**S2 Appendix: Additional Explanation of Population Modelling Procedures**

For our analyses, population size *n* was defined as the number of females in a population and was the sum of the numbers in each age class

*n = n1 + n2 + n3 …. ni*(1)

where n*i* is the number of females in age class *i*. A population with three age classes can be described by a set of difference equations, where *Pi* are age-specific productivities i.e. the number of female young produced at each age class *i* and *Si* are age-specific survival probabilities of age class *i*, such that

*n*1(*t*+1) = *P*2*n*2(*t*) + *P*3*n*3(*t*)

*n*2(*t*+1) = *S*1*n*1(*t*) (2)

*n*3(*t*+1) = *S*2*n*2(*t*) + *S*3*n*3(*t*)

So a generic age structured bat population can be represented as a life-cycle graph thus



The difference equations describing the bat population can be written more simply in matrix form

**n**(*t*+1) = **An**(*t*), (3)

where **n** is an age distribution vector

**n**(*t*) =, (4)

and **A** is a population projection matrix as described by [1,2]. The elements of the matrix can be obtained from the difference equations or from the life-cycle graph, so that

= **** (5)

This is known as an extended Leslie matrix; it assumes the vital rates in the matrix are equal after age class two.

By repeatedly multiplying the matrix **A** by the vector **n** a series of vectors are obtained that stabilise out after a few generations, so they differ from each other by a scalar factor. This factor λ, is the dominant eigenvector of the population projection matrix **A**, and is equal to the population growth rate, so that

**n**(*t*+1) = λ**n**(*t*), (6)

where

λ = *er* , (7)

*r* being the per capita rate or intrinsic rate of population growth. Successively multiplying **n** by **A** also results in the vector **w** that is proportional to the stable age structure of the population.

**w** =, (8)

The stable age structure **w** and λ can be used to determine the population size **n** at any time *t*, thus

**n**(*t*) = λ*t***w**. (9)

Substituting this into (3) gives

λt+1**w** = λ*t***Aw**. (10)

A scalar λ and vector **w** that satisfy this equation are the dominant eigenvalue and right eigenvector of the matrix **A**, respectively. These must satisfy

(**A** - λ**I**) **w** = 0, (11)

where **I** is the identity matrix. **w** has a nonzero solution only if the determinant of the matrix.

**A** - λ**I** equals zero, hence the value of λ and **w** can be solved using the characteristic equation

det (**A** - λ**I**) = 0, (12)

The characteristic equation can be solved using polynomial expansion, which for bats here with a matrix of three by three, produces seven eigenvalues and their associated eigenvectors, where λ is the dominant eigenvalue. This also produces a solution for the left eigenvector **v** of the matrix **A** which is known as the reproductive stage vector and represents the contribution of each age class to the population, so that

**v**\***A** = λ**v**\*, (13)

where **v**\* is the transpose of **v**.The left and right eigenvectors can be scaled so that their scalar product equals one, so for the eigenvalue λ

〈**w,v**〉 = **v**\***w** = 1, (14)

where 〈**w,v**〉 is the scalar product of **w** and **v**. This allows calculation of the effect that a small additive change in any of the vital rates of the population projection matrix **A** would have on λ, so that

 (15)

where sij is the sensitivity of λ to changes in the matrix element a*ij*. The sensitivity of λ to changes in other parameters can then be calculated for any other vital rate (*x*).

 (16)

Often more important in conservation and management is to determine the proportional effect that a small change in each vital rate would have on λ. This can be examined using elasticity analysis, where

 (17)

where the elasticity to other parameters can be calculated by

 (18)

 (19)

**Estimating vital rates for soprano pipistrelles**

Several studies provide some information on litter size for *P. pygmaeus* in the United Kingdom [3-6]. Collectively, they recorded three sets of twins from 78 females i.e. a 3.8% rate of twinning, which we used here. We assumed an equal sex ratio at birth and that female bats can start breeding at one year of age, such that

 (20)

where *Pi* is the productivity of age class *i* (where *i* = {1, 2}), *Li* is the mean litter size and *alphai* is the proportion of age class *i* breeding*.* The productivity of bats in their first year (i.e. infants) will be 0. In one study 110 of 118 (93%) female *P. pygmaeus* examined at several points overwinter had been inseminated [7]. While we did not know the proportion of second year versus older individuals breeding in a given year, we assumed this was 93% for both age-classes, and productivity was expressed as

 (21)

There were no robust survival estimates for *P. pygmaeus*; the data available [e.g. 8] are now believed to be underestimates of true survival due to methodological drawbacks of the approaches used at the time. Limitations of these data include unreliable assumptions about age-distribution and population stability, and a failure to account for variable recapture probabilities [9]. So we used survival estimates for the closely related *P. pipistrellus* [9]; standard errors of these data suggest that annual survival varied by about 5-10%. Looking at studies on other species of bats, first year survival varied by about 5% between year for Leisler’s bat *Nyctalus leisleri* in Switzerland, and adult survival by 12% [10]. Adult survival varied by about 10% between years for the long-fingered bat *Myotis capaccinii* in Greece [11], while adult survival varied by about 16% in the Isabelline serotine *Eptesicus isabellinus* in Spain [12]. Since the annual variation in survival was broadly similar across studies, we assumed a mean of 11%.

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