**S2 Appendix: Additional explanation of population modelling procedures.**

For our analyses, population size *n* was defined as the number of females in a population and was the sum of the numbers in each age class

*n = n1 + n2 + n3 …. ni*(1)

where n*i* is the number of females in age class *i*. A population with three age classes can be described by a set of difference equations, where *Pi* are age-specific productivities i.e. the number of female young produced at each age class *i* and *Si* are age-specific survival probabilities of age class *i*, such that

*n*1(*t*+1) = *P*2*n*2(*t*) + *P*3*n*3(*t*)

*n*2(*t*+1) = *S*1*n*1(*t*) (2)

*n*3(*t*+1) = *S*2*n*2(*t*) + *S*3*n*3(*t*)

So a generic age structured bat population can be represented as a life-cycle graph thus



The difference equations describing the bat population can be written more simply in matrix form

**n**(*t*+1) = **An**(*t*), (3)

where **n** is an age distribution vector

**n**(*t*) =, (4)

and **A** is a population projection matrix as described by [1,2]. The elements of the matrix can be obtained from the difference equations or from the life-cycle graph, so that

= **** (5)

This is known as an extended Leslie matrix [1,2]; it assumes the vital rates in the matrix are equal after age class two.

By repeatedly multiplying the matrix **A** by the vector **n**, a series of vectors are obtained that stabilise out after a few generations, so they differ from each other by a scalar factor. This factor λ, is the dominant eigenvector of the population projection matrix **A**, and is equal to the population growth rate, so that

**n**(*t*+1) = λ**n**(*t*), (6)

where

λ = *er* , (7)

*r* being the per capita rate or intrinsic rate of population growth. Successively multiplying **n** by **A** also results in the vector **w** that is proportional to the stable age structure of the population.

**w** =, (8)

The stable age structure **w** and λ can be used to determine the population size **n** at any time *t*, thus

**n**(*t*) = λ*t***w**. (9)

Substituting this into (3) gives

λt+1**w** = λ*t***Aw**. (10)

A scalar λ and vector **w** that satisfy this equation are the dominant eigenvalue and right eigenvector of the matrix **A**, respectively. These must satisfy

(**A** - λ**I**) **w** = 0, (11)

where **I** is the identity matrix. **w** has a nonzero solution only if the determinant of the matrix.

**A** - λ**I** equals zero, hence the value of λ and **w** can be solved using the characteristic equation

det (**A** - λ**I**) = 0, (12)

The characteristic equation can be solved using polynomial expansion, which for bats here with a matrix of three by three, produces seven eigenvalues and their associated eigenvectors, where λ is the dominant eigenvalue. This also produces a solution for the left eigenvector **v** of the matrix **A** which is known as the reproductive stage vector and represents the contribution of each age class to the population, so that

**v**\***A** = λ**v**\*, (13)

where **v**\* is the transpose of **v**.The left and right eigenvectors can be scaled so that their scalar product equals one, so for the eigenvalue λ

〈**w,v**〉 = **v**\***w** = 1, (14)

where 〈**w,v**〉 is the scalar product of **w** and **v**. This allows calculation of the effect that a small additive change in any of the vital rates of the population projection matrix **A** would have on λ, so that

 (15)

where sij is the sensitivity of λ to changes in the matrix element a*ij*. The sensitivity of λ to changes in other parameters can then be calculated for any other vital rate (*x*).

 (16)

Often more important in conservation and management is to determine the proportional effect that a small change in each vital rate would have on λ. This can be examined using elasticity analysis, where

 (17)

so the elasticity to other parameters can be calculated by

 (18)

 (19)

**Estimating vital rates for Natterer’s bat**

Productivity is assumed to be 1 for *M. nattereri*. Twins have been recorded but the level of twinning in *M. nattereri* in the British Isles is believed to be negligible [3]. We assume an equal sex ratio of females at birth, and that female bats can start breeding at one year of age, such that

 (20)

where *Pi* is the productivity of age class *i* (where *i* = {1, 2}), *Li* is the mean litter size and *alphai* is the proportion of age class *i* breeding*.* The productivity of bats in their first year (i.e. infants) will be 0. In relation to the proportion of individuals breeding in any given year, our data recorded from *M. nattereri* maternity roosts during this study show that 170 of 233 adult females (73%) and 5 of 18 first year females (28%) showed signs of pregnancy, lactation or post-lactation. We used these as an estimate of the likely number of individuals of each age-class breeding, with the caveat that these figures may underestimate the true proportion of each age-class breeding because it is difficult on occasions to confirm in the hand whether a bat has bred. Productivity was expressed as

 (21)

We use estimates of first year, second year and adult survival in [4]. An estimate of female adult survival for Natterer's bat produced in [5] is comparable with that recorded in [4].

**References**

1. Leslie PH (1945) On the use of matrices in certain population mathematics. Biometrika 33: 183-212. doi: 10.2307/2332297

2. Leslie PH (1948) Some further notes on the use of matrices in population mathematics. Biometrika 35: 213-245. doi: 10.1093/biomet/35.3-4.213

3. Smith PG, Rivers NM. Natterer’s bat *Myotis nattereri*. In: Harris S, Yalden DW, editors. Mammals of the British Isles: Handbook.4th edition. Southampton: The Mammal Society; 2008. pp. 323-328.

4. Mortimer G. Foraging, roosting and survival of Natterer’s bats *Myotis nattereri* in a commercial coniferous plantation. Ph.D. Thesis, University of St. Andrews. 2006.

5. Rivers NM, Butlin RK, Altringham JD (2006) Autumn swarming behaviour of Natterer’s bats in the UK: population size, catchment area and dispersal. Biol Conserv 127: 215-226. doi: doi:10.1016/j.biocon.2005.08.010