

Supporting Information: Table S3**List of reactions and their catalysts in ACS(36,28) (referred in Fig. 14 of main text)**

The table lists all the reactions with their respective catalysts in the example of a catalyzed chemistry, quoted in the main text, containing a cascade of nested ACSs for $f = 2$ generated using Algorithm 4. The steady state concentrations for this chemistry are displayed in Fig. 14. This chemistry was generated with $g = 7$ and $n_k = 3$.

The molecules in various generations are as follows: $P_0 = \{(1, 0), (0, 1)\}$, $P_1 = \{(1, 1), (0, 2), (2, 0)\}$, $P_2 = \{(1, 3), (2, 2), (3, 0)\}$, $P_3 = \{(2, 6), (2, 3), (5, 2)\}$, $P_4 = \{(4, 6), (7, 4), (7, 8)\}$, $P_5 = \{(8, 12), (6, 12), (14, 8)\}$, $P_6 = \{(14, 24), (22, 20), (11, 12)\}$, $P_7 = \{(29, 28), (36, 28), (24, 26)\}$.

The catalyst for a reaction listed under generation P_k is added at step k of algorithm. It is apparent from the reaction table that the ACSs are maximally overlapping, *i.e.*, any ACS of generation k contains all the reactions of generation $k - 1$.

Reaction	Catalyst added in generation						
	P_1	P_2	P_3	P_4	P_5	P_6	P_7
$(0, 1) + (0, 1) \rightleftharpoons (0, 2)$	(1, 1)	(1, 3)	(2, 3)	(7, 4)	(8, 12)	(11, 12)	(29, 28)
$(0, 1) + (1, 0) \rightleftharpoons (1, 1)$	(0, 2)	(1, 3)	(2, 6)	(4, 6)	(14, 8)	(22, 20)	(36, 28)
$(1, 0) + (1, 0) \rightleftharpoons (2, 0)$	(1, 1)	(3, 0)	(5, 2)	(7, 4)	(14, 8)	(14, 24)	(29, 28)
$(1, 0) + (2, 0) \rightleftharpoons (3, 0)$		(2, 2)	(5, 2)	(4, 6)	(14, 8)	(14, 24)	(24, 26)
$(1, 1) + (0, 2) \rightleftharpoons (1, 3)$		(1, 3)	(2, 6)	(7, 8)	(8, 12)	(14, 24)	(24, 26)
$(0, 2) + (2, 0) \rightleftharpoons (2, 2)$		(1, 3)	(2, 6)	(7, 8)	(6, 12)	(22, 20)	(29, 28)
$(0, 1) + (2, 2) \rightleftharpoons (2, 3)$			(2, 6)	(7, 4)	(14, 8)	(22, 20)	(24, 26)
$(3, 0) + (2, 2) \rightleftharpoons (5, 2)$			(2, 3)	(7, 8)	(6, 12)	(11, 12)	(29, 28)
$(1, 3) + (1, 3) \rightleftharpoons (2, 6)$			(5, 2)	(7, 8)	(14, 8)	(22, 20)	(24, 26)
$(2, 0) + (2, 6) \rightleftharpoons (4, 6)$				(4, 6)	(14, 8)	(22, 20)	(36, 28)
$(2, 2) + (5, 2) \rightleftharpoons (7, 4)$				(7, 4)	(6, 12)	(14, 24)	(36, 28)
$(5, 2) + (2, 6) \rightleftharpoons (7, 8)$				(7, 4)	(14, 8)	(14, 24)	(36, 28)
$(2, 6) + (4, 6) \rightleftharpoons (6, 12)$					(14, 8)	(11, 12)	(29, 28)
$(4, 6) + (4, 6) \rightleftharpoons (8, 12)$					(14, 8)	(22, 20)	(36, 28)
$(7, 4) + (7, 4) \rightleftharpoons (14, 8)$					(8, 12)	(11, 12)	(36, 28)
$(3, 0) + (8, 12) \rightleftharpoons (11, 12)$						(22, 20)	(36, 28)

continued on next page ...

...continued from previous page

Reaction	Catalyst in generation						
	P_1	P_2	P_3	P_4	P_5	P_6	P_7
$(8, 12) + (6, 12) \rightleftharpoons (14, 24)$						$(22, 20)$	$(24, 26)$
$(14, 8) + (8, 12) \rightleftharpoons (22, 20)$						$(11, 12)$	$(29, 28)$
$(2, 6) + (22, 20) \rightleftharpoons (24, 26)$							$(29, 28)$
$(7, 8) + (22, 20) \rightleftharpoons (29, 28)$							$(29, 28)$
$(14, 8) + (22, 20) \rightleftharpoons (36, 28)$							$(29, 28)$