# **Appendix. Mathematical formalism of the wavelet leader based scaling analysis**

## **Wavelet leaders**

Assuming a discrete process *X(t)* and a mother wavelet function *ψ0(t)* with a compact time support, the coefficients of the discrete wavelet transform (DWT) of *X*, *dX(j,k)* [54] are given by:

 (1)

Let *λ = λ j,k =*[*k2j,* (*k+1*)*2j*)be a dyadic intervals of indices and  the union of the interval *λ* with its adjacent dyadic intervals. The wavelet leaders [55] are the quantities *LX(j,k)* defined from the discrete wavelet coefficients as:

 (2)

*LX(j,k)* is the largest discrete wavelet coefficient computed over all finer scales 2*j’* ≤ 2*j* in the time neighborhood 3*λ* (S1 Fig.).

## **Log-cumulants**

For a given analysis scale, the time averages (across the *nj* samples of *X(t)* at the scale 2*j*) of the *q*th power of the wavelet leaders *LX(j,k)* are referred to as the structure functions:

 (3)

Under mild uniform Hölder regularity condition on *X*(*t*), it has been demonstrated [55] that the structure functions are related to the scaling exponents  by a power law in the limit 2*j* → 0:

 (4)

The structure functions *SL*(*j*,*q*) could be expressed as the sample mean estimators for the ensemble averages . Using the second characteristic functions of the distributions of random variables ln *LX*(*j*,∙), eq. (4) could be rewritten as:

 (5)

where  are the cumulants of order p ≥ 1 of ln *LX*(*j*,∙).

Eq. (5) implies that:

 (6)

Replacing  in eq. (5) by their expression in eq. (6) yields:

 (7)

 is denoted the log-cumulant (or simply the cumulant) of order *p*.