S1 Appendix

For weighted networks, the generalized clustering coefficient $C_{wu}$ [1] and the global efficiency $E_{global}$ [2] are defined as follows:

\[ C_{wu} = \frac{1}{p} \sum_i D_{ii} (D_{ii} - 1) \sum_{j,k} (W_{ij}W_{ik}W_{jk})^{1/3}, \]  
\[ E_{global} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{l_{ij}}, \]

where $D_{ii} = \sum_j W_{ij}$ is the degree of vertex $i$; $l_{ij}$ is the shortest weighted path length between vertices $i$ and $j$; $N$ denotes the number of vertices in the network. Notice that the subscript ‘wu’ indicates a weighted undirected network.

There are multiple ways of specifying the modularity. One way to define the modularity of a partition of the network into $c$ communities is [3]

\[ Q = \sum_{i=1}^c (e_{ii} - a_i^2), \]

where $e_{ii}$ is the fraction of edges with both end vertices in the same community $i$ and $a_i$ is the fraction of ends of edges adjacent to vertices in community $i$. If we define $B_{ij}$ as the difference between $W_{ij}$ and the expected weight between vertices $i$ and $j$ if edge weights are distributed at random, then the modularity can also be viewed as the normalized summation of $B_{ij}$ over all pairs of $i, j$ that are in the same community.

References

