**S4 Text. Coupled oscillator model for morning and evening oscillators**

**M oscillator**

$$\frac{dM\_{p1m}}{dt}=v\_{s1m}\frac{B\_{N}^{mm}}{k\_{a1m}^{mm}+B\_{Nm}^{mm}}-v\_{1m}\frac{M\_{p1m}}{k\_{e1m}+M\_{p1m}}-k\_{d1m}M\_{p1m}+L+v\_{cm1}\frac{ AVP}{k\_{cm}+AVP} (1)$$

$$\frac{dP\_{1cm}}{dt}=k\_{1m}M\_{p1m}-v\_{2m}\frac{P\_{1cm}}{k\_{e2m}+P\_{1cm}}-k\_{d2m}P\_{1cm} (2)$$

$$\frac{dP\_{1nm}}{dt}=k\_{2m}P\_{1cm}-v\_{3m}\frac{P\_{1nm}}{k\_{e3m}+P\_{1nm}}+k\_{p1m}-k\_{d3}P\_{1n} (3)$$

$$\frac{dM\_{p2m}}{dt}=v\_{s2m}\frac{B\_{Nm}^{mm}}{k\_{a2m}^{mm}+B\_{Nm}^{mm}}-v\_{4m}\frac{M\_{p2m}}{k\_{e4m}+M\_{p2m}}-k\_{d4m}M\_{p2m}+L +v\_{cm2}\frac{ AVP}{k\_{cm}+AVP} (4)$$

$$\frac{dP\_{2cm}}{dt}=k\_{3m}M\_{p2m}-v\_{5m}\frac{P\_{2cm}}{k\_{e5m}+P\_{2cm}}-k\_{d5m}P\_{2cm} (5)$$

$$\frac{dP\_{2nm}}{dt}=k\_{4m}P\_{2cm}-v\_{6m}\frac{P\_{2nm}}{k\_{e6m}+P\_{2nm}}+k\_{p3m}PB\_{2m}-k\_{p4m}P\_{2nm}B\_{Nm}-k\_{d6m}P\_{2nm} (6)$$

$$\frac{dM\_{Bm}}{dt}=v\_{s3m}\frac{k\_{I1m}^{2}}{k\_{I1m}^{2}+R\_{m}^{2}+(\frac{R\_{m}}{k\_{xm}})}+v\_{s4m}\frac{P\_{2nm}^{wm}}{k\_{a3m}^{wm}+P\_{2nm}^{wm}}-v\_{7m}\frac{M\_{Bm}}{k\_{e7m}+M\_{Bm}}-k\_{d7m}M\_{Bm} (7)$$

$$\frac{dB\_{cm}}{dt}=k\_{5m}M\_{Bm}-v\_{8m}\frac{B\_{cm}}{k\_{e8m}+B\_{cm}}-k\_{d8m}B\_{cm} (8)$$

$$\frac{dB\_{Nm}}{dt}=k\_{6m}B\_{cm}-v\_{9m}\frac{B\_{Nm}}{k\_{e9m}+B\_{Nm}}+k\_{p1m}PB\_{1m}-k\_{p2m}P\_{1nm}B\_{Nm}+k\_{p3m}PB\_{2m}-k\_{p4m}P\_{2nm}B\_{Nm}-k\_{d9m}B\_{Nm} (9)$$

$$\frac{dM\_{Rm}}{dt}=v\_{s5m}\frac{B\_{Nm}^{sm}}{k\_{a4m}^{sm}+B\_{Nm}^{sm}}-v\_{10m}\frac{M\_{Rm}}{k\_{e10m}+M\_{Rm}}-k\_{d10m}M\_{Rm} (10) $$

$$\frac{dR\_{m}}{dt}=k\_{7m}M\_{Rm}-v\_{11m}\frac{R\_{m}}{k\_{e11m}+R\_{m}}-k\_{d11m}R\_{m} (11)$$

$$\frac{dPB\_{1m}}{dt}=-k\_{p1m}PB\_{1m}+k\_{p2m}P\_{1nm}B\_{Nm}-k\_{d12m}PB\_{1m} (12)$$

$$\frac{dPB\_{2m}}{dt}=-k\_{p3m}PB\_{2m}+k\_{p4m}P\_{2nm}B\_{Nm}-k\_{d13m}PB\_{2m} (13)$$

**E oscillator**

$$\frac{dM\_{p1e}}{dt}=v\_{s1e}\frac{B\_{Ne}^{me}}{k\_{a1e}^{me}+B\_{Ne}^{me}}-v\_{1}\frac{M\_{p1e}}{k\_{e1e}+M\_{p1e}}-k\_{d1e}M\_{p1e}+\frac{β+VIP \left(v\_{ce1}+L\right)}{k\_{ce}+VIP} (14)$$

$$\frac{dP\_{1ce}}{dt}=k\_{1e}M\_{p1e}-v\_{2e}\frac{P\_{1ce}}{k\_{e2e}+P\_{1ce}}-k\_{d2e}P\_{1ce} (15)$$

$$\frac{dP\_{1ne}}{dt}=k\_{2e}P\_{1ce}-v\_{3e}\frac{P\_{1ne}}{k\_{e3e}+P\_{1ne}}+k\_{p1e}PB\_{1e}-k\_{p2e}P\_{1ne}B\_{Ne}-k\_{d3e}P\_{1ne} (16)$$

$$\frac{dM\_{p2e}}{dt}=v\_{s2e}\frac{B\_{Ne}^{me}}{k\_{a2e}^{me}+B\_{Ne}^{me}}-v\_{4e}\frac{M\_{p2e}}{k\_{e4e}+M\_{p2e}}-k\_{d4e}M\_{p2e}+\frac{ β+VIP \left(v\_{ce2}+L\right)}{k\_{ce}+VIP} (17)$$

$$\frac{dP\_{2ce}}{dt}=k\_{3e}M\_{p2e}-v\_{5e}\frac{P\_{2ce}}{k\_{e5e}+P\_{2ce}}-k\_{d5e}P\_{2ce} (18)$$

$$\frac{dP\_{2ne}}{dt}=k\_{4e}P\_{2ce}-v\_{6e}\frac{P\_{2ne}}{k\_{e6e}+P\_{2ne}}+k\_{p3e}PB\_{2e}-k\_{p4e}P\_{2ne}B\_{Ne}-k\_{d6e}P\_{2ne} (19)$$

$$\frac{dM\_{Be}}{dt}=v\_{s3e}\frac{k\_{I1e}^{2}}{k\_{I1e}^{2}+R\_{e}^{2}+(\frac{R\_{e}}{k\_{xe}})}+v\_{s4e}\frac{P\_{2ne}^{we}}{k\_{a3e}^{we}+P\_{2ne}^{we}}-v\_{7e}\frac{M\_{Be}}{k\_{e7e}+M\_{Be}}-k\_{d7e}M\_{Be} (20)$$

$$\frac{dB\_{ce}}{dt}=k\_{5e}M\_{Be}-v\_{8e}\frac{B\_{ce}}{k\_{e8e}+B\_{ce}}-k\_{d8e}B\_{ce} (21)$$

$$\frac{dB\_{Ne}}{dt}=k\_{6e}B\_{ce}-v\_{9e}\frac{B\_{Ne}}{k\_{e9e}+B\_{Ne}}+k\_{p1e}PB\_{1e}-k\_{p2e}P\_{1ne}B\_{Ne}+k\_{p3e}PB\_{2e}-k\_{p4e}P\_{2ne}B\_{Ne}-k\_{d9e}B\_{Ne} (22)$$

$$\frac{dM\_{Re}}{dt}=v\_{s5e}\frac{B\_{Ne}^{se}}{k\_{a4e}^{se}+B\_{Ne}^{se}}-v\_{10e}\frac{M\_{Re}}{k\_{e10e}+M\_{Re}}-k\_{d10e}M\_{Re} (23) $$

$$\frac{dR\_{e}}{dt}=k\_{7e}M\_{Re}-v\_{11e}\frac{R\_{e}}{k\_{e11e}+R\_{e}}-k\_{d11e}R\_{m} (24)$$

$$\frac{dPB\_{1m}}{dt}=-k\_{p1m}PB\_{1m}+k\_{p2m}P\_{1nm}B\_{Nm}-k\_{d12m}PB\_{1m} (25)$$

$$\frac{dPB\_{2m}}{dt}=-k\_{p3m}PB\_{2m}+k\_{p4m}P\_{2nm}B\_{Nm}-k\_{d13m}PB\_{2m} (26)$$

**AVP and VIP**

$$\frac{dVIP}{dt}=\left\{\begin{array}{c}0, under DD\\k\_{vs1}\left(P\_{1nm}+P\_{2nm}\right)-v\_{v1}\frac{VIP}{k\_{ev1}+VIP}-k\_{dv1} VIP , under LL and LD \end{array}\right. (27)$$

$$\frac{dAVP}{dt}=k\_{vs2}\left(P\_{1ne}+P\_{2ne}\right)-v\_{v2}\frac{AVP}{k\_{ev2}+AVP}-k\_{dv2} AVP (28)$$