Supplementary: Quantifying systemic risk by mean-variance portfolio ground-states

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In the article a uniform distribution of investors along the efficient frontier was assumed, in order to capture an average over the a general market. Yet, it is possible that $\rho(\lambda)$ takes other forms. We study the case of ρ being a δ-function around a certain value in Fig. (1). This would be the case when every investor is only interested in one risk-level λ. We tested for $\lambda = 0\%, 1\%, 10, \%, 50\%, 100\%$. In general even one certain risk-level exhibits the distinctive peak when reweighing the portfolio takes place. It is also possible, that most of the investors are packed around a certain λ. In order to mimic that, we show in Fig. (2) the results of choosing ρ as a normal distribution centered around $\mu_\lambda = 0\%, 10\%, 25\%, 50\%, 100\%$ and a standard deviation of $\sigma_\lambda = 1/8$. Choosing a broader distribution leads to values similar as in the uniform case.

As our parameters we set ω to be 12 months, with τ and ζ equal to 10^{-4} and restricting the investors to $N_d = 5$ US indcies.

As a last step, we compare in Fig. (3) our early warning indicator to the indicator proposed by Zheng on our dataset. The sum of normalised eigenvalues α_i is

$$\Lambda_k = \sum_{i=0}^{k} \alpha_i \tag{1}$$

where the normed eigenvalues α_i are sorted in decreasing order. In order to capture between 85% and 95% of the market, we set $k = 5$ and plot the changes in Λ_5 normalised by the absolute maximum. The same was done with M. Comparing both indicators reveals, that the eigenvalues and the average investing strategy signal November 2007 as a critical date.

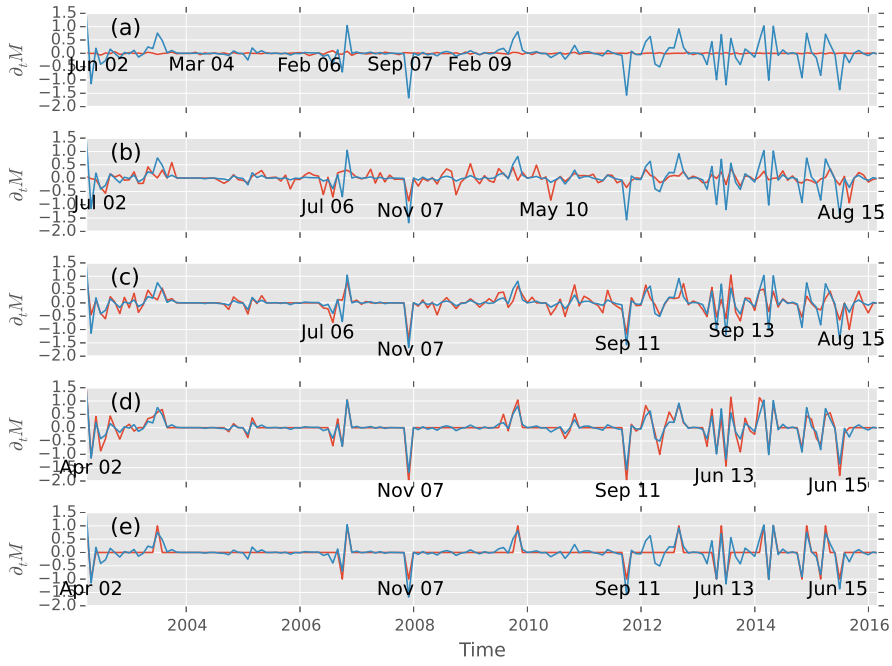

Figure 1: Time derivative of the investing strategy with $\rho(\lambda) = \delta(\lambda - k)$ for different risk-levels: (a) $\lambda = 0\%$ (b) $\lambda = 1\%$ (c) $\lambda = 10\%$ (d) $\lambda = 50\%$ (e) $\lambda = 100\%$. In all risk-levels the distinct peak in November 2007 is present as well as the higher volatility after 2009.

Figure 2: The time derivative of M with a normal distribution centered around different λ's with a scaling factor of $\sigma_\lambda = 1/8$. In (a) only very risk aware investors are considered by setting $\mu_\lambda = 0\%$; form (b) with $\mu_\lambda = 10\%$; (c) $\mu_\lambda = 25\%$; (d) $\mu_\lambda = 50\%$ to (e) with $\mu_\lambda = 100\%$ one can read the points in time where reshuffling is about to take place. Increasing σ_λ leads to similar distributions as a uniform density function.

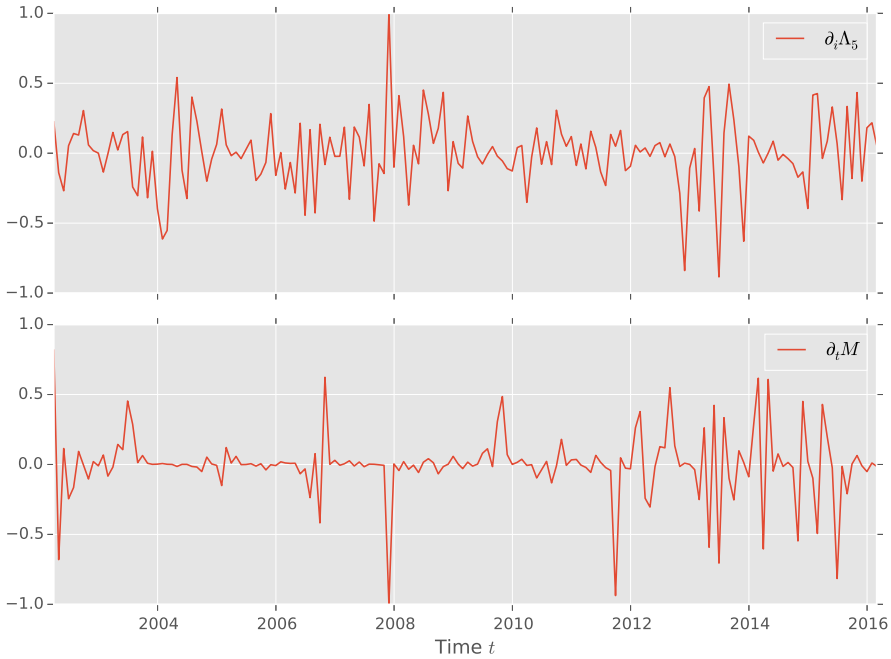

Figure 3: Comparison between the changes in the 5 largest eigenvalues (upper) with the average investing strategy (lower). The strongest increase in coupling happened in November 2007. This coincides with the steepest decrease of M.In order to explain between 85% and 95% of the data, the five greatest eigenvalues were chosen to calculate the changes in the cross-correlation matrix.