Supporting Material, File S2 Text

Homeostatic Controllers Compensating for Growth and Perturbations

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Steady state of transporter-generated compound $A$ without negative feedback

We start with Eq. 15 where transporter $T$ pumps external $A (A_{\text{ext}})$ into a constantly growing cell ($\dot{V}=$constant)

$$\dot{A} = \frac{k_2 \cdot T}{V} - A \left( \frac{\dot{V}}{V} \right)$$  \hspace{1cm} (S1)

We assume that the surface concentration of $T$ is constant and that the pump rate is zero-order with respect to the external $A$ concentration.

The steady state of $A$ is given by setting Eq. S1 to zero, which gives

$$\dot{A} = \frac{k_2 \cdot T}{V} - A \left( \frac{\dot{V}}{V} \right) = 0 \Rightarrow A_{ss} = \frac{k_2 \cdot T}{V}$$ \hspace{1cm} (S2)

independent of the initial concentration of $A$.

In case there is a first-order removal of cellular $A$ with respect to $A$ the rate equation becomes

$$\dot{A} = \frac{k_2 \cdot T}{V} - k_3 \cdot A - A \left( \frac{\dot{V}}{V} \right) = 0$$ \hspace{1cm} (S3)

Setting Eq. S3 to zero leads to

$$A_{ss} = \frac{k_2 \cdot T}{k_3 \cdot V + V} \rightarrow 0 \quad \text{as} \quad V \rightarrow \infty$$ \hspace{1cm} (S4)

In case the removal of cellular $A$ is zero-order with respect to $A$ (for example by an enzyme removing $A$ at maximum velocity $V_{\text{max}}$), then in this case the steady state condition

$$\dot{A} = \frac{k_2 \cdot T}{V} - V_{\text{max}} - A \left( \frac{\dot{V}}{V} \right) = 0$$ \hspace{1cm} (S5)
gives

\[ A_{ss} = \frac{1}{V} (k_2 T - V_{\text{max}} V) \]  \hspace{1cm} (S6)

As the volume \( V \) grows there will be a critical volume \( V_{\text{crit}} = k_2 T/V_{\text{max}} \) at which \( A_{ss} \) becomes zero.