RESEARCH ARTICLE

Cyclic connectivity index of fuzzy incidence graphs with applications in the highway system of different cities to minimize road accidents and in a network of different computers

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Abstract

A parameter is a numerical factor whose values help us to identify a system. Connectivity parameters are essential in the analysis of connectivity of various kinds of networks. In graphs, the strength of a cycle is always one. But, in a fuzzy incidence graph (FIG), the strengths of cycles may vary even for a given pair of vertices. Cyclic reachability is an attribute that decides the overall connectedness of any network. In graph the cycle connectivity (CC) from vertex a to vertex b and from vertex b to vertex a is always one. In fuzzy graph (FG) the CC from vertex a to vertex b and from vertex b to vertex a is always same. But if someone is interested in finding CC from vertex a to an edge ab , then graphs and FGs cannot answer this question. Therefore, in this research article, we proposed the idea of CC for FIG. Because in FIG, we can find CC from vertex a to vertex b and also from vertex a to an edge ab . Also, we proposed the idea of CC of fuzzy incidence cycles (FICs) and complete fuzzy incidence graphs (CFIGs). The fuzzy incidence cyclic cut-vertex, fuzzy incidence cyclic bridge, and fuzzy incidence cyclic cut pair are established. A condition for CFIG to have fuzzy incidence cyclic cut-vertex is examined. Cyclic connectivity index and average cyclic connectivity index of FIG are also investigated. Three different types of vertices, such as cyclic connectivity increasing vertex, cyclically neutral vertex and, cyclic connectivity decreasing vertex, are also defined. The real-life applications of CC of FIG in a highway system of different cities to minimize road accidents and a computer network to find the best computers among all other computers are also provided.

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1 Introduction

Graphs are convenient tools to explain associations between different types of entities under examination. Vertices or nodes denote entities, and edges or arcs explain the vertices' connections. A mathematical structure to describe unpredictability and equivocity in daily life

circumstances was first addressed by Zadeh. He presented the perception of the fuzzy set (FS) [1]. He aimed to develop a mathematical theory to tackle unreliability and inexactness. The major difference between sets and FSs is that the sets classified the universal set (US) into two values: members and non-members. On the other hand, FS designates a sequence of membership values to elements of the US from $[0, 1]$ closed interval. Also, FSs are beneficial to learn about quality variables, including reasoning, prettiness, uniformity, etc. There is a deficiency in describing the entities and their association, and we need to draw a FG model. Rosenfeld [2] was the first who proposed the crucial idea of FGs after the revolutionary work of Zadeh's on FSs.

FGs can be expressed as the strength of relationships between objects. The study of FGs plays an essential role for many students as well as mathematicians to take part in this field, such as Bhutani and Rosenfeld [3] suggested the notion of geodesics in FGs. An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted; this idea was provided by Bhutani and Rosenfeld [4]. The notion of fuzzy end nodes in FGs was presented by Bhutani and Rosenfeld [5]. Gani and Ahamad [6] presented a way to compute different types of degrees, order, and size of FGs and compare the relationship among degree, order, and size of FGs. Bhutani et al. [7] explained degrees of end nodes and cut-nodes in FGs. Al-Hawary [8] described different kinds of operations on FGs. Moreover, he introduced and studied the notion of balanced FG and gave necessary and sufficient conditions for the preceding products of two fuzzy balanced graphs to be balanced. Al-Hawary and Horani [9] provided three new products on product FGs and gave sufficient conditions for each one of them to be strong. They also discussed balanced and co-balanced product FGs. Al-Hawary [10] investigated several classes of FGs and provided two new operations on FGs, such as parallel connection and series connection. He also showed that parallel connection and series connection of balanced FGs need not be balanced. Woznaik et al. [11] presented electronic modules, infrastructure, and fuzzy rules control model with implemented software for new generation home environment. Zielonka et al. [12] presented IoT convection installation for a small house with the developed remote platform control system. Connectivity analysis of cyclically balanced FGs was a fascinating idea provided by Jicy, and Mathew [13]. Binu et al. [14, 15] gave the idea to calculate the connectivity index and Wiener index of a FG and its applications to human trafficking and illegal immigration networks, respectively. The concept of CC of a FG, CC of fuzzy trees, fuzzy cycles, and complete FGs was examined by Mathew and Sunitha [16]. They also initiated the idea of cyclic cut nodes and cyclic bridges in FGs. Mathew and Sunitha [17] provided the concept of node-strength sequence, fuzzy node connectivity, and fuzzy arc connectivity. They also furnished a new clustering technique based on fuzzy arc connectivity. The criterion for connectivity of a FG and the structure of the complement of a fuzzy cycle was investigated by Narayan and Sunitha [18]. Bipolar FGs, various methods of their construction, the idea of isomorphism of bipolar FGs, and some of their key properties were proposed by Akram [19]. Akram and Alshehri [20] proposed an idea of intuitionistic fuzzy cycles and intuitionistic fuzzy trees. Depending on the strength of an arc, Mathew and Sunitha [21] classified arcs of a FG into three different types, namely α -strong, β -strong, and δ -arc. Mathew and Sunitha [22] initiated the idea of the strongest strong cycle and θ -FGs. Different kinds of operations, including Cartesian product, composition, union, and join in FGs, were given by Mordeson and Chang-Shyh [23]. Fuzzy tolerance, fuzzy tolerance graph, fuzzy bounded tolerance graph, fuzzy interval containment graph and regular representation of fuzzy tolerance graph, fuzzy unit tolerance graph, and proper tolerance graph were discussed by Samanta and Pal [24]. A fuzzy planar graph is a very important subclass of FG that was presented by Samanta and Pal [25]. Mordeson and Nair [26] brought the idea of fuzzy hypergraphs. Three kinds of operations, including direct product, semi-strong product, and

strong product for interval-valued FGs was provided by Rashmanlou, and Jun [27]. Sunitha and Vijayakumar [28] defined complement of a FG. Mordeson and Nair [29] introduced and examined the concepts of chords, twigs, 1-chains with boundary zero, cycle vectors, coboundary, and cocycles for FGs. They have also shown that although the set of cycle vectors, fuzzy cycle vectors, cocycles, and fuzzy cocycles do not necessarily form vector spaces over the field Z_2 of integers modulo 2, they nearly do. Later on, different mathematicians participated in the development of graphs and FGs. Their achievements can be seen in [30–35].

There is a flaw in FGs because they do not give any clue of the impact of a vertex on edge. This lack of FGs become the fundamental cause to establish the scheme of FIG. The proposal of FIGs was first initiated by Dinesh [36]. For example, in a highway system, if vertices represent various cities and edges serve as highways, introducing the degree of connection between city L and the highway LM joining cities L and M permits a profound analysis of the highway system. This connection could be the ramp system joining L and LM . We indicate this relationship by the ordered pair (L, LM) . Malik et al. [37] applied FIGs in different types of applications. Mathew and Mordeson [38] proposed the idea of cut pairs and fuzzy incidence trees in FIGs. They also discussed some vital properties of FIGs. Three different types of nodes, including fuzzy incidence connectivity enhancing node, fuzzy incidence connectivity reducing node, and fuzzy incidence connectivity neutral node in FIGs was introduced by Fang et al. [39]. Like node and edge connectivity in graphs, Mathew et al. [40] discussed these concepts for FIGs. Mordeson and Mathew [41] developed fuzzy end nodes and fuzzy incidence cut vertices in FIGs. Nazeer et al. [42] presented the idea of intuitionistic fuzzy incidence graphs (IFIGs) as a generalization of FIGs along with their certain properties. They introduced a variety of operations in IFIGs. They also provided a fascinating application of the product of IFIGs. The idea of order, size, domination, strong fuzzy incidence domination, and weak fuzzy incidence domination in FIGs was proposed by Nazeer et al. [43]. Nazeer and Rashid [44] presented the idea of picture FIGs. They introduced picture fuzzy cut-vertices, picture fuzzy bridges, picture fuzzy incidence cut pairs, and picture fuzzy incidence cut-vertices. More extensive and comprehensive work on FIGs, can be seen [45–48].

Connectivity parameters are connectivity measures of any system. In graphs, the connectivity between any two vertices is 1, and in FGs, it is from closed interval $[0, 1]$. There are certain motives to propose the concept of CC in FIGs. Firstly, in FGs, we can only compute the CC from vertex l to vertex m and from vertex m to the vertex l but if someone is interested in examining the CC from vertex l to an edge lm , then FGs are not enough to answer this question. Therefore, we propose the concept of CC in FIGs because FIGs permit us to find the CC from vertex l to an edge lm due to the presence of an incidence pair in FIGs. Secondly, in FIGs, the CC from vertex l to an edge lm and vertex m to an edge lm may or may not be the same. Thirdly, we cannot apply graphs and FGs to the applications of the highway systems of different cities and networks of different computers provided in Section 5 due to the non-availability of the influence of a vertex on and edge. Fourthly, the objective to introduce these ideas to FIGs is that Mathew and Sunitha [16] initiated the notion of CC in FGs. Later, Binu et al. [49] initiated an idea of cyclic connectivity index (CCI) and average cyclic connectivity index (ACCI) of FGs. We extended their work for FIGs. This paper establishes CC, CCI and ACCI of FIGs.

The other part of this article is constructed as follows. Section 2 consists of some introductory outcomes essential to comprehend the remaining portion of the article. CC, fuzzy incidence cyclic cut-vertex (FICCV), fuzzy incidence cyclic bridge (FICB) and fuzzy incidence cyclic cut pair (FICCP) of FIG are explained in Section 3. The formula to determine CCI, the way to manipulate ACCI of FIG, and three different types of vertices, namely, cyclic connectivity increasing vertex (CCIV), cyclically neutral vertex (CNV), and cyclic connectivity

decreasing vertex (CCDV) are described in Section 4. The real-life applications of CC of a FIG in a highway system of different cities to reduce road accidents and a computer network to find the best computers sharing the maximum amount of data among all other computers are discussed in Section 5. A comparative analysis of our study with the existing study is provided in Section 6. Section 7 carries some conclusions and future directions.

2 Preliminaries

This section carries some elementary and rudimentary definitions and results of FIGs. These will be useful to understand the contents of the article. $\wedge$ indicates the minimum operator, and $\vee$ denotes the maximum operator in this article.

Definition 1. [41] A fuzzy subset (FSS) of a set is a function of the set into the closed interval $[0, 1]$. A FG on (V, E) is a pair (σ, τ) , where σ is a FSS of V and τ is a FSS of E such that for every $l, m \in V$, $\tau(lm) \leq \sigma(l) \wedge \sigma(m)$.

Definition 2. [41, Definition 2.1] Let $G = (V, E, I)$, where $I \subseteq V \times E$. Then G is called an incidence graph (IG).

Definition 3. [41] Let $G = (V, E, I)$ be an IG. A sequence of distinct vertices $P_1: k_0, (k_0, k_0 k_1), k_0 k_1, (k_1, k_0 k_1), k_1, \dots, k_{n-1}, (k_{n-1}, k_{n-1} k_n), k_{n-1} k_n, (k_n, k_{n-1} k_n), k_n$ is called an incidence path and vertices k_0 and k_n are said to be connected. The incidence strength (I_s) of P_1 is defined as $\eta(k_0, k_0 k_1) \wedge \eta(k_1, k_0 k_1) \wedge \dots \wedge \eta(k_{n-1}, k_{n-1} k_n)$ and is expressed by $(I_{s_{P_1}})$. A sequence $P_2: k_0, (k_0, k_0 k_1), k_0 k_1, (k_1, k_0 k_1), k_1, \dots, k_{n-1}, (k_{n-1}, k_{n-1} k_n), k_{n-1} k_n, (k_n, k_{n-1} k_n), k_n, (k_n, k_n k_{n+1}), k_n k_{n+1}$ is another incidence path between k_0 and $k_n k_{n+1}$. The I_s of P_2 is defined as $\eta(k_0, k_0 k_1) \wedge \eta(k_1, k_0 k_1) \wedge \dots \wedge \eta(k_n, k_n k_{n+1})$. Let $l, m \in V \cup E$. We define $\text{ICONN}_{\tilde{G}}(l, m)$ to be the I_s of the strongest path from l to m .

Definition 4. [41, Definition 2.9] Let $G = (\sigma, \tau)$, where σ is a FSS of V and τ is a FSS of E . If η is a fuzzy incidence of G , then $\tilde{G} = (\sigma, \tau, \eta)$ is said to be a FIG.

Example 1. In Fig 1 a FIG $\tilde{G} = \{\sigma, \tau, \eta\}$ is shown with $\sigma^* = \{a, b, c, d, e, f\}$ and $\sigma(a) = 0.5$, $\sigma(b) = 0.6$, $\sigma(c) = 0.9$, $\sigma(d) = 0.3$, $\sigma(e) = 0.5$, $\sigma(f) = 0.4$, $\tau(ab) = 0.4$, $\tau(ac) = 0.4$, $\tau(bc) = 0.6$, $\tau(cd) = 0.3$, $\tau(de) = 0.2$, $\tau(ef) = 0.2$; $\eta(a, ab) = 0.3$, $\eta(b, ab) = 0.4$, $\eta(a, ac) = 0.03$, $\eta(c, ac) = 0.05$, $\eta(b, bc) = 0.5$, $\eta(c, bc) = 0.3$, $\eta(c, cd) = 0.2$, $\eta(d, cd) = 0.1$, $\eta(d, de) = 0.07$, $\eta(e, de) = 0.05$, $\eta(e, ef) = 0$, $\eta(f, ef) = 0$. There are two possible incidence paths from vertex a to d namely, $P_1: a, (a, ab), ab, (b, ab), b, (b, bc), bc, (c, bc), c, (c, cd), cd, (d, cd), d$ and $P_2: a, (a, ac), ac, (c, ac), c, (c, cd), cd, (d, cd), d$ with $I_{s_{P_1}} = \eta(a, ab) \wedge \eta(b, ab) \wedge \eta(b, bc) \wedge \eta(c, bc) \wedge \eta(c, cd) \wedge \eta(d, cd) = 0.3 \wedge 0.4 \wedge 0.3 \wedge 0.5 \wedge 0.3 \wedge 0.2 \wedge 0.1 = 0.1$. Similarly, $I_{s_{P_2}} = 0.03$ and the strength of the strongest path from a to d is $\text{ICONN}_{\tilde{G}}(a, d) = \vee\{I_{s_{P_1}}, I_{s_{P_2}} = 0.1, 0.03 = 0.1\}$.

Definition 5. [38] Any $l \in V$ is said to be in the support of σ ($\text{Supp}(\sigma)$) if $\sigma(l) > 0$, $lm \in E$ is said to be in the support of τ ($\text{Supp}(\tau)$) if $\tau(lm) > 0$ and $(l, lm) \in V \times E$ is said to be in the support of η ($\text{Supp}(\eta)$) if $\eta(l, lm) > 0$. The σ^* , τ^* and η^* are indicating the support of σ , τ and η respectively. Any edge lm is an edge of the FIG $\tilde{G} = (\sigma, \tau, \eta)$ if $lm \in \tau^*$. Similarly, any (l, lm) is a incidence pair (I_p) of $\tilde{G}$ if $(l, lm) \in \eta^*$.

Definition 6. [41, Definition 2.11] Let $\tilde{G} = (\sigma, \tau, \eta)$ be a FIG. Then an IG $\tilde{H} = (\iota, \kappa, \varsigma)$ is known as partial fuzzy incidence subgraph (PFISG) of $\tilde{G}$ if $\iota \subseteq \sigma$, $\kappa \subseteq \tau$ and $\varsigma \subseteq \eta$. $\tilde{H}$ is named as fuzzy incidence subgraph (FISG) of $\tilde{G}$ if $\iota = \sigma$, $\kappa = \tau$ and $\varsigma = \eta$.

Definition 7. [38, Definition 3] If $(\sigma^*, \tau^*, \eta^*)$ is a cycle then the FIG $\tilde{G} = (\sigma, \tau, \eta)$ is a cycle. If $(\sigma^*, \tau^*, \eta^*)$ is a cycle and $\nexists$ a single $lm \in \tau^*$ such that $\tau(lm) = \wedge\{\tau(uv) | uv \in \tau^*\}$ then the FIG $\tilde{G}$ is a fuzzy cycle. The FIG $\tilde{G} = (\sigma, \tau, \eta)$ is a fuzzy incidence cycle (FIC), and $\nexists$ a single $(l, lm) \in \eta^*$ such that $\eta(l, lm) = \wedge\{\eta(u, uv) | (u, uv) \in \eta^*\}$.

Fig 1. A FIG.

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Definition 8. [38, Definition 7] Consider a FIG $\tilde{G} = (\sigma, \tau, \eta)$. Then an $I_p(l, lm)$ is said to be a strong incidence pair (I_p^s) if $\eta(l, lm) \geq \text{ICONN}_{\tilde{G}-(l,lm)}$ where $\text{ICONN}_{\tilde{G}-(l,lm)}$ indicates the maximum incidence strength of $l - lm$. Particularly, it is said to be $\alpha - I_p^s$ if $\eta(l, lm) > \text{ICONN}_{\tilde{G}-(l,lm)}$ and $\beta - I_p^s$ if $\eta(l, lm) = \text{ICONN}_{\tilde{G}-(l,lm)}$. A pair is said to be I_p^s if it is either $\alpha - I_p^s$ or $\beta - I_p^s$.

Definition 9. [38, Definition 8] Consider a FIG $\tilde{G} = (\sigma, \tau, \eta)$. If $\eta(l, lm) < \text{ICONN}_{\tilde{G}-(l,lm)}$ then (l, lm) is called δ -incidence pair ($\delta - \text{IPr}$).

Proposition 1. [38, Proposition 1] Each FIC is a strong cycle.

Definition 10. [38] If $\eta(l, lm) = \sigma(l) \wedge \tau(lm)$ for each $(l, lm) \in \eta^*$ then a FIG $\tilde{G} = (\sigma, \tau, \eta)$ is named as a CFIG.

Proposition 2. [38, Proposition 9] A CFIG has no $\delta - \text{IPr}$.

3 Cycle connectivity of fuzzy incidence graphs

In this section, we present the novel idea of connectivity named as CC of a FIG. The CC of any FIG is denoted by Ω . The idea of Ω of a FIC is also illuminated. It is also proved that the Ω of a FIC $\tilde{G}$ is equal to the I_s of $\tilde{G}$. We have also introduced three innovative ideas, namely FICCV, FICB, and FICCP. On deleting the FICCV, FICB, and FICCP from the FIG, there will be an effect in the connectivity and Ω of a FIG. For easiness, in the coming sections, we will take $\sigma(a) = 1$ for every $a \in \sigma^*$ unless otherwise specifies.

Definition 11. Assume $\tilde{G} = \{\sigma, \tau, \eta\}$ is a FIC. The I_s of FIC is the lowest η value of all I_p in it.

Definition 12. Let $\tilde{G} = \{\sigma, \tau, \eta\}$ be a FIG. Then for any two vertices l and m of $\tilde{G}$, there associated a set say $v(l, m)$ named the v -estimation of l and m and is defined as $v(l, m) = \{\chi: \chi \in (0, 1]\}$ where χ is the I_s of a FIC passing through l and m .

Definition 13. Consider a FIG $\tilde{G} = \{\sigma, \tau, \eta\}$. Then Ω between l and m is defined as $\vee\{\chi: \chi \in v(l, m) | l, m \in \sigma^*\}$ and it is indicated by $\Omega_{l,m}^{\tilde{G}}$.

Remark 1. If $v(l, m) = \emptyset$ for some pair of vertices l and m , then $\Omega_{l,m}^{\tilde{G}}(l, m) = 0$.

Example 2. In Fig 2 a FIG $\tilde{G} = \{\sigma, \tau, \eta\}$ is shown with $\sigma^* = \{a, b, c, d\}$ and $\tau(ab) = 0.7, \tau(ac) = 0.3, \tau(ad) = 0.3, \tau(bc) = 0.3, \tau(cd) = 0.9; \eta(a, ab) = 0.6, \eta(b, ab) = 0.5, \eta(a, ac) = 0.2, \eta(c, ac) = 0.2, \eta(a, ad) = 0.2, \eta(d, ad) = 0.2, \eta(b, bc) = 0.3, \eta(c, bc) = 0.2, \eta(c, cd) = 0.9, \eta(d, cd) = 0.8$ a FIG. Here in $\tilde{G}$, $abca, abcd$ and $adca$ are all FICs. There are three FICs passing through a and c comprising, $abca, adca$ and $abcd$ with $I_s = \wedge\{\eta(a, ab) = 0.6, \eta(b, ab) = 0.5, \eta(b, bc) = 0.3, \eta(c, bc) = 0.2, \eta(c, ac) = 0.2, \eta(a, ac) = 0.2\} = 0.2; \wedge\{\eta(a, ad) = 0.2, \eta(d, ad) = 0.2, \eta(d, cd) = 0.8, \eta(c, cd) = 0.9, \eta(c, ac) = 0.2, \eta(a, ac) = 0.2\} = 0.2; \wedge\{\eta(a, ab) = 0.6, \eta(b, ab) = 0.5, \eta(b, bc) = 0.3, \eta(c, bc) = 0.2, \eta(c, cd) = 0.9, \eta(d, cd) = 0.8, \eta(d, ad) = 0.2, \eta(a, ad) = 0.2\} = 0.2$, respectively. Thus $v(a, c) = \vee\{0.2, 0.2, 0.2\} = 0.2$, this implies that $\Omega_{a,c}^{\tilde{G}} = 0.2$.

Definition 14. Let $\tilde{G} = \{\sigma, \tau, \eta\}$ be a FIG. Then the Ω of $\tilde{G}$ is defined as

$$\Omega(\tilde{G}) = \vee\{\Omega_{l,m}^{\tilde{G}} | l, m \in \sigma^*\}$$

That is Ω of a FIG $\tilde{G}$ is defined as the largest Ω of different pairs of vertices of $\tilde{G}$.

Example 3. Assume $\tilde{G} = \{\sigma, \tau, \eta\}$ is a FIG given in Fig 2 with $\sigma^* = \{a, b, c, d\}$ and $\tau(ab) = 0.7, \tau(ac) = 0.3, \tau(ad) = 0.3, \tau(bc) = 0.3, \tau(cd) = 0.9; \eta(a, ab) = 0.6, \eta(b, ab) = 0.5, \eta(a, ac) = 0.2, \eta(c, ac) = 0.2, \eta(a, ad) = 0.2, \eta(d, ad) = 0.2, \eta(b, bc) = 0.3, \eta(c, bc) = 0.2, \eta(c, cd) = 0.9$,

Fig 2. A FIG with $\Omega_{a,c}^{\tilde{G}} = 0.2$.

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$\eta(d, cd) = 0.8$. Then $\Omega_{a,b}^{\tilde{G}} = 0.2, \Omega_{a,c}^{\tilde{G}} = 0.2, \Omega_{a,d}^{\tilde{G}} = 0.2, \Omega_{b,c}^{\tilde{G}} = 0.2, \Omega_{b,d}^{\tilde{G}} = 0.2$ and $\Omega_{c,d}^{\tilde{G}} = 0.2$. Hence $\Omega(\tilde{G}) = \vee\{\Omega_{a,b}^{\tilde{G}} = 0.2, \Omega_{a,c}^{\tilde{G}} = 0.2, \Omega_{a,d}^{\tilde{G}} = 0.2, \Omega_{b,c}^{\tilde{G}} = 0.2, \Omega_{b,d}^{\tilde{G}} = 0.2, \Omega_{c,d}^{\tilde{G}} = 0.2\} = 0.2$.

Next, we will propose a fascinating result related to FIC in the form of a proposition. We can easily calculate the Ω of any FIC by just applying this result. This proposition will help us to save time and energy. Also, this will be helpful to avoid very long calculations.

Proposition 3. The Ω of a FIC $\tilde{G}$ is the I_s of $\tilde{G}$.

Proof. It follows by Proposition 1 that each I_p is a I_p^s in a FIC. Therefore, the Ω of a FIC $\tilde{G}$ is the I_s of $\tilde{G}$.

Now, we are going to introduce an actual result in the form of a theorem. With the help of this theorem, we will be able to compute Ω of any CFIG. By applying this theorem, we do not have to need to do complicated calculations. We have to use the theorem and get the required result.

Theorem 2. Let $\tilde{G}$ be a CFIG with vertices $h_1, h_2, \dots, h_n$ such that $\sigma(h_i) = j_i$ and $j_1 \leq j_2 \leq \dots \leq j_{n-2} \leq j_{n-1} \leq j_n$ with $\tau(xy) = \sigma(x) \wedge \sigma(y)$ for each $x, y \in \sigma^*$. Then $\Omega(\tilde{G}) = j_{n-2}$.

Proof. Suppose the conditions of the Theorem. Since any three vertices of $\tilde{G}$ are adjacent because $\tilde{G}$ is a CFIG also any three vertices are in 3 vertices FIC. Since $\tilde{G}$ is a CFIG, by Proposition 2 all I_p are I_p^s in CFIG. Therefore, to calculate the smallest I_s of FIC in $\tilde{G}$, it is enough to calculate the smallest I_s of every 3 vertices FIC in $\tilde{G}$. Since $\tilde{G}$ is a CFIG therefore to examine a 4 vertices FIC, $C = abcd$ in $\tilde{G}$ (it is notable that the case is same for n vertices FIC) there will be parts of two 3 vertices FIC in C , namely $C_1 = abca$ and $C_2 = acda$. Let the $I_s(C) = j$. For all $I_p(x, xy)$ in C , $\eta(x, xy) \geq j$. Particularly, $\eta(a, ab) \geq j$ and $\eta(b, bc) \geq j$. Since $\tilde{G}$ is a CFIG, then by Proposition 2, $\tilde{G}$ does not have any $\delta - IPr$. This means $\eta(a, ac) \geq \wedge\{\eta(a, ab), \eta(b, bc)\} \geq j$. Thus $\eta(a, ac) \geq j$.

Consider, $\eta(a, ac) = j$, then $I_s(C_1) = I_s(C_2) = I_s(C) = j$. Suppose $\eta(a, ac) > j$, since $I_s(C) = j$ then either C_1 or C_2 will have I_s equal to j . Now, $I_s(C) = \wedge\{I_s(C_1), I_s(C_2)\}$. Thus the I_s of a 4 vertices FIC is same as the I_s of a 3 vertices FIC in $\tilde{G}$. From all 3 vertices FIC, the 3 vertices FIC devised by three vertices with largest vertices strength will have the greatest strength. Therefore, the FIC $C = h_{n-2}, (h_{n-2}, h_{n-2}, h_{n-1}), h_{n-2}, h_{n-1}, (h_{n-1}, h_{n-2}, h_{n-1}), h_{n-1}, (h_{n-1}, h_{n-1}, h_n), h_{n-1}, h_n, (h_n, h_{n-1}, h_n), h_n, (h_n, h_n, h_{n-2}), h_n, h_{n-2}, (h_{n-2}, h_n, h_{n-2}), h_{n-2}$ is a FIC with largest I_s in $\tilde{G}$. Also I_s of $C = j_{n-2} \wedge j_{n-1} \wedge j_n = j_{n-2}$. Hence $\Omega(\tilde{G}) = j_{n-2}$.

Example 4. Assume $\tilde{G} = \{\sigma, \tau, \eta\}$ is a CFIG as shown in Fig 3 with $\sigma^* = \{a, b, c, d\}$ and $\tau(ab) = 0.3, \tau(ac) = 0.3, \tau(ad) = 0.3, \tau(bc) = 0.5, \tau(bd) = 0.5, \tau(cd) = 0.7; \eta(a, ab) = 0.3, \eta(b, ab) = 0.3, \eta(a, ac) = 0.3, \eta(c, ac) = 0.3, \eta(a, ad) = 0.3, \eta(d, ad) = 0.3, \eta(b, bc) = 0.5, \eta(c, bc) = 0.5, \eta(b, bd) = 0.5, \eta(d, bd) = 0.5, \eta(c, cd) = 0.7, \eta(d, cd) = 0.7$. Then $\Omega_{a,b}^{\tilde{G}} = 0.3, \Omega_{a,c}^{\tilde{G}} = 0.3, \Omega_{a,d}^{\tilde{G}} = 0.3, \Omega_{b,c}^{\tilde{G}} = 0.5, \Omega_{b,d}^{\tilde{G}} = 0.5$ and $\Omega_{c,d}^{\tilde{G}} = 0.5$. Hence $\Omega(\tilde{G}) = \vee\{\Omega_{a,b}^{\tilde{G}} = 0.3, \Omega_{a,c}^{\tilde{G}} = 0.3, \Omega_{a,d}^{\tilde{G}} = 0.3, \Omega_{b,c}^{\tilde{G}} = 0.5, \Omega_{b,d}^{\tilde{G}} = 0.5, \Omega_{c,d}^{\tilde{G}} = 0.5\} = 0.5$. Theorem 2 is verified.

Definition 15. A vertex l in a FIG $\tilde{G}$ is said to be a FICCV if

$$\Omega(\tilde{G} - l) < \Omega(\tilde{G}).$$

Definition 16. An edge (l, m) in a FIG $\tilde{G}$ is said to be a FICB if

$$\Omega(\tilde{G} - (l, m)) < \Omega(\tilde{G}).$$

Fig 3. A CFIG.

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Definition 17. A pair (l, lm) in a FIG $\tilde{G}$ is said to be a FICCP if

$$\Omega(\tilde{G} - (l, lm)) < \Omega(\tilde{G}).$$

Definition 18. A FIG $\tilde{G}$ is called cyclically balanced if $\tilde{G}$ is without FICCV, FICB and FICCP.

Example 5. Assume $\tilde{G} = \{\sigma, \tau, \eta\}$ is a FIG as provided in Fig 4 with $\sigma^* = \{a, b, c, d, e, f\}$ and $\tau(ab) = 0.2$, $\tau(ac) = 0.2$, $\tau(af) = 0.5$, $\tau(bc) = 0.3$, $\tau(cd) = 0.1$, $\tau(ce) = 0.1$, $\tau(de) = 0.2$, $\tau(ef) = 0.1$; $\eta(a, ab) = 0.07$, $\eta(b, ab) = 0.15$, $\eta(a, ac) = 0.07$, $\eta(c, ac) = 0.1$, $\eta(a, af) = 0.4$, $\eta(f, af) = 0.3$, $\eta(c,$

Fig 4. A FIG with FICCVs, FICBs and FICCPs.

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$cd) = 0.1, \eta(d, cd) = 0.09, \eta(c, ce) = 0.06, \eta(e, ce) = 0.1, \eta(d, de) = 0.06, \eta(e, de) = 0.2, \eta(e, ef) = 0.06, \eta(f, ef) = 0.1$. Now, $\Omega_{a,b}^{\tilde{G}} = 0.07, \Omega_{a,c}^{\tilde{G}} = 0.07, \Omega_{a,d}^{\tilde{G}} = 0.06, \Omega_{a,e}^{\tilde{G}} = 0.06, \Omega_{a,f}^{\tilde{G}} = 0.06, \Omega_{b,c}^{\tilde{G}} = 0.07, \Omega_{b,d}^{\tilde{G}} = 0.06, \Omega_{b,e}^{\tilde{G}} = 0.06, \Omega_{b,f}^{\tilde{G}} = 0.06, \Omega_{c,d}^{\tilde{G}} = 0.06, \Omega_{c,e}^{\tilde{G}} = 0.06, \Omega_{c,f}^{\tilde{G}} = 0.06, \Omega_{d,e}^{\tilde{G}} = 0.06, \Omega_{d,f}^{\tilde{G}} = 0.06$, and $\Omega_{e,f}^{\tilde{G}} = 0.06$. Hence $\Omega(\tilde{G}) = 0.07$. Since $\Omega(\tilde{G} - a) = \Omega(\tilde{G} - b) = \Omega(\tilde{G} - c) = 0.06 < \Omega(\tilde{G}) = 0.07$. Therefore, a, b and c are FICCV. In the same way, $\Omega(\tilde{G} - (a, b)) = \Omega(\tilde{G} - (a, c)) = \Omega(\tilde{G} - (b, c)) = 0.06 < \Omega(\tilde{G}) = 0.07$ therefore edges $(a, b), (a, c)$ and (b, c) are FICBs. Similarly, $(a, ab), (b, ab), (a, ac), (c, ac), (b, bc)$ and (c, bc) are FICCPs.

Proposition 4. If an edge (l, m) in a FIG is an FICB, then both the vertices l and m are FICCVs.

Proof. Let $\tilde{G} = (\sigma, \tau, \eta)$ be a FIG and (l, m) be a FICB in $\tilde{G}$. Then by definition of FICB, $\Omega(\tilde{G} - (l, m)) < \Omega(\tilde{G})$. Hence $\Omega(\tilde{G} - l) \leq \Omega(\tilde{G} - (l, m)) < \Omega(\tilde{G})$ and $\Omega(\tilde{G} - m) \leq \Omega(\tilde{G} - (l, m)) < \Omega(\tilde{G})$. This implies that both vertices l and m are FICCVs.

Now, we are going to establish a handy result for CFIG. With the help of this result, we can easily find the FICCV in any CFIG.

Theorem 3. Let $\tilde{G}$ be a CFIG with $|\sigma^*| \geq 4$. Let $h_1, h_2, \dots, h_n \in \sigma^*$ and $\sigma(h_i) = k_i$ for $i = 1, 2, \dots, n$, and $k_1 \leq k_2 \leq \dots \leq k_n$. Then $\tilde{G}$ has a FICCV if and only if $k_{n-3} < k_{n-2}$.

Proof. Let $h_1, h_2, \dots, h_n \in \sigma^*$ and $\sigma(h_i) = k_i$ for $i = 1, 2, \dots, n$, and $k_1 \leq k_2 \leq \dots \leq k_n$. Assume that $\tilde{G}$ has a FICCV say l . Then by definition of FICCV, $\Omega(\tilde{G} - l) < \Omega(\tilde{G})$. That is l belongs to a distinctive FIC C . Suppose I_s of FIC, $C = \lambda > I_s$ of C' for any other FIC, C' in $\tilde{G}$. Since $k_1 \leq k_2 \leq \dots \leq k_n$, it means that the I_s of the FIC $h_{n-2} h_{n-1} h_{n-3}$ is λ . Hence $l \in \{h_{n-2}, h_{n-1}, h_{n-3}\}$.

To show that $k_{n-3} < k_{n-2}$. Assume that $k_{n-3} = k_{n-2}$. Then $C_1 = h_n h_{n-1} h_{n-2}$ and $C_2 = h_n h_{n-1} h_{n-3}$ have the equal I_s , and hence the deleting of h_{n-2}, h_{n-1} or h_{n-3} will not lessen $\Omega(\tilde{G})$. This contradiction shows that $k_{n-3} < k_{n-2}$.

Conversely, assume that $k_{n-3} < k_{n-2}$. Now, we have to show that $\tilde{G}$ has a FICCV. Since $k_{n-2} \leq k_{n-1} \leq \dots \leq k_n$ and $k_{n-3} < k_{n-2}$, all FICs of $\tilde{G}$ have I_s less than that of I_s of $h_{n-2} h_{n-1} h_{n-3}$. Hence the removal of h_{n-1}, h_{n-2} or h_{n-3} will become the cause of reduction of $\Omega(\tilde{G})$ therefore, $\tilde{G}$ has a FICCV.

4 Average cyclic connectivity index of fuzzy incidence graph

Now, we are going to initiate a new idea of CCI and ACCI of a FIG. The CCI of any FIG is denoted by Ω_I . In this section, we formulate the formula to compute the Ω_I of a FIG. It is also shown that Ω_I of any PFISG $\tilde{H}$ is always $\leq \Omega_I$ of any strong fuzzy incidence graph (SFIG) $\tilde{G}$. A lower and upper bound of Ω_I of a CFIG is also provided.

Definition 19. In a FIG $\tilde{G}$, the Ω_I is defined as

$$\Omega_I(\tilde{G}) = \sum_{l, m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l, m}^{\tilde{G}},$$

where $\Omega_{l, m}^{\tilde{G}}$ is the Ω between vertices l and m in $\tilde{G}$.

Example 6. Fig 5 is indicating the FIG $\tilde{G} = \{\sigma, \tau, \eta\}$ with $\sigma^* = \{a, b, c, d, e, f, g\}$ and $\tau(ab) = 0.2$, $\tau(ac) = 0.2$, $\tau(af) = 0.5$, $\tau(ag) = 0.2$, $\tau(bc) = 0.3$, $\tau(cd) = 0.1$, $\tau(ce) = 0.1$, $\tau(cg) = 0.2$, $\tau(de) = 0.2$, $\tau(ef) = 0.1$, $\tau(fg) = 0.2$; $\eta(a, ab) = 0.07$, $\eta(b, ab) = 0.15$, $\eta(a, ac) = 0.07$, $\eta(c, ac) = 0.1$, $\eta(a, af) = 0.4$, $\eta(f, af) = 0.3$, $\eta(a, ag) = 0.2$, $\eta(g, ag) = 0.3$, $\eta(b, bc) = 0.3$, $\eta(c, bc) = 0.2$, $\eta(c, cd) = 0.1$, $\eta(d, cd) = 0.09$, $\eta(c, ce) = 0.06$, $\eta(e, ce) = 0.1$, $\eta(c, cg) = 0.01$, $\eta(g, cg) = 0.01$, $\eta(d, de) = 0.06$, $\eta(e, de) = 0.2$, $\eta(e, ef) = 0.06$, $\eta(f, ef) = 0.1$, $\eta(f, fg) = 0.03$, $\eta(g, fg) = 0.03$. Thus by computing, $\Omega_{a, b}^{\tilde{G}} = 0.07$, $\Omega_{a, c}^{\tilde{G}} = 0.07$, $\Omega_{a, d}^{\tilde{G}} = 0.06$, $\Omega_{a, e}^{\tilde{G}} = 0.06$, $\Omega_{a, f}^{\tilde{G}} = 0.06$, $\Omega_{a, g}^{\tilde{G}} = 0.03$, $\Omega_{b, c}^{\tilde{G}} = 0.07$, $\Omega_{b, d}^{\tilde{G}} = 0.06$, $\Omega_{b, e}^{\tilde{G}} = 0.06$, $\Omega_{b, f}^{\tilde{G}} = 0.06$, $\Omega_{b, g}^{\tilde{G}} = 0.03$, $\Omega_{c, d}^{\tilde{G}} = 0.06$, $\Omega_{c, e}^{\tilde{G}} = 0.06$, $\Omega_{c, f}^{\tilde{G}} = 0.06$, $\Omega_{c, g}^{\tilde{G}} = 0.03$, $\Omega_{d, e}^{\tilde{G}} = 0.06$, $\Omega_{d, f}^{\tilde{G}} = 0.06$, $\Omega_{d, g}^{\tilde{G}} = 0.03$, $\Omega_{e, f}^{\tilde{G}} = 0.06$, $\Omega_{e, g}^{\tilde{G}} = 0.03$ and $\Omega_{f, g}^{\tilde{G}} = 0.06$. Hence $\Omega_I(\tilde{G}) = 3(0.07) + 13(0.06) + 5(0.03) = 1.14$.

Remark 4. Assume $\tilde{G}$ is a FIG with vertex set t . Then $0 \leq \Omega_I(\tilde{G}) \leq \binom{t}{2}$.

Definition 20. A FIG $\tilde{G}$ is called SFIG if each of its I_p is a I_p^s .

Proposition 5. Let $\tilde{G} = (\sigma, \tau, \eta)$ be a SFIG. Then PFISG $\tilde{H} = (\iota, \kappa, \varsigma)$ of $\tilde{G}$, $\Omega_I(\tilde{H}) \leq \Omega_I(\tilde{G})$.

Proof. Assume $\tilde{G} = (\sigma, \tau, \eta)$ is a SFIG and let $\tilde{H} = (\iota, \kappa, \varsigma)$ be a PFISG of $\tilde{G} = (\sigma, \tau, \eta)$. Let $l, m \in \iota^*$. Then by the definition of PFISG $\iota(l) \leq \sigma(l)$ and ν -estimation of l and m fulfills the relation $\nu(l, m)|_{\tilde{H}} \subseteq \nu(l, m)|_{\tilde{G}}$. This implies $\Omega_{l, m}^{\tilde{H}} \leq \Omega_{l, m}^{\tilde{G}}$ and

$$\sum_{l, m \in \iota^*} \iota(l)\iota(m)\Omega_{l, m}^{\tilde{H}} \leq \sum_{l, m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l, m}^{\tilde{G}}$$

Fig 5. A FIG with $\Omega_I = 1.14$.

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Hence,

$$\Omega_I(\tilde{H}) \leq \Omega_I(\tilde{G}).$$

Example 7. Assume $\tilde{H} = \{\sigma, \tau, \eta\}$ is a PFISG shown in Fig 6 of FIG provided in Fig 5 with $\iota^* = \{a, b, c, f, g\}$ and $\tau(ab) = 0.2, \tau(af) = 0.5, \tau(ag) = 0.2, \tau(bc) = 0.3, \tau(cg) = 0.2, \tau(gf) = 0.2; \eta(a, ab) = 0.07, \eta(b, ab) = 0.15, \eta(a, af) = 0.4, \eta(f, af) = 0.3, \eta(a, ag) = 0.2, \eta(g, ag) = 0.3, \eta(b, bc) =$

Fig 6. A PFISG $\tilde{H}$ with $\Omega_I = 0.16$.

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$0.3, \eta(c, bc) = 0.2, \eta(c, cg) = 0.01, \eta(g, cg) = 0.01, \eta(g, gf) = 0.03, \eta(f, gf) = 0.03$. Now, $\Omega_{a,b}^{\tilde{H}} = 0.01, \Omega_{a,c}^{\tilde{H}} = 0.01, \Omega_{a,f}^{\tilde{H}} = 0.03, \Omega_{a,g}^{\tilde{H}} = 0.03, \Omega_{b,c}^{\tilde{H}} = 0.01, \Omega_{b,f}^{\tilde{H}} = 0.01, \Omega_{b,g}^{\tilde{H}} = 0.01, \Omega_{c,f}^{\tilde{H}} = 0.01, \Omega_{c,g}^{\tilde{H}} = 0.01$ and $\Omega_{g,f}^{\tilde{H}} = 0.03$. Hence $\Omega_I(\tilde{H}) = 7(0.01) + 3(0.03) = 0.16 < \Omega_I(\tilde{G}) = 1.14$.

Proposition 6. In any FIG $\tilde{G}$, $\Omega_I(\tilde{G} - l) < \Omega_I(\tilde{G})$, where l is a FICCV of $\tilde{G}$.

Proof. Consider that l is a FICCV of $\tilde{G}$. Consequently deletion of l will lessen $\Omega(\tilde{G})$. Also

$$\Omega_l(\tilde{G}) = \sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} + \sum_{x,y \in \sigma^* \setminus \{l\}} \sigma(x)\sigma(y)\Omega_{x,y}^{\tilde{G}} \geq \sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} +$$

$$\Omega_l(\tilde{G} - l) > \Omega_l(\tilde{G} - l). \text{ If } \sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} > 0, \text{ then the statement } \Omega_l(\tilde{G}) =$$

$$\sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} + \sum_{x,y \in \sigma^* \setminus \{l\}} \sigma(x)\sigma(y)\Omega_{x,y}^{\tilde{G}} \geq \sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} + \Omega_l(\tilde{G} - l) > \Omega_l(\tilde{G} - l)$$

is true otherwise it is not true. On the contrary, if $\sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} = 0$, then $\Omega_{l,m}^{\tilde{G}} = 0$ for each $m \in \sigma^* \setminus \{l\}$. This means l is not a part of any FIC in $\tilde{G}$ but this is impossible because l is a FICCV of $\tilde{G}$. Hence we may come to an end that $\sum_{m \in \sigma^* \setminus \{l\}} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} > 0$ now this implies

$$\Omega_l(\tilde{G} - l) < \Omega_l(\tilde{G}).$$

Now, we are going to discuss lower and upper bounds of Ω_l of a CFIG. For this, we have presented two fundamental theorems. With the help of these theorems, we can quickly know about the Ω_l of a CFIG.

Theorem 5. Assume $\tilde{G}$ is a CFIG having $|\sigma^*| = n \geq 3$, $\sigma(h_i) = k_i$ for $i = 1, 2, \dots, n$ and $0 < \sigma(h_i) < 1$. Then $\Omega_l(\tilde{G}) < \vee\{\sigma(h_i)\}$.

Proof. Since $\tilde{G}$ is a CFIG with $|\sigma^*| = n \geq 3$ therefore by Proposition 2 $\tilde{G}$ does not have any $\delta - IPr$. This means all I_p in $\tilde{G}$ are I_p^s and according to Proposition 1 each FIC is a strong FIC in $\tilde{G}$. Suppose l and m are any two vertices of $\tilde{G}$ then we have to calculate I_s of all FICs contain vertices l and m . After this, we have to compute Ω which is the maximum value of I_s of all FICs containing pair of vertices l and m . Similarly, we have to compute Ω up to n (total number of vertices) and take the minimum value of all Ω of the CFIG $\tilde{G}$. It is clear that $\Omega_l(\tilde{G})$ can never be exceed the maximum membership value of all the vertices by definition of

$$\Omega_l(\tilde{G}) = \sum_{l,m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}}.$$

Also, multiplication of all Ω to the membership values of the corresponding vertices and addition of all these Ω of the graph $\tilde{G}$ provide the $\Omega_l(\tilde{G})$ less than $\vee\{\sigma(h_i)\}$. This implies that $\Omega_l(\tilde{G}) < \vee\{\sigma(h_i)\}$.

Example 8. Assume $\tilde{G} = \{\sigma, \tau, \eta\}$ is a CFIG as given in Fig 7 with $\sigma^* = \{a, b, c, d, e\}$ and $\sigma(a) = 0.2$, $\sigma(b) = 0.05$, $\sigma(c) = 0.09$, $\sigma(d) = 0.3$, $\sigma(e) = 0.01$; $\tau(ab) = 0.03$, $\tau(ac) = 0.03$, $\tau(ad) = 0.03$, $\tau(ae) = 0.01$, $\tau(bc) = 0.04$, $\tau(bd) = 0.04$, $\tau(be) = 0.01$, $\tau(cd) = 0.07$, $\tau(ce) = 0.01$, $\tau(de) = 0.01$; $\eta(a, ab) = 0.03$, $\eta(b, ab) = 0.03$, $\eta(a, ac) = 0.03$, $\eta(c, ac) = 0.03$, $\eta(a, ad) = 0.03$, $\eta(d, ad) = 0.03$, $\eta(a, ae) = 0.01$, $\eta(e, ae) = 0.01$, $\eta(b, bc) = 0.04$, $\eta(c, bc) = 0.04$, $\eta(b, bd) = 0.04$, $\eta(d, bd) = 0.04$, $\eta(b, be) = 0.01$, $\eta(e, be) = 0.01$, $\eta(c, cd) = 0.07$, $\eta(d, cd) = 0.07$, $\eta(c, ce) = 0.01$, $\eta(e, ce) = 0.01$, $\eta(d, de) = 0.01$, $\eta(e, de) = 0.01$. Then $\Omega_l(\tilde{G}) = .004564 < \vee\{0.2, 0.05, 0.09, 0.3, 0.01\} = 0.3$.

Theorem 6. Let $\tilde{G}$ be a CFIG having $|\sigma^*| = n \geq 3$, $\sigma(h_i) = k_i$ for $i = 1, 2, \dots, n$. Then

$$\binom{n}{2}(\wedge(\Omega))^3 \leq \Omega_l(\tilde{G}) \leq \binom{n}{2}(\vee(\sigma_i))^3.$$

Proof. By given statement of theorem. $\tilde{G}$ is a CFIG with $|\sigma^*| = n \geq 3$ therefore Proposition 2 yields that $\tilde{G}$ will be without any $\delta - IPr$. This means all I_p in $\tilde{G}$ are I_p^s and as stated in Proposition 1 every FIC is a strong FIC in $\tilde{G}$. Suppose l and m are any two vertices of $\tilde{G}$ then we have to calculate I_s of all FICs contain vertices l and m . After this, we have to compute Ω which is the maximum value of I_s of all FICs containing pair of vertices l and m . Similarly, we have to compute Ω up to n (total number of vertices) and take the minimum value of all Ω of the CFIG $\tilde{G}$. Also, the total number of edges for a CFIG is always equal to $\binom{n}{2} = \frac{n!}{(n-2)!2!}$. Since

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$\Omega_I(\tilde{G}) = \sum_{l,m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}}$ therefore, $\binom{n}{2}(\wedge(\Omega))^3$ can never be exceed than the $\Omega_I(\tilde{G})$. This implies,

$$\binom{n}{2}(\wedge(\Omega))^3 \leq \Omega_I(\tilde{G}) \quad (1)$$

Now, by definition of $\Omega_I(\tilde{G}) = \sum_{l,m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}}$ it can be seen that $\Omega_I(\tilde{G})$ will always remains less than or equal to the $\binom{n}{2}(\vee(\sigma_i))^3$. This means

$$\Omega_I(\tilde{G}) \leq \binom{n}{2}(\vee(\sigma_i))^3 \quad (2)$$

Hence from Eqs (1) and (2) it can be concluded that

$$\binom{n}{2}(\wedge(\Omega))^3 \leq \Omega_I(\tilde{G}) \leq \binom{n}{2}(\vee(\sigma_i))^3.$$

Example 9. A CFIG $\tilde{G} = \{\sigma, \tau, \eta\}$ is shown in Fig 7 with $\sigma^* = \{a, b, c, d, e\}$ and $\sigma(a) = 0.2, \sigma(b) = 0.05, \sigma(c) = 0.09, \sigma(d) = 0.3, \sigma(e) = 0.01; \tau(ab) = 0.03, \tau(ac) = 0.03, \tau(ad) = 0.03, \tau(ae) = 0.01, \tau(bc) = 0.04, \tau(bd) = 0.04, \tau(be) = 0.01, \tau(cd) = 0.07, \tau(ce) = 0.01, \tau(de) = 0.01; \eta(a, ab) = 0.03, \eta(b, ab) = 0.03, \eta(a, ac) = 0.03, \eta(c, ac) = 0.03, \eta(a, ad) = 0.03, \eta(d, ad) = 0.03, \eta(a, ae) = 0.01, \eta(e, ae) = 0.01, \eta(b, bc) = 0.04, \eta(c, bc) = 0.04, \eta(b, bd) = 0.04, \eta(d, bd) = 0.04, \eta(b, be) = 0.01, \eta(e, be) = 0.01, \eta(c, cd) = 0.07, \eta(d, cd) = 0.07, \eta(c, ce) = 0.01, \eta(e, ce) = 0.01, \eta(d, de) = 0.01, \eta(e, de) = 0.01$. Then $\binom{n}{2}(\wedge(\Omega))^3 = .00001 \leq \Omega_I(\tilde{G}) = .004564 \leq \binom{n}{2}(\vee(\sigma_i))^3 = 0.27$.

Here, we are going to present a very foundational concept of ACCI of a FIG. In enormous networks, the sturdy flow among different vertices is mandatory to sustain trustability and devotedness. To guarantee the firmness of the exchange of data in the complete or portion of the network, measuring the average value of the cyclic data exchange is vital. Therefore, we discuss the ACCI of a FIG. The ACCI of any FIG is denoted by Ω_{AI} . Also, we define three different types of vertices.

Definition 21. Assume a FIG $\tilde{G} = (\sigma, \tau, \eta)$. Then the Ω_{AI} is defined by

$$\Omega_{AI}(\tilde{G}) = \frac{1}{\binom{t}{2}} \sum_{l,m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}},$$

where t is the number of vertices in $\tilde{G}$.

Definition 22. Consider a FIG $\tilde{G} = (\sigma, \tau, \eta)$. Then a vertex y is known as CCIV if $\Omega_{AI}(\tilde{G}) < \Omega_{AI}(\tilde{G} - y)$. If $\Omega_{AI}(\tilde{G}) = \Omega_{AI}(\tilde{G} - y)$, then y is said to be CNV. If $\Omega_{AI}(\tilde{G}) > \Omega_{AI}(\tilde{G} - y)$ then y is named as CCDV.

In a FIG an isolated vertex is always a CNV.

Example 10. In Fig 8 a FIG $\tilde{G} = \{\sigma, \tau, \eta\}$ is provided with $\sigma^* = \{a, b, c, d\}$ and $\tau(ab) = 0.7, \tau(ad) = 0.3, \tau(bc) = 0.2, \tau(bd) = 0.3, \tau(cd) = 0.2; \eta(a, ab) = 0.5, \eta(b, ab) = 0.6, \eta(a, ad) = 0.07, \eta(d, ad) = 0.05, \eta(b, bc) = 0.2, \eta(c, bc) = 0.1, \eta(b, bd) = 0.2, \eta(d, bd) = 0.05, \eta(c, cd) = 0.03, \eta(d, cd) = 0.03$. There are three FICs in $\tilde{G}$ namely, $C_1: a, b, d, a, C_2: b, c, d, c$ and $C_3: a, b, c, d, a$ with I_s are 0.05, 0.03 and 0.03 respectively. Now, $\Omega_{a,b}^{\tilde{G}} = 0.05, \Omega_{a,c}^{\tilde{G}} = 0.03, \Omega_{a,d}^{\tilde{G}} = 0.05, \Omega_{b,c}^{\tilde{G}} = 0.03, \Omega_{b,d}^{\tilde{G}} = 0.05$ and $\Omega_{c,d}^{\tilde{G}} = 0.03$. Therefore, $\Omega_I(\tilde{G}) = 0.03 + 0.03 + 0.05 + 0.03 + 0.05 + 0.03 = 0.22$. Also $\Omega_{AI}(\tilde{G}) = \frac{1}{\binom{4}{2}} \sum_{l,m \in \sigma^*} \sigma(l)\sigma(m)\Omega_{l,m}^{\tilde{G}} = \frac{1}{\binom{4}{2}} 0.22 = 0.036$. Also after calculation $\Omega_{AI}(\tilde{G} - a) = 0.03 < \Omega_{AI}(\tilde{G}) = 0.036, \Omega_{AI}(\tilde{G} - b) = 0 < \Omega_{AI}(\tilde{G}) = 0.036, \Omega_{AI}(\tilde{G} - d) = 0 <$

Fig 8. A FIG with CCDV and CCIV.

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$\Omega_{AI}(\tilde{G}) = 0.036$ and $\Omega_{AI}(\tilde{G} - c) = 0.05 > \Omega_{AI}(\tilde{G}) = 0.036$. Now, a, b and d are CCDV and c is a CCIV.

Now, we are going to present an essential proposition. With the help of this proposition, we will be able to find CNV, CCIV, and CCDV of FIG.

Proposition 7. Let $\tilde{G} = (\sigma, \tau, \eta)$ be a FIG having $|\sigma^*| = t \geq 3$ and assume $x \in \sigma^*$. Let $r = \frac{\Omega_I(\tilde{G})}{\Omega_I(\tilde{G}-x)}$ and $s = \frac{t}{t-2}$. Then x is said to be a CNV if and only if $s = r$, x is named as CCIV if and only if $r < s$ and x is called CCDV if and only if $r > s$.

Proof. Consider a FIG $\tilde{G} = (\sigma, \tau, \eta)$ having $|\sigma^*| = t \geq 3$ and consider $x \in \sigma^*$. Suppose that x is a CCIV, then by definition of CCIV, the $\Omega_{AI}(\tilde{G} - x) > \Omega_{AI}(\tilde{G})$. Clearly, $\frac{\Omega_I(\tilde{G}-x)}{\binom{t-1}{2}} > \frac{\Omega_I(\tilde{G})}{\binom{t}{2}}$ and also we have $\frac{\Omega_I(\tilde{G}-x)}{\binom{t-1}{2}} > \frac{\Omega_I(\tilde{G})}{t}$. Thus, $\frac{\Omega_I(\tilde{G})}{\Omega_I(\tilde{G}-x)} < \frac{t}{t-2}$. If $r = \frac{\Omega_I(\tilde{G})}{\Omega_I(\tilde{G}-x)}$ and $s = \frac{t}{t-2}$, then $r < s$. Converse part is trivial. Therefore, x is CCIV if and only if $r < s$. Remaining cases may be proved in a similar way.

5 Real-life applications of cycle connectivity

In daily life, Ω has various uses. Here, we have proposed two critical real-life applications of Ω of FIGs. In the first application, we have taken a highway system of different cities and apply the idea of Ω of FIG to find the roads which are becoming the leading cause of maximum accidents. In the second application, we have taken a network of different computers sharing data. We have applied the idea of Ω to the network of different computers and find which computer/computers are transferring the maximum amount of data to other computers.

5.1 Application of cycle connectivity in highway system

Due to the huge traffic on roads, the percentage of accidents is increasing day by day. To minimize these accidents government should take some serious steps to lessen the percentage of road accidents. Here, we are presenting a graphical model of FIG to tackle this problem. This can be done by calculating the Ω between each pair of vertices of FIG. The roads having a maximum Ω are the roads of maximum traffic flow and become a source of the highest road accidents. Government can work on these roads by making speed breakers, speed bumps and

Fig 9. A highway system of different cities.

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deploying more traffic wardens to minimize road accidents. Here, we include an application of Ω of FIG in a highway system of different cities. As an explanatory case, consider a network of FIG consisting of eight vertices expressing different cities $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ and c_8 , edges are indicating the roads joining these cities and I_p are expressing flow of traffic from one city to another city. For example, an $I_p(c_1, c_1 c_2)$ is showing the flow of traffic from city c_1 to city c_2 and an $I_p(c_2, c_1 c_2)$ is proclaiming the flow of traffic from city c_2 to city c_1 through road $c_1 c_2$. The membership value (MSV) of the edges is showing the traffic flow (bikes, cars, vehicles, heavy vehicles) among different cities and the MSV of $(c_1, c_1 c_2)$ is specifying the flow of traffic from city c_1 to city c_2 and the MSV of $(c_2, c_1 c_2)$ is proclaiming the flow of traffic from city c_2 to city c_1 through road $c_1 c_2$.

Assume $\tilde{G} = (\sigma, \tau, \eta)$ is a FIG as shown in Fig 9 representing a highway system with $\sigma^* = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$ and $\tau(c_1 c_2) = 0.7, \tau(c_1 c_8) = 0.9, \tau(c_2 c_3) = 0.3, \tau(c_2 c_8) = 0.7,$

$\tau(c_3 c_4) = 0.6, \tau(c_3 c_6) = 0.5, \tau(c_4 c_5) = 0.5, \tau(c_5 c_6) = 0.8, \tau(c_6 c_7) = 0.8, \tau(c_7 c_8) = 0.3; \eta(c_1, c_1 c_2) = 0.5, \eta(c_2, c_1 c_2) = 0.7, \eta(c_1, c_1 c_8) = 0.5, \eta(c_8, c_1 c_8) = 0.9, \eta(c_2, c_2 c_3) = 0.2, \eta(c_3, c_2 c_3) = 0.25, \eta(c_2, c_2 c_8) = 0.3, \eta(c_8, c_2 c_8) = 0.3, \eta(c_3, c_3 c_4) = 0.2, \eta(c_4, c_3 c_4) = 0.4, \eta(c_3, c_3 c_6) = 0.1, \eta(c_6, c_3 c_6) = 0.1, \eta(c_4, c_4 c_5) = 0.5, \eta(c_5, c_4 c_5) = 0.3, \eta(c_5, c_5 c_6) = 0.6, \eta(c_6, c_5 c_6) = 0.5, \eta(c_6, c_6 c_7) = 0.8, \eta(c_7, c_6 c_7) = 0.3, \eta(c_7, c_7 c_8) = 0.3, \eta(c_8, c_7 c_8) = 0.2$. There are eight FICs in $\tilde{G}$ namely, $C_1: c_1, c_2, c_8, c_1, C_2: c_1, c_2, c_3, c_6, c_7, c_8, c_1, C_3: c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_1, C_4: c_3, c_4, c_5, c_6, c_3, C_5: c_3, c_4, c_5, c_6, c_7, c_8, c_2, c_3, C_6: c_3, c_4, c_5, c_6, c_7, c_8, c_1, c_2, c_3, C_7: c_3, c_6, c_7, c_8, c_2, c_3$, and $C_8: c_3, c_6, c_7, c_8, c_1, c_2, c_3$. Then $\Omega_{c_1, c_2}^{\tilde{G}} = \Omega_{c_1, c_8}^{\tilde{G}} = \Omega_{c_2, c_8}^{\tilde{G}} = 0.3$ and $\Omega_{c_1, c_3}^{\tilde{G}} = \Omega_{c_1, c_4}^{\tilde{G}} = \Omega_{c_1, c_5}^{\tilde{G}} = \Omega_{c_1, c_6}^{\tilde{G}} = \Omega_{c_1, c_7}^{\tilde{G}} = \Omega_{c_2, c_3}^{\tilde{G}} = \Omega_{c_2, c_4}^{\tilde{G}} = \Omega_{c_2, c_5}^{\tilde{G}} = \Omega_{c_2, c_6}^{\tilde{G}} = \Omega_{c_2, c_7}^{\tilde{G}} = \Omega_{c_3, c_4}^{\tilde{G}} = \Omega_{c_3, c_5}^{\tilde{G}} = \Omega_{c_3, c_6}^{\tilde{G}} = \Omega_{c_3, c_7}^{\tilde{G}} = \Omega_{c_3, c_8}^{\tilde{G}} = \Omega_{c_4, c_5}^{\tilde{G}} = \Omega_{c_4, c_6}^{\tilde{G}} = \Omega_{c_4, c_7}^{\tilde{G}} = \Omega_{c_4, c_8}^{\tilde{G}} = \Omega_{c_5, c_6}^{\tilde{G}} = \Omega_{c_5, c_7}^{\tilde{G}} = \Omega_{c_5, c_8}^{\tilde{G}} = \Omega_{c_6, c_7}^{\tilde{G}} = \Omega_{c_6, c_8}^{\tilde{G}} = \Omega_{c_7, c_8}^{\tilde{G}} = 0.2$. Hence $\Omega(\tilde{G}) = \bigvee \{ \Omega_{c_1, c_2}^{\tilde{G}} = \Omega_{c_1, c_8}^{\tilde{G}} = \Omega_{c_2, c_8}^{\tilde{G}} = 0.3, \Omega_{c_1, c_3}^{\tilde{G}} = \Omega_{c_1, c_4}^{\tilde{G}} = \Omega_{c_1, c_5}^{\tilde{G}} = \Omega_{c_1, c_6}^{\tilde{G}} = \Omega_{c_1, c_7}^{\tilde{G}} = \Omega_{c_2, c_3}^{\tilde{G}} = \Omega_{c_2, c_4}^{\tilde{G}} = \Omega_{c_2, c_5}^{\tilde{G}} = \Omega_{c_2, c_6}^{\tilde{G}} = \Omega_{c_2, c_7}^{\tilde{G}} = \Omega_{c_3, c_4}^{\tilde{G}} = \Omega_{c_3, c_5}^{\tilde{G}} = \Omega_{c_3, c_6}^{\tilde{G}} = \Omega_{c_3, c_7}^{\tilde{G}} = \Omega_{c_3, c_8}^{\tilde{G}} = \Omega_{c_4, c_5}^{\tilde{G}} = \Omega_{c_4, c_6}^{\tilde{G}} = \Omega_{c_4, c_7}^{\tilde{G}} = \Omega_{c_4, c_8}^{\tilde{G}} = \Omega_{c_5, c_6}^{\tilde{G}} = \Omega_{c_5, c_7}^{\tilde{G}} = \Omega_{c_5, c_8}^{\tilde{G}} = \Omega_{c_6, c_7}^{\tilde{G}} = \Omega_{c_6, c_8}^{\tilde{G}} = \Omega_{c_7, c_8}^{\tilde{G}} = 0.2 \} = 0.3$. Thus $\Omega(\tilde{G}) = 0.3$ is representing that the roads joining cities $c_1 c_2, c_1 c_8$ and $c_2 c_8$ are the main roads which are becoming a main cause of highest percentage of road accidents. So, the government should focus on these roads by making more speed breakers, speed bumps and deploying more traffic wardens on these roads. In this way, they can minimize the percentage of road accidents.

We have used FIGs in our application. The FIGs are more instrumental and effective than graphs. We cannot use graphs to explain the above phenomenon because graphs do not show the impact of a vertex on an edge. Another thing, in graphs, the Ω between each pair of vertices is always equal to 1, and we are unable to find which roads are becoming the main cause of maximum road accidents, but in FIGs, the Ω between each pair of vertices will be different. Therefore, FIGs are more helpful and useful than graphs.

5.2 Application of cycle connectivity in a computer network

In a network of different computers, computers are sharing data with each other. We want to find which computer/computers are best in performance among all other computers and sharing maximum data with all other computers in a network. This can be done by computing Ω between each pair of computers in a network. The pair of computers which have a maximum Ω will be the required computers transferring maximum data to all other computers in a network. Here, we are presenting a graphical model of FIG to explain this phenomenon. As an example, assume a network of FIG comprising of eight vertices. The vertices are showing the eight distinct computers in a network. The MSV of the vertices is indicating data store in each of these computers, the MSV of the edges is demonstrating the total amount of data that can be transferred from one computer to another computer and the MSV of the I_p is representing the amount of data which one computer is transferring to another computer. For example, an $I_p(a, ab)$ is indicating the transfer of data from computer a to computer b and an $I_p(b, ab)$ is showing the transfer of data from computer b to computer a .

Let $\tilde{G} = (\sigma, \tau, \eta)$ be a FIG as shown in Fig 10 representing a network of different computers with $\sigma(a) = 0.5, \sigma(b) = 0.2, \sigma(c) = 0.7, \sigma(d) = 0.3, \sigma(e) = 0.05, \sigma(f) = 0.1, \sigma(g) = 1, \sigma(h) = 0.8; \tau(ab) = 0.2, \tau(ag) = 0.4, \tau(ah) = 0.4, \tau(bc) = 0.2, \tau(bh) = 0.2, \tau(cd) = 0.05, \tau(ch) = 0.6, \tau(de) = 0.04, \tau(ef) = 0.04, \tau(eh) = 0.04, \tau(fg) = 0.06, \tau(gh) = 0.5; \eta(a, ab) = 0.15, \eta(b, ab) = 0.1, \eta(a, ag) = 0.2, \eta(g, ag) = 0.3, \eta(a, ah) = 0.4, \eta(h, ah) = 0.2, \eta(b, bc) = 0.1, \eta(c, bc) = 0.2, \eta(b, bh) = 0.1, \eta(h, bh) = 0.1, \eta(c, cd) = 0.04, \eta(d, cd) = 0.05, \eta(c, ch) = 0.6, \eta(h, ch) = 0.5, \eta(d, de) = 0.04, \eta(e, de) = 0.04, \eta(e, ef) = 0.01, \eta(f, ef) = 0.01, \eta(e, eh) = 0.02, \eta(h, eh) = 0.02, \eta(f, fg) = 0.05, \eta(g, fg) = 0.06, \eta(g, gh) = 0.4, \eta(h, gh) = 0.5$. Now, by computing Ω between each pair of vertices we get

Fig 10. A network of different computers.

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$\Omega_{a,b}^{\hat{G}} = \Omega_{a,c}^{\hat{G}} = \Omega_{a,g}^{\hat{G}} = \Omega_{a,h}^{\hat{G}} = \Omega_{b,c}^{\hat{G}} = \Omega_{b,g}^{\hat{G}} = \Omega_{b,h}^{\hat{G}} = \Omega_{c,g}^{\hat{G}} = \Omega_{c,h}^{\hat{G}} = 0.1$, $\Omega_{a,d}^{\hat{G}} = \Omega_{a,e}^{\hat{G}} = \Omega_{b,d}^{\hat{G}} = \Omega_{b,e}^{\hat{G}} = \Omega_{c,d}^{\hat{G}} = \Omega_{c,e}^{\hat{G}} = \Omega_{d,e}^{\hat{G}} = \Omega_{d,g}^{\hat{G}} = \Omega_{d,h}^{\hat{G}} = \Omega_{e,g}^{\hat{G}} = \Omega_{e,h}^{\hat{G}} = 0.02$, $\Omega_{a,f}^{\hat{G}} = \Omega_{b,f}^{\hat{G}} = \Omega_{c,f}^{\hat{G}} = \Omega_{d,f}^{\hat{G}} = \Omega_{e,f}^{\hat{G}} = \Omega_{f,g}^{\hat{G}} = \Omega_{f,h}^{\hat{G}} = 0.01$ and $\Omega_{g,h}^{\hat{G}} = 0.2$. Thus $\Omega_{g,h}^{\hat{G}} = 0.2$ is representing the maximum Ω between computers g and h . Therefore, computers g and h are best computers in performance among all other computers and sharing maximum data with all other computers in a network.

6 Comparative analysis

Here, we are going to compare our model with the existing model. In Fig 9 a FIG is indicating a highway system of eight different cities $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ and c_8 in which the edges are indicating the roads joining these cities and I_p are expressing flow of traffic from one city to another city. For example, an $I_p(c_1, c_1 c_2)$ is showing the flow of traffic from city c_1 to city c_2 and an $I_p(c_2, c_1 c_2)$ is showing the flow of traffic from city c_2 to city c_1 through road $c_1 c_2$. The MSV of the edges is showing the traffic flow (bikes, cars, vehicles, heavy vehicles) among different cities and the MSV of $(c_1, c_1 c_2)$ is specifying the flow of traffic from city c_1 to city c_2 and $(c_2, c_1 c_2)$ is proclaiming the flow of traffic from city c_2 to city c_1 through road $c_1 c_2$. Now, if we assign MSVs to all the edges and all I_p of FIG provided in Fig 9 is equal to 1 we get a graph. In

the case of graph the Ω between each pair of vertices is $\Omega_{c_1,c_2}^{\tilde{G}} = \Omega_{c_1,c_8}^{\tilde{G}} = \Omega_{c_2,c_8}^{\tilde{G}} = \Omega_{c_1,c_3}^{\tilde{G}} = \Omega_{c_1,c_4}^{\tilde{G}} = \Omega_{c_1,c_5}^{\tilde{G}} = \Omega_{c_1,c_6}^{\tilde{G}} = \Omega_{c_1,c_7}^{\tilde{G}} = \Omega_{c_2,c_3}^{\tilde{G}} = \Omega_{c_2,c_4}^{\tilde{G}} = \Omega_{c_2,c_5}^{\tilde{G}} = \Omega_{c_2,c_6}^{\tilde{G}} = \Omega_{c_2,c_7}^{\tilde{G}} = \Omega_{c_3,c_4}^{\tilde{G}} = \Omega_{c_3,c_5}^{\tilde{G}} = \Omega_{c_3,c_6}^{\tilde{G}} = \Omega_{c_3,c_7}^{\tilde{G}} = \Omega_{c_3,c_8}^{\tilde{G}} = \Omega_{c_4,c_5}^{\tilde{G}} = \Omega_{c_4,c_6}^{\tilde{G}} = \Omega_{c_4,c_7}^{\tilde{G}} = \Omega_{c_4,c_8}^{\tilde{G}} = \Omega_{c_5,c_6}^{\tilde{G}} = \Omega_{c_5,c_7}^{\tilde{G}} = \Omega_{c_5,c_8}^{\tilde{G}} = \Omega_{c_6,c_7}^{\tilde{G}} = \Omega_{c_6,c_8}^{\tilde{G}} = 1$. Since in case of graph the Ω between each pair of vertices is equal to 1.

Therefore, we are unable to find the roads which are becoming a main reason of maximum accidents. Hence our model is better than the previous one.

Similarly, in Fig 10 a FIG is representing a network of different computers. Computers are sharing data. We want to find which computer/computers are best in performance among all other computers and sharing maximum data with all other computers in a network. This can be done by computing Ω between each pair of computers in a network. The pair of computers which have a maximum Ω will be the required computers transferring maximum data to all other computers in a network. The vertices are showing the eight distinct computers in a network. The MSV of the vertices is indicating data store in each of these computers, the MSV of the edges is demonstrating the total amount of data that can be transferred from one computer to another computer and the MSV of the I_p is representing the amount of data which one computer is transferring to another computer. For example, an $I_p(a, ab)$ is indicating the transfer of data from computer a to computer b and an $I_p(b, ab)$ is showing the transfer of data from computer b to computer a .

Now, if we assign MSVs to all the edges and all I_p of the FIG shown in Fig 10 we get a graph. In the case of graph the Ω between each pair of vertices is $\Omega_{a,b}^{\tilde{G}} = \Omega_{a,c}^{\tilde{G}} = \Omega_{a,g}^{\tilde{G}} = \Omega_{a,h}^{\tilde{G}} = \Omega_{b,c}^{\tilde{G}} = \Omega_{b,g}^{\tilde{G}} = \Omega_{b,h}^{\tilde{G}} = \Omega_{c,g}^{\tilde{G}} = \Omega_{c,h}^{\tilde{G}} = \Omega_{a,d}^{\tilde{G}} = \Omega_{a,e}^{\tilde{G}} = \Omega_{b,d}^{\tilde{G}} = \Omega_{b,e}^{\tilde{G}} = \Omega_{c,d}^{\tilde{G}} = \Omega_{c,e}^{\tilde{G}} = \Omega_{d,e}^{\tilde{G}} = \Omega_{d,g}^{\tilde{G}} = \Omega_{d,h}^{\tilde{G}} = \Omega_{e,g}^{\tilde{G}} = \Omega_{e,h}^{\tilde{G}} = \Omega_{a,f}^{\tilde{G}} = \Omega_{b,f}^{\tilde{G}} = \Omega_{c,f}^{\tilde{G}} = \Omega_{d,f}^{\tilde{G}} = \Omega_{e,f}^{\tilde{G}} = \Omega_{f,g}^{\tilde{G}} = \Omega_{f,h}^{\tilde{G}} = \Omega_{g,h}^{\tilde{G}} = 1$. Therefore, previous model is not helpful to find which computer/computers are transferring maximum amount of data. Thus, our model is more effective and beneficial than the previous one.

7 Conclusion

In this article, we advanced the theory of FIGs. The notion of connectivity is indivisible from the theory of FIGs. There are a variety of parameters that command the connectivity of a network. In this article, the authors attempted to make up a new connectivity idea named as Ω , $FICCV$, $FICB$, and $FICCP$ in FIGs. Ω of various FIG theoretic structures are examined. As the number of FICs grows, the cyclic accessibility from one vertex to another vertex enhances. This benefits to upgrading the trustability of any network. The formula to compute Ω_I of any FIG is discussed with a variety of examples. The Ω_I and Ω_{AI} of FIGs are two frameworks associated with the cyclic accessibility of networks. Ω_{AI} represents the average strength of the cyclic flow in a network. $CCIV$, CNV , and $CCDV$ three different types of vertices are also introduced. The criterion to check $CCIV$, CNV , and $CCDV$ is developed. An application of Ω of FIG in highway systems of different cities to reduce road accidents and in a computer network to find the best computers among all other computers in a network is also provided. A comparative analysis of our study with the existing study is also provided. More related ideas will be contemplated in the upcoming papers.

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