 + \frac{10303182674104320}{354149237420452970496000} r^6$$

$$\Phi''(r) = r^{31} + \frac{142590263789341}{8309364148800} r^{30} + \frac{685751104869983351}{4697039160253440} r^{29} + \frac{317970272348671658719761481}{387350728428620436480000} r^{28} + \frac{614045446315686238195542724177}{178956036534022641653760000} r^{27} + \frac{782883035835001850913227050807}{2027468204090641240495620587659} r^{26} + \frac{68829244820777939097600000}{56074922376008233328640000} r^{25} + \frac{62336794853655369318718185089022867}{540599738388527595424381013403073} r^{24} + \frac{85898897536330867993804800000}{36813813229856086283059200000} r^{23} + \frac{54122335023003970350539078924384011}{1733545798098181700601364139420724667} r^{22} + \frac{206157354087194083185131520000}{4123147081743881663702630400000} r^{21} + \frac{1677617314994594652141277695694600403}{13378648977067195339236595661589558203} r^{20} + \frac{274876472116258775801753600000}{1649258832697552665481052160000} r^{19} + \frac{106641778813417172713997454149404519}{9374593978090858152054746159000953163} r^{18} + \frac{107094729395944978277990400000}{8246294163487763327405260800000} r^{17} + \frac{39696361687927740615186917132468293}{39696361687927740615186917132468293} r^{16} + \frac{32723389537649854473830400000}{32723389537649854473830400000} r^{15} + \frac{9374593978090858152054746159000953163}{106641778813417172713997454149404519} r^{14} + \frac{8246294163487763327405260800000}{107094729395944978277990400000} r^{13} + \frac{13378648977067195339236595661589558203}{1677617314994594652141277695694600403} r^{12} + \frac{1649258832697552665481052160000}{274876472116258775801753600000} r^{11} + \frac{1733545798098181700601364139420724667}{54122335023003970350539078924384011} r^{10} + \frac{4123147081743881663702630400000}{206157354087194083185131520000} r^{9} + \frac{540599738388527595424381013403073}{623367948536553693187185089022867} r^{8} + \frac{36813813229856086283059200000}{85898897536330867993804800000} r^{7} + \frac{2027468204090641240495620587659}{782883035835001850913227050807} r^{6} + \frac{65074922376008233328640000}{68829244820777939097600000} r^{5} + \frac{614045446315686238195542724177}{317970272348671658719761481} r^{4} + \frac{178956036534022641653760000}{387350728428620436480000} r^{3} + \frac{685751104869983351}{142590263789341} r^{2} + \frac{4697039160253440}{8309364148800} r + 1$$

$$\Delta'(r) = r^7(r-1) \left(-\frac{1}{304} r^{27} - \frac{25197109151333}{398849479142400} r^{26} - \frac{79418092024874684869}{15143937856292402262196129} r^{25} - \frac{130663089367050240000}{3873507284286204364800000} r^{24} - \frac{2366973910210561053038186471}{8455314295207479062300968873111} r^{23} - \frac{124943850961935807990988800}{114531863381774490658406400000} r^{22} - \frac{991524478799860112838020328209849}{5521164844468759159410730585869037} r^{21} - \frac{4123147081743881663702630400000}{8246294163487763327405260800000} r^{20} - \frac{985364585564805190812437620862539}{111731529120083126099350945280644519} r^{19} - \frac{610836604702797283511500800000}{32985176653951053309621043200000} r^{18} - \frac{205460937088057311653205176835961321}{11073449367030373276155500115787513} r^{17} - \frac{32985176653951053309621043200000}{10995058884650351103207014400000} r^{16} - \frac{158032724138319990637134255263110231}{199802720995875508460288885619407237} r^{15} - \frac{10995058884650351103207014400000}{10995058884650351103207014400000} r^{14} - \frac{224538609583176412587468836353029011}{2409466399232446388630691852040643} r^{13} - \frac{10995058884650351103207014400000}{1178042023355394761057894400000} r^{12} - \frac{150767863647126646217505642437799077}{53451052002645000374177015848862851} r^{11} - \frac{8246294163487763327405260800000}{3665019628216783701069004800000} r^{10} - \frac{341577973854758089828094884500692837}{6134982182129497498119956432252281} r^{9} - \frac{32985176653951053309621043200000}{942433618684315808846315520000} r^{8} - \frac{5639056379567408901354673628901377}{18840773531540731523638914353570237} r^{7} - \frac{1570722697807193014743859200000}{10995058884650351103207014400000} r^{6} - \frac{218101089181656220622956529171233}{85136118048946532310061747931273} r^{5} - \frac{314144539561438602948771840000}{366501962821678370106900480000} r^{4} - \frac{132994911304258723701851574251}{79621768891657352281009056253} r^{3} - \frac{2157586123361528866406400000}{6544677907529970894766080000} r^{2} - \frac{562493367795228941534181}{1060703020501283} r - \frac{354149237420452970496000}{10303182674104320})$$

$$\Delta''(r) = r^{37} + \frac{142590263789341}{8309364148800} r^{36} + \frac{690448144030236791}{4697039160253440} r^{35} + \frac{324617284268784317447995081}{387350728428620436480000} r^{34} + \frac{640351350752994828494392138177}{178956036534022641653760000} r^{33} + \frac{10927345392515915835176820297601}{306957477135913987153449321197} r^{32} + \frac{894780182670113208268800000}{8837335137482599587840000} r^{31} + \frac{34845476760990740918893922667907}{46792604023787574552501267885526031} r^{30} + \frac{410999509743209894707200000}{257696692608992603981414400000} r^{29} + \frac{51140920333603397111269496525056973}{827208499195876349134187545906993721} r^{28} + \frac{147255252919424345132236800000}{1374382360581293887900876800000} r^{27} + \frac{7889974227256354217659306239108441937}{11624280269542628969132052974659677181} r^{26} + \frac{8246294163487763327405260800000}{8246294163487763327405260800000} r^{25} + \frac{190566613632525673986701170263208391}{78568005593981519178754523078759401} r^{24} + \frac{98170168612949563421491200000}{31235962740483951997747200000} r^{23} + \frac{1058860227495618987685429305456054977}{738362330204465247116888965793050837} r^{22} + \frac{34359590145323471975219200000}{2061573540871940831851315200000} r^{21} + \frac{1716965580898187107499130859630048009}{34278818580615368782242631553834901977} r^{20} + \frac{434015482288829648810803200000}{8246294163487763327405260800000} r^{19} + \frac{34278818580615368782242631553834901977}{1716965580898187107499130859630048009} r^{18} + \frac{8246294163487763327405260800000}{434015482288829648810803200000} r^{17} + \frac{738362330204465247116888965793050837}{1058860227495618987685429305456054977} r^{16} + \frac{2061573540871940831851315200000}{34359590145323471975219200000} r^{15} + \frac{78568005593981519178754523078759401}{190566613632525673986701170263208391} r^{14} + \frac{31235962740483951997747200000}{98170168612949563421491200000} r^{13} + \frac{11624280269542628969132052974659677181}{7889974227256354217659306239108441937} r^{12} + \frac{8246294163487763327405260800000}{8246294163487763327405260800000} r^{11} + \frac{827208499195876349134187545906993721}{5114092033360339711269496525056973} r^{10} + \frac{1374382360581293887900876800000}{147255252919424345132236800000} r^{9} + \frac{4679260402378757452501267885526031}{34845476760990740918893922667907} r^{8} + \frac{257696692608992603981414400000}{410999509743209894707200000} r^{7} + \frac{306957477135913987153449321197}{10927345392515915835176820297601} r^{6} + \frac{8837335137482599587840000}{894780182670113208268800000} r^{5} + \frac{640351350752994828494392138177}{324617284268784317447995081} r^{4} + \frac{178956036534022641653760000}{387350728428620436480000} r^{3} + \frac{690448144030236791}{142590263789341} r^{2} + \frac{4697039160253440}{8309364148800} r + 1$$

Graph ℓ_{10}

$$\begin{aligned} \Phi'(r) = & r^{49} + \\ & 63998337467225103992090545275949 r^{48} + \\ & 2453784351049966935520268248500 r^{47} + \\ & 1467791915119732826876236247120465601078713 r^{46} + \\ & 4347173063939489764811377306367332725000 r^{45} + \\ & 35863545786298851175250332766194774847587066497001 r^{44} + \\ & 12401832675459773375542137748470045164516250000 r^{43} + \\ & 10107225020718561540118243112729380257297990160772275379 r^{42} + \\ & 548483451904883937306726584063836217445895672500000 r^{41} + \\ & 17397736112593212118849985482353558760254598400369231894071 r^{40} + \\ & 186745556243805721511575956002687093082769240875000000 r^{39} + \\ & 1584820175953684739314764508568609751473509134206333820165933161 r^{38} + \\ & 4072981476967482288033181896925997680709028481279687500000 r^{37} + \\ & 517274463035147513860432834351215563141952618120529963719956531977 r^{36} + \\ & 3747142958810083704990527345171917866252306202777312500000000 r^{35} + \\ & 4952120295598135430003008711918788531699336080751711979186627645861149 r^{34} + \\ & 1166544768492718163948366802457468114678020588917252812500000000 r^{33} + \\ & 232819629505635204261122698697497213530179530184438758651042497936165479 r^{32} + \\ & 202621861840806084073867442740280375889036785127231785156250000000 r^{31} + \\ & 1407051619422293599314085668376462617712634457962573176050687488215725213 r^{30} + \\ & 50788120100002722646071499687092699283413941116170792968750000000 r^{29} + \\ & 704776109036054971829235346720766616789810509486535754147964424687649474571 r^{28} + \\ & 11732055951029389312425164277184135344686203978354531757812500000000 r^{27} + \\ & 560923897879890207748212897673461209350427363232082881495578421471399410965213 r^{26} + \\ & 4751482660166902671532191532259574814597912611233585361914062500000000 r^{25} + \\ & 502667498451061439286536771249640753000401734211654584203734622109856518280909 r^{24} + \\ & 23757413300834513357660957661297874072989563056167926809570312500000000 r^{23} + \\ & 2642982401545335688749171370572651127697436882617726500323253117106612145203 r^{22} + \\ & 7602372526267044274451506451615319703356660177973736579062500000000 r^{21} + \\ & 20823387297836818281526008435918371919883630252711427765368887472917082446989053 r^{20} + \\ & 395956883472418892943492945496456788315938426946544682617187500000000 r^{19} + \\ & 218249476472181748593498178533656026352573696800474212113593696562977826835140353 r^{18} + \\ & 296967666260431416970761970766223425912369538202099085119628906250000000 r^{17} + \\ & 12073616682732932701560441487145827994410542880111327873960403995706625725384629 r^{16} + \\ & 126909259085654451669690682511377069483434595649662354064941406250000000 r^{15} + \\ & 135789940566315840585532565857821740547661945056372353589849669605940440768457227 r^{14} + \\ & 118787066504172566788304788306489370364947815280839634047851562500000000 r^{13} + \\ & 126393866501084509152825413120795020467290201523310371883484596474040356624342127 r^{12} + \\ & 989892220868104723235873235887411419707898460673663617065429687500000000 r^{11} + \\ & 3893768339258599798434728937074950725159562999271918086462520859044296962887753 r^{10} + \\ & 2934463105340231392991719078717622785695351168004931671142578125000000 r^{9} + \\ & 1216824486353361237463418998567150903579908174072483322171283158910558540587207 r^{8} + \\ & 948172625352590731068844095677597145314078985319601165771484375000000 r^{7} + \\ & 34295065886761457556477444874331500786589425104481846989084698332117314061325299 r^{6} + \\ & 296967666260431416970761970766223425912369538202099085119628906250000000 r^{5} + \\ & 13662088991753822285217823124817295526976539096842805176370201484115198050916799 r^{4} + \\ & 141413174409729246176553319412487345672556922953380516723632812500000000 r^{3} + \\ & 10610372870896170772509656575515409475380951963052424452593627351709782825503313 r^{2} + \\ & 141413174409729246176553319412487345672556922953380516723632812500000000 r^{1} + \\ & 496044793580204554525678229969923834502352531064764912680884504837714700470547 r^{0} \\ & 918835601053315027756070454103414065322925551367880832672119140625000 r^{0} + \\ & 772359485654188724829120426699784984692635338460009360833608747322067262809039 r^{0} + \\ & 2151939610582836354860593991059590042843257523203616558837890625000000 r^{0} + \\ & 652516365227989567609612927273551680629147392704119960328347885247676692854839209 r^{0} + \\ & 296967666260431416970761970766223425912369538202099085119628906250000000 r^{0} + \\ & 366302474000671446200946961697442757335305613890313285591333774174703031631539261 r^{0} + \\ & 296967666260431416970761970766223425912369538202099085119628906250000000 r^{0} + \\ & 1645494418668165225995421450087912710711608999941997419463709015853368829896609 r^{0} + \\ & 26049795286002755874628243049668721571260485807201674133300781250000000 r^{0} + \\ & 3483355672276308965948183203469013565617881463433844672444674298813595385516279 r^{0} + \\ & 118787066504172566788304788306489370364947815280839634047851562500000000 r^{0} + \\ & 3636607175932414505549205751279497624573418272160719024002888028025852967255521 r^{0} + \\ & 296967666260431416970761970766223425912369538202099085119628906250000000 r^{0} + \\ & 1566098453886886032512195177922507302933744244599563429255680654923560486349 r^{0} + \\ & 343712576690314140012455984683128965176353632178355422592163085937500 r^{0} + \\ & 9849998529036909725835193690470491230633425936733439335488624857067513903961 r^{0} + \\ & 6599281472454031488239154905916076131385989737824424113769531250000000 r^{0} + \\ & 761224467164339496002495975771535194184958698344817283774206365930539589 r^{0} + \\ & 17957228496473555070038516750792043894927863232175303710937500000000 r^{0} + \\ & 1780331892413176694593527982155770314406131540035651123394684892300797027 r^{0} + \\ & 1740469838889753375575793158459981005853159748108371289062500000000 r^{0} + \\ & 666823956195516846359952872220117109274829807880728490758393042227 r^{0} + \\ & 32694089206142112098386011380595437223355235743605468750000000 r^{0} + \\ & 502416727669024719925142702253603868291369674070233164892842561 r^{0} + \\ & 155686139076867200468504816098073510587405884493359375000000 r^{0} + \\ & 20309000357168189131016179693765660499567247658979879117 r^{0} + \\ & 5344694490858694730872157429537304821434882812500000 r^{0} + \\ & 2231094028186541795006656736377928879201619060106379 r^{0} + \\ & 7535835871022928497217850902161732770498046875000 r^{0} + \\ & 2160930461505967730083659877408854787 r^{0} + \\ & 18869431352372327224847699296875000 r^{0} \end{aligned}$$

$$\begin{aligned}
\Phi''(r) = & r^{49} + 66452121818275070927610813524449 r^{48} + \\
& \frac{2453784351049966935520268248500}{31710659041031378883917151919165713711668601} r^{47} + \\
& \frac{8694346127878979529622754612734665450000}{1615514668089610786321341609853576292618189857722729} r^{46} + \\
& \frac{49607330701839093502168550993880180658065000000}{2068529647022090441678077299141302609199588004130495071} r^{45} + \\
& \frac{95388426418240684748995927663275863903634030000000}{450435638313535005425039251624357069084344950220101681650037} r^{44} + \\
& \frac{3921656681119920151743095076056428954738154058375000000}{6867963997466308046319209185518123663135600673726493180976212363} r^{43} + \\
& \frac{13625974395673031654511008527897883150008386191917500000000}{269061418914533471263783319921967679903930817801083532938975715087} r^{42} + \\
& \frac{14274830319276509352344866076845401395246880772485000000000}{10872394332592777284288878621556700840415990768352322461450204898552609} r^{41} + \\
& \frac{177314804810893160920151753973535153431059129515422427500000000}{38291101140138887359782676914615116290259958340221391656013420168256811577} r^{40} + \\
& \frac{217210635893344122127185898617580562953047433656392473687500000000}{98514958000354677455643814676264534931115549410475725682221654907248284623} r^{39} + \\
& \frac{217210635893344122127185898617580562953047433656392473687500000000}{26732729318515826831791274995616579065637953912223549102534868243984792082723} r^{38} + \\
& \frac{253412408542234809148383548387177323445220059324578859687500000000}{425183741527331139825902027524200329764363303966235727482073723400678572174677} r^{37} + \\
& \frac{190059306406676106861287661290382992583916504449343414476562500000000}{82807264851056126318968209929741089832356566113607542045269837442115035231749} r^{36} + \\
& \frac{190059306406676106861287661290382992583916504449343414476562500000000}{1490429730095426434065316373615468433612748954602965427264845089050815525174899} r^{35} + \\
& \frac{190059306406676106861287661290382992583916504449343414476562500000000}{4155048488190944638832776774327706161798547725172699805670965166909440410048507} r^{34} + \\
& \frac{316765510677935114354794354839716543065275074155723574609375000000000}{22130456707641639004248146630609315027823814070629995136603247025845612921651603} r^{33} + \\
& \frac{10798824227652051526209526209680851851358892298258148549804687500000000}{1784422236438075188537138900658095059296529954702208506947548731323074538577597981} r^{32} + \\
& \frac{593935332520862833941523941532446851824739076404198170239257812500000000}{3287151019212848217807984745591777366372276860411583588494571887934733431113199393} r^{31} + \\
& \frac{79191377669448377858869858870992913576631876853893089365234375000000000}{12891830160861887042312930556116527397598369437831253112410533891470144462896563509} r^{30} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{1143364706849144471368732701620706675895368660811433194915842968849736588740046217} r^{29} + \\
& \frac{169695809291675095411863983294984814807068307544056620068359375000000000}{1894223361894024164709180344193508276149281595591591986658211519315537547323846207} r^{28} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{21428097029880373218311112435358229670656891082066760972501955864501432138378896531} r^{27} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{801022019708817904455746198578617530054038040854412364759411591253792343583646439} r^{26} + \\
& \frac{8192211483046383916434812986654439335513642433161354072265625000000000}{6043831092625088541215047733673129681002510108096187925439945057336154334932897427} r^{25} + \\
& \frac{593935332520862833941523941532446851824739076404198170239257812500000000}{6043831092625088541215047733673129681002510108096187925439945057336154334932897427} r^{24} + \\
& \frac{593935332520862833941523941532446851824739076404198170239257812500000000}{801022019708817904455746198578617530054038040854412364759411591253792343583646439} r^{23} + \\
& \frac{8192211483046383916434812986654439335513642433161354072265625000000000}{21428097029880373218311112435358229670656891082066760972501955864501432138378896531} r^{22} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{1894223361894024164709180344193508276149281595591591986658211519315537547323846207} r^{21} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{1143364706849144471368732701620706675895368660811433194915842968849736588740046217} r^{20} + \\
& \frac{169695809291675095411863983294984814807068307544056620068359375000000000}{12891830160861887042312930556116527397598369437831253112410533891470144462896563509} r^{19} + \\
& \frac{237574133008345133576609576612978740729895630561679268095703125000000000}{3287151019212848217807984745591777366372276860411583588494571887934733431113199393} r^{18} + \\
& \frac{79191377669448377858869858870992913576631876853893089365234375000000000}{1784422236438075188537138900658095059296529954702208506947548731323074538577597981} r^{17} + \\
& \frac{593935332520862833941523941532446851824739076404198170239257812500000000}{22130456707641639004248146630609315027823814070629995136603247025845612921651603} r^{16} + \\
& \frac{10798824227652051526209526209680851851358892298258148549804687500000000}{4155048488190944638832776774327706161798547725172699805670965166909440410048507} r^{15} + \\
& \frac{316765510677935114354794354839716543065275074155723574609375000000000}{1490429730095426434065316373615468433612748954602965427264845089050815525174899} r^{14} + \\
& \frac{1900593064066761068612876612903829925839165044493434144765625000000000}{82807264851056126318968209929741089832356566113607542045269837442115035231749} r^{13} + \\
& \frac{1900593064066761068612876612903829925839165044493434144765625000000000}{425183741527331139825902027524200329764363303966235727482073723400678572174677} r^{12} + \\
& \frac{1900593064066761068612876612903829925839165044493434144765625000000000}{26732729318515826831791274995616579065637953912223549102534868243984792082723} r^{11} + \\
& \frac{2534124085422348091483835483871773234452200593245788596875000000000}{98514958000354677455643814676264534931115549410475725682221654907248284623} r^{10} + \\
& \frac{2172106358933441221271858986175805629530474336563924736875000000000}{38291101140138887359782676914615116290259958340221391656013420168256811577} r^9 + \\
& \frac{2172106358933441221271858986175805629530474336563924736875000000000}{10872394332592777284288878621556700840415990768352322461450204898552609} r^8 + \\
& \frac{1773148048108931609201517539735351534310591295154224275000000000}{269061418914533471263783319921967679903930817801083532938975715087} r^7 + \\
& \frac{142748303192765093523448660768454013952468807724850000000000}{6867963997466308046319209185518123663135600673726493180976212363} r^6 + \\
& \frac{136259743956730316545110085278978831500083861919175000000000}{450435638313535005425039251624357069084344950220101681650037} r^5 + \\
& \frac{39216566811199201517430950760564289547381540583750000000}{2068529647022090441678077299141302609199588004130495071} r^4 + \\
& \frac{95388426418240684748995927663275863903634030000000}{1615514668089610786321341609853576292618189857722729} r^3 + \\
& \frac{49607330701839093502168550993880180658065000000}{31710659041031378883917151919165713711668601} r^2 + \\
& \frac{8694346127878979529622754612734665450000}{66452121818275070927610813524449} r + 1 \\
& 2453784351049966935520268248500
\end{aligned}$$

$\Delta'(r) = (r-1)r^9 \left(-\frac{81}{26800}r^{45} - \right.$
563319538758565590920518332973 $r^{44} -$
6524782699847965260642688600000 $r^{43} -$
185118512906802958216749583829138613138971 $r^{42} -$
15027264912383421409224514145467323000000 $r^{41} -$
11292851072467750513499229152350823090599713979537 $r^{40} -$
964586985869093484764388491547670179462375000000 $r^{39} -$
27951381645948785379888787463235668294504668152854207 $r^{38} -$
335217853505001795200297386666566567318112500000000 $r^{37} -$
60827942349203178778068863211998358870186907289615783102254391 $r^{36} -$
12810745158325072495694110581784334585477969924025000000000 $r^{35} -$
856291220210090311905987439041329717463210575569086154366093523 $r^{34} -$
380144937850298346883096977046426450199509324919437500000000 $r^{33} -$
44621655705123136077347662614843260751907658760768760237116789155653 $r^{32} -$
487128584645310881648768554872349322612799806361050625000000000 $r^{31} -$
80492686432889156295605195823804626469146709501499514533353144038439997 $r^{30} -$
2468302680606183205990748847927051851739175382459005382812500000000 $r^{29} -$
2157246710904829367097195187318479384875565969609496310367800179931386703 $r^{28} -$
208856380666770405060995179015197720702379169773004554687500000000 $r^{27} -$
23941595327872587256493987343365062085807713765396772351115918617324986847 $r^{26} -$
8122192581481884908602036807281324469398141215783906601562500000000 $r^{25} -$
28641097615802981958553267469679468109543713475889363519184564264830177497 $r^{24} -$
37413249292652776941198358521728935548015059930973113085937500000000 $r^{23} -$
1823892724320361051151326792463966345135150426030137599700252876097945396887 $r^{22} -$
100031213898250582558572453310727890833640265499654428671875000000000 $r^{21} -$
70499700652415621486993348713301146159860420872071990636425357270453464200547 $r^{20} -$
1759808392654408396683774641577620301702930596753179763671875000000000 $r^{19} -$
19373207286072881740428451172741520568818585588851393956298385872467385979712781 $r^{18} -$
237574133008345133576609576612978740729895630561679268095703125000000000 $r^{17} -$
183229707773937520910155652073606115259697013470625694328906663299547816303896081 $r^{16} -$
118780665041725667883047883064893703649478152808396340478515625000000000 $r^{15} -$
80690365377769302841410373659439059544672078289902985512877385073728956242240459 $r^{14} -$
296967666260431416970761970766223425912369538202099085119628906250000000 $r^{13} -$
265118588319449904088785046907749369413286006479697580445186959644220145865382569 $r^{12} -$
593935332520862833941523941532446851824739076404198170239257812500000000 $r^{11} -$
208357604885427163154003094265059665124071798476779690396417115314565417433 $r^{10} -$
304331109623322516929120505968381398028364839680590820312500000000 $r^9 -$
19426468826311685890111431991671272181216667262683895205966558507624928619658101 $r^8 -$
197978444173620944647174647177482283941579692134732723413085937500000000 $r^7 -$
617476897107861419038912615027599943737329385627295727458353708683921405584637 $r^6 -$
4695140968544370228786750525948196457112561868807890673828125000000000 $r^5 -$
1959429974773129382885912492383125905939334993697276431408852173926975516152184711 $r^4 -$
1187870665041725667883047883064893703649478152808396340478515625000000000 $r^3 -$
20012471149919090534576487714214848267424443570957198256689224909784074027410747 $r^2 -$
103293101307976145033308511570860322056476361113773594824218750000000000 $r^1 -$
1101215958042911677802089636529865845490314087034606537101778851663521743695293 $r^0 -$
51646550653988072516654255785430161028238180556886797412109375000000000 $r^{-1} -$
326615826789904548993876020347565973369126840229159564065219595833069862596943361 $r^{-2} -$
1484838331302157084853809853831117129561847691010495425598144531250000000 $r^{-3} -$
289509871858714149262817301609610759031626458944642575540032303015534612546931 $r^{-4} -$
136067659225856319345137214554970641884247211089163383789062500000000 $r^{-5} -$
1763090921596426518287022431299317526595637721076822566564048965898951483251201 $r^{-6} -$
91374666541671205221772914081914900280729088677568949267578125000000000 $r^{-7} -$
5410535208441819792331252631216415905145343333898279336690202899776497173666487 $r^{-8} -$
329964073622701574411957745295803806569299486891221205688476562500000000 $r^{-9} -$
1549649686748338734454987051806469178896730084238877170009170310032316093437520631 $r^{-10} -$
1187870665041725667883047883064893703649478152808396340478515625000000000 $r^{-11} -$
1135420222193602957320608891428099418884365703673049006806329037681831956344373 $r^{-12} -$
1170315926149483416633544712379205619359091776165907724609375000000000 $r^{-13} -$
11330668168315445571882115643884537226967185502728708803240987316670851550989 $r^{-14} -$
16837049298262613823801900512606394007873427064228662109375000000000 $r^{-15} -$
84578352318555854318068425149237494160595130472327482696493859485784343700267 $r^{-16} -$
194828713308467388532564848788731130662535370314645947265625000000000 $r^{-17} -$
5136997041921364612112546442178345709278066935905959887612494862149982516339931 $r^{-18} -$
197978444173620944647174647177482283941579692134732723413085937500000000 $r^{-19} -$
2178334152914974787743317152025992846465590849994367955140829072881017947763137 $r^{-20} -$
15229111090278534203628819013652483380121514779594824877929687500000000 $r^{-21} -$
3966112505277605479181531357920169806017945409553794150895806492299908416797 $r^{-22} -$
548416742863215913150068274729867822552852332783193139648437500000000 $r^{-23} -$
1099102683812100152919952408177060036078549127470602338328818688702513211800499 $r^{-24} -$
329964073622701574411957745295803806569299486891221205688476562500000000 $r^{-25} -$
5489104698106140057625570314166626863533545889798027408520383285191057226547333 $r^{-26} -$
395956888347241889294349294354964567883159384269465446826171875000000000 $r^{-27} -$
613110683251160291925645271049607211243871882884379663696704067168400911092199 $r^{-28} -$
118787066504172566788304788306489370364947815280839634047851562500000000 $r^{-29} -$
2689414305873757566525592643024364570485198203754824699833483392541589960819 $r^{-30} -$
15838275533889675571773971774198582715326375370778617873046875000000000 $r^{-31} -$
164922623283682397754680273310812198254876823713073900932151067207404659961 $r^{-32} -$
3393916185833501908237279665899696296141366150881132401367187500000000 $r^{-33} -$
20625289867463619046424933470290161899000367214515775332338073846727947 $r^{-34} -$
1740469838889753375575793158459981005853159748108371289062500000000 $r^{-35} -$
352291059682303397502433427535140117512763041819685007703823551579 $r^{-36} -$
147123401427639504442737051212679467505098560846224609375000000 $r^{-37} -$
29962903292647976025325503049075929992044225870272449515704223 $r^{-38} -$
77843069538433600234252408049036755293702942246679687500000 $r^{-39} -$
239277682879825336161377672215173406914951652136521829783 $r^{-40} -$
5184353615613293388894599270665118567679183632812500000 $r^{-41} -$
34572296001809112450941603979281195831409275758813 $r^{-42} -$
9419794783877866062152231362770216596312255859375 $r^{-43} -$
273987326268735002858812178111979787 $r^{-44} -$
1886943135237232727224847699296875000 $r^{-45} -$

$$\Delta''(r) = r^{57} +$$

66452121818275070927610813524449 $r^{56} +$
2453784351049966935520268248500
317976025023101686344677467377844833771186601 $r^{55} +$
8694346127878979529622754612734665450000
162894906945201046374046948661769225591462519982279 $r^{54} +$
49607330701839093502168550993880180658065000000
48378563499458279070496997046791674658021454156922194833 $r^{53} +$
2193933807619535749226906336255344869783582690000000
231656564107404127657533499876345428450010894244108883079531 $r^{52} +$
19608283405599600758715475380282144773690770291875000000
7168431160720568553056298409926485056517907420131597702535267113 $r^{51} +$
13625974395673031654511008527897883150008386191917500000000
285926016781552491757251758712965627095091610902054659627165571887 $r^{50} +$
14274830319276509352344866076845401395246880772485000000000
1180522279537344870098094733640434340820666060934047270481119207967299 $r^{49} +$
1773148048108931609201517539735351534310591295154224275000000000
2940817097346045092988068120528287372278483923112119641403310046791028651 $r^{48} +$
149800438547133877329093723184538319277963747349236188750000000000
18829392186780260998012126575519180137135872634107599504106048177001370983 $r^{47} +$
362017726488906870211976497695967604921745722760654122812500000000
47561407068113171420696418508451533053175046368911395678068968300303173325997 $r^{46} +$
38011861281335221372257532258076598516783300889868682895312500000000
524037357309638139993594284647384667702228606494496816682135156818346707335427 $r^{45} +$
190059306406676106861287661290382992583916504449343414476562500000000
8459313447387326074946974058059972437796587091359021440657895512613663035609 $r^{44} +$
15084071937037786258832354070665316871739405115027255117187500000000
287775124630634889566487180871029887926987759533231616327271370672552413629167 $r^{43} +$
2715132948666801526589823732719757036913092920704905921093750000000000
2578756730434961337682727762586159353112520159903506447727051475200597178416409 $r^{42} +$
1377241350773015267110780154278137627419684814850314597656250000000000
492369024007770206158782745957266597900056372641367109988420049606172193809987419 $r^{41} +$
15838275533889675571773971774198582715326375370778617873046875000000000
827254739411915971204146165247084942401593952605011862683522913598100754467256291 $r^{40} +$
169695809291675095411863983294984814807068307544056620068359375000000000
8616210554080518945933099499459364853363012529112111429131028867463619051637282857 $r^{39} +$
1187870665041725667883047883064893703649478152808396340478515625000000000
24431515620756683731121214566700356307778212944866802042856090003534524137775807833 $r^{38} +$
237574133008345133576609576612978740729895630561679268095703125000000000
5521962711456028760426891066551076472363379779569429960353154893961457922388134417 $r^{37} +$
39595888347241889294349294354964567883159384269465446826171875000000000
21561564951167919750832487402775766820265336541427813068300970403451271092619370889 $r^{36} +$
118787066504172566788304788306489370364947815280839634047851562500000000
180094645405691272853636043341981269754390520984184869175609769139777152496452261 $r^{35} +$
791913776694483778588698588709929135766318768538930893652343750000000000
6531767766325335711894511940209678389193319191213490439059004031459876355124802259 $r^{34} +$
237574133008345133576609576612978740729895630561679268095703125000000000
6361723402465704601391008974020835271870013536882077258218330638936049738047840343 $r^{33} +$
197978444173620944647174647177482283941579692134732723413085937500000000
3322181371831713186062696934390631441994401878066104511021752322523938976696259767 $r^{32} +$
9137466654167120522177291408191490028072908867756894926757812500000000
4117461692411435385433658921230255753386348979644534206373486498604632172945064269 $r^{31} +$
10329310130797614503330851157086032205647636111377359482421875000000000
10066712375173857530179267712688226692532520092976644231277321324831547378171258549 $r^{30} +$
237574133008345133576609576612978740729895630561679268095703125000000000
58239281417937548181056119188246707626409658757859402832597412456373577529537229 $r^{29} +$
1333188176253339694593768667861833561896159542994833154296875000000000
58239281417937548181056119188246707626409658757859402832597412456373577529537229 $r^{28} +$
133318817625333969459376866786183356189615954299483315429687500000000
10066712375173857530179267712688226692532520092976644231277321324831547378171258549 $r^{27} +$
237574133008345133576609576612978740729895630561679268095703125000000000
4117461692411435385433658921230255753386348979644534206373486498604632172945064269 $r^{26} +$
10329310130797614503330851157086032205647636111377359482421875000000000
3322181371831713186062696934390631441994401878066104511021752322523938976696259767 $r^{25} +$
9137466654167120522177291408191490028072908867756894926757812500000000
6361723402465704601391008974020835271870013536882077258218330638936049738047840343 $r^{24} +$
197978444173620944647174647177482283941579692134732723413085937500000000
6531767766325335711894511940209678389193319191213490439059004031459876355124802259 $r^{23} +$
23757413300834513357660957661297874072989563056167926809570312500000000
1800946454056912728536360433419812697543905209841848691756097691339777152496452261 $r^{22} +$
79191377669448377858869858870992913576631876853893089365234375000000000
21561564951167919750832487402775766820265336541427813068300970403451271092619370889 $r^{21} +$
118787066504172566788304788306489370364947815280839634047851562500000000
5521962711456028760426891066551076472363379779569429960353154893961457922388134417 $r^{20} +$
39595888347241889294349294354964567883159384269465446826171875000000000
24431515620756683731121214566700356307778212944866802042856090003534524137775807833 $r^{19} +$
23757413300834513357660957661297874072989563056167926809570312500000000
8616210554080518945933099499459364853363012529112111429131028867463619051637282857 $r^{18} +$
118787066504172566788304788306489370364947815280839634047851562500000000
827254739411915971204146165247084942401593952605011862683522913598100754467256291 $r^{17} +$
16969580929167509541186398329498481480706830754405662006835937500000000
492369024007770206158782745957266597900056372641367109988420049606172193809987419 $r^{16} +$
15838275533889675571773971774198582715326375370778617873046875000000000
2578756730434961337682727762586159353112520159903506447727051475200597178416409 $r^{15} +$
137724135077301526711078015427813762741968481485031459765625000000000
287775124630634889566487180871029887926987759533231616327271370672552413629167 $r^{14} +$
271513294866680152658982373271975703691309292070490592109375000000000
8459313447387326074946974058059972437796587091359021440657895512613663035609 $r^{13} +$
15084071937037786258832354070665316871739405115027255117187500000000
524037357309638139993594284647384667702228606494496816682135156818346707335427 $r^{12} +$
190059306406676106861287661290382992583916504449343414476562500000000
47561407068113171420696418508451533053175046368911395678068968300303173325997 $r^{11} +$
38011861281335221372257532258076598516783300889868682895312500000000
18829392186780260998012126575519180137135872634107599504106048177001370983 $r^{10} +$
362017726488906870211976497695967604921745722760654122812500000000

$$\begin{aligned}
& \frac{2940817097346045092988068120528287372278483923112119641403310046791028651}{14980043854713387732909372318453831927796374734923618875000000000} r^9 + \\
& \frac{11805222279537344870098094733640434340820666060934047270481119207967299}{1773148048108931609201517539735351534310591295154224275000000000} r^8 + \\
& \frac{285926016781552491757251758712965627095091610902054659627165571887}{142748303192765093523448660768454013952468807724850000000000} r^7 + \\
& \frac{7168431160720568553056298409926485056517907420131597702535267113}{136259743956730316545110085278978831500083861919175000000000} r^6 + \\
& \frac{231656564107404127657533499876345428450010894244108883079531}{19608283405599600758715475380282144773690770291875000000} r^5 + \\
& \frac{48378563499458279070496997046791674658021454156922194833}{2193933807619535749226906336255344869783582690000000} r^4 + \\
& \frac{162894906945201046374046948661769225591462519982279}{49607330701839093502168550993880180658065000000} r^3 + \\
& \frac{3179760250231016863446774673778448377118601}{8694346127878979529622754612734665450000} r^2 + \\
& \frac{66452121818275070927610813524449}{2453784351049966935520268248500} r + 1
\end{aligned}$$

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