S3 Appendix: Continuous Approximation of Non-differential Function in ODEs

The optimization algorithms implemented in \textit{PSOPT} require the derivatives of the function \( f(x(t), u(t), \Theta_{G_b}) \) exists. We notice that there are discontinuities in Eqs. (S1)-(S9).

The smooth approximation of the Renal exertion function \( E(t) \) in Eq. (S6) by using a Heaviside function is,

\[
E(t) = k_{c1}(G_p(t) - k_{c2}) \times H(G_p(t), k_{c2}, k), \quad \text{(S12)}
\]

where,

\[
H(G_p(t), k_{c2}, k) = \frac{1}{1 + e^{-k(G_p - k_{c2})}}, \quad k \in \mathbb{Z}. \quad \text{(S13)}
\]

Here a larger \( k \) corresponds to a sharper transition around \( G_p(t) = k_{c2} \).

We define a continuous approximation of the Dirac delta function \( \delta(t - \tau_D) \) in Eq. (S3c),

\[
\delta(t - \tau_D) = \frac{d}{dt} H(t, \tau_D, k), \quad \text{(S14)}
\]

where \( H(t, \tau_D, k) = \frac{1}{1 + e^{-k(t - \tau_D)}}, \quad k \in \mathbb{Z}. \) Here a larger \( k \) corresponds to a sharper transition at \( t = \tau_D \).

We also define continuous approximation of the \( \max(.) \) function, e.g. in Eq. (S4d), as

\[
\max(H(t) - H_b, 0) = (H(t) - H_b) \times H(H(t), H_b, k), \quad \text{(S15)}
\]

where \( H(H(t), H_b, k) = \frac{1}{1 + e^{-k(H - H_b)}} \), \( k \in \mathbb{Z}. \) Here a larger \( k \) corresponds to a sharper transition at \( H(t) = H_b \). In all our approximation we set \( k = 4 \).