RESEARCH ARTICLE

Optimal designs of the side sensitive synthetic chart for the coefficient of variation based on the median run length and expected median run length

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Abstract

The side sensitive synthetic chart was proposed to improve the performance of the synthetic chart to monitor shifts in the coefficient of variation (γ), by incorporating the side sensitivity feature where successive non-conforming samples must fall on the same side of the control limits. The existing side sensitive synthetic- γ chart is only evaluated in terms of the average run length (ARL) and expected average run length ($EARL$). However, the run length distribution is skewed to the right, hence the actual performance of the chart may be frequently different from what is shown by the ARL and $EARL$. This paper evaluates the entire run length distribution by studying the percentiles of the run length distribution. It is shown that false alarms frequently happen much earlier than the in-control ARL (ARL_0), and small shifts are often detected earlier compared to the ARL_1 . Subsequently, this paper proposes an alternative design based on the median run length (MRL) and expected median run length ($EMRL$). The optimal design based on the MRL shows smaller out-of-control MRL (MRL_1), which shows a quicker detection of the out-of-control condition, compared to the existing design, while the results from the optimal design based on the $EMRL$ is similar to that of the existing designs. Comparisons with the synthetic- γ chart without side sensitivity shows that side sensitivity reduces the median number of samples required to detect a shift and reduces the variability in the run length. Finally, the proposed designs are implemented on an actual industrial example.

1. Introduction

Control charts are useful tools to detect the presence of assignable cause(s) which results in an out-of-control process. By convention, control charts monitor the process mean (μ) and/or standard deviation (σ), where shifts in μ and/or σ shows the presence of assignable cause(s). For example, readers can refer to the studies by Aslam et al. [1], Riaz et al. [2], Abujiya et al. [3], and many others.

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However, conventional charts in the preceding paragraph could not be used to monitor all types of processes. This is because not all processes have a constant μ . Furthermore, σ may change according to μ . For such processes, monitoring μ and/or σ will result in dubious conclusions, since shifts in μ and/or σ does not mean that the process is out-of-control. Kang et al. [4] proposed to monitor such processes through the coefficient of variation (γ), where $\gamma = \frac{\sigma}{\mu}$. Assignable cause(s) is detected when there is a shift in the linear relationship between σ and μ . Kang et al. [4] monitored γ only through the present sample, hence the chart is not sensitive towards small and moderate shifts in γ .

Since the first γ chart was proposed by Kang et al. [4], numerous new and better charts are proposed to monitor γ . Hong et al. [5] was the first to propose an Exponentially Weighted Moving Average (EWMA) γ chart, followed by numerous improvements on the EWMA chart by Castagliola et al. [6], Zhang et al. [7] and Zhang et al. [8]. Besides the EWMA chart, other charts proposed to monitor γ include the run rules chart [9], side sensitive group runs (SSGR) chart [10] and run sum chart [11]. γ charts with variable charting parameters was first proposed by Castagliola et al. [12], who varied the sampling interval. Subsequently, the variable sample size γ chart [13, 14], variable sample size and sampling interval γ chart [15], variable sampling interval EWMA γ chart [16], variable parameters γ chart [17], and variable sample size EWMA γ chart [18] are proposed. Charts monitoring γ that is designed by considering measurement errors are also proposed Yeong et al. [19], Tran et al. [20], Tran et al. [21] and Saha et al. [22].

Calzada and Scariano [23] proposed the synthetic chart to monitor γ . The synthetic chart waits until the second point to fall outside the control limits before deciding whether the process is in-control or out-of-control. Samples that fall inside the control limits are called conforming samples, while those that fall outside the control limits are called non-conforming samples. If two successive non-conforming samples happen too close to each other, the process is out-of-control. Calzada and Scariano [23] showed that the synthetic- γ chart outperforms the Shewhart- γ chart for all shift sizes, but does not outperform the Exponentially Weighted Moving Average (EWMA)- γ chart, proposed by Castagliola et al. [6], for small and moderate shift sizes.

The synthetic- γ chart does not differentiate between non-conforming samples that fall outside the upper control limit (UCL) and those that fall below the lower control limit (LCL). Hence, as long as the samples fall either outside the UCL or below the LCL , they are considered to be non-conforming samples. Successive non-conforming samples do not have to fall on the same side of the control limits. In other words, if the first non-conforming sample falls above the UCL , the next non-conforming sample can either fall above the UCL or below the LCL .

Subsequently, Yeong et al. [24] proposed the side sensitive synthetic- γ chart. In the side sensitive synthetic- γ chart, successive non-conforming samples must fall on the same side of the control limits. For example, if the first non-conforming sample falls above the UCL , the next non-conforming sample must also fall above the UCL . Samples that fall below the LCL are not considered to be non-conforming in this case. The side sensitive synthetic- γ chart is shown to result in a significant improvement over the performance of the synthetic- γ chart proposed by Calzada and Scariano [23]. Furthermore, unlike the synthetic- γ chart, the side sensitive synthetic- γ chart showed comparable or better performance than the EWMA- γ chart for most shift sizes, except for very small shift sizes.

In Yeong et al. [24], the performance of the side sensitive synthetic- γ chart is evaluated only in terms of the average run length (ARL) and expected average run length ($EARL$). There are two types of ARL , the in-control ARL (ARL_0), which is the average number of samples taken until a false alarm occurs, and the out-of-control ARL (ARL_1), which is the average number of

samples taken until a shift of a specific magnitude is detected. The ARL_1 requires the shift size to be specified. It is difficult to specify the shift size in a lot of practical scenarios, hence the side sensitive synthetic- γ chart is also evaluated in terms of the $EARL$, which is the expected average number of samples taken to detect a shift that is specified as a range of values.

Evaluating a chart's performance based solely on the ARL might lead to confusion on the actual performance of the chart [25]. This is because, for in-control processes or out-of-control processes with small shift sizes, the run length distribution is highly skewed to the right [26]. For a right-skewed distribution, the median is smaller than the mean, hence the median run length (MRL) will be smaller than the ARL . When the in-control MRL (MRL_0) is smaller than the ARL_0 , more than 50% of the in-control run lengths will be smaller than the ARL_0 . In other words, for more than 50% of the time, the false alarm will happen before what is indicated by the ARL_0 . When this happens, practitioners would have reduced confidence towards the ARL_0 as a performance measure. Hence, in this paper, the performance of the side sensitive synthetic- γ chart is evaluated based on its' entire run length distribution to have a clearer idea of its' actual performance. An analysis of the in-control and out-of-control percentiles (for known shift sizes) and expected percentiles (for unknown shift sizes) will be conducted.

The optimal charting parameters of the existing side sensitive synthetic- γ chart is obtained based on two designs, where the first design optimizes the ARL_1 (for known shift sizes) while the second optimizes the $EARL$ (for unknown shift size). For both designs, constraints in ARL_0 needs to be satisfied. However, since the run length distribution is skewed to the right (for in-control and out-of-control run lengths for small shift sizes), alternative designs are proposed to obtain the optimal charting parameters of the side sensitive synthetic- γ chart. In this paper, alternative designs are proposed where the optimal charting parameters are obtained to minimize the out-of-control MRL (MRL_1) and the expected MRL ($EMRL$), subject to constraints in the MRL_0 . This is because the MRL is a more accurate measure of performance when the run length is skewed.

Designs based on the MRL are available for several charts in the literature, among some of the more recent ones are studies on the MRL performance of the synthetic $\bar{X}$ chart by Hu et al. [27], optimal designs based on the MRL and/or $EMRL$ for the one-sided exponential CUSUM chart [28], one-sided exponential EWMA chart [29], EWMA- γ chart [30], EWMA $\bar{X}$ chart [31], variable sample size $\bar{X}$ chart [32], synthetic np chart [33], multivariate synthetic $|S|$ chart [34], double sampling $\bar{X}$ chart [35], and many others. However, the design of the side sensitive synthetic- γ chart is not available in the literature. This paper will fill this gap.

The rest of the paper is organized as follows. The next section gives an overview of the side sensitive synthetic- γ chart, as well as the formulae to evaluate the ARL , standard deviation of the run length ($SDRL$), $EARL$ and the percentiles of the run length distribution. Section 3 analyses the percentiles of the run length distribution by adopting the optimal charting parameters by Yeong et al. [24]. Subsequently, Section 4 proposes the algorithms to obtain the optimal charting parameters based on the MRL_1 and $EMRL$, and illustrates the optimal charting parameters, MRL_1 and $EMRL$ based on several numerical examples. This is followed by a comparison with the synthetic- γ chart without side sensitivity in Section 5, and the implementation of the proposed designs on an actual industrial example in Section 6. Finally, some concluding remarks are given in Section 7.

2. Side sensitive synthetic- γ chart

The synthetic- γ chart works by waiting until the second sample to fall outside the control limits before deciding whether the process is in-control or out-of-control. For the synthetic- γ chart without the side sensitivity feature, when the sample coefficient of variation ($\hat{\gamma}$) falls

Fig 1. The CRL sub-chart of the side-sensitive synthetic- γ chart.

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within the UCL and LCL , the sample is a conforming sample, while if $\hat{\gamma} > UCL$ or $\hat{\gamma} < LCL$, the sample is a non-conforming sample. The number of conforming samples between two successive non-conforming samples (including the ending non-conforming sample) is referred to as the conforming run length (CRL). For example, when there are three conforming samples between two successive non-conforming samples, the $CRL = 4$. When $CRL \leq L$, where L is a pre-determined threshold set by the practitioner, the synthetic- γ chart will produce an out-of-control signal. To give the synthetic- γ chart a head-start, the first CRL counts the number of conforming samples until the first non-conforming sample appears.

The main difference between the side sensitive synthetic- γ chart and the synthetic- γ chart is that successive non-conforming samples must fall on the same side of the control limits. For example, by referring to the CRL sub-chart of the side sensitive synthetic- γ chart in Fig 1, since the first non-conforming sample (Sample 2) falls above the UCL , only samples that fall above the UCL are considered to be non-conforming samples. Although Sample 5 falls outside the control limits, it is still a conforming sample as it falls below the LCL . The next non-conforming only occurs in Sample 7. Hence, the $CRL = 5$.

The LCL and UCL of the side sensitive synthetic- γ chart are computed as

$$LCL = \mu_0(\hat{\gamma}) - K\sigma_0(\hat{\gamma}), \quad (1)$$

and

$$UCL = \mu_0(\hat{\gamma}) + K\sigma_0(\hat{\gamma}), \quad (2)$$

where K represents the control limit coefficient, while $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$ are the in-control mean

and standard deviation of $\hat{\gamma}$, respectively. Although $\mu_0(\hat{\gamma})$ and $\sigma_0(\hat{\gamma})$ have no closed forms, the following approximations by Reh and Scheffler [36] will be used, i.e.

$$\mu_0(\hat{\gamma}) \approx \gamma_0 \left[1 + \frac{1}{n} \left(\gamma_0^2 - \frac{1}{4} \right) + \frac{1}{n^2} \left(3\gamma_0^4 - \frac{\gamma_0^2}{4} - \frac{7}{32} \right) + \frac{1}{n^3} \left(15\gamma_0^6 - \frac{3\gamma_0^4}{4} - \frac{7\gamma_0^2}{32} - \frac{19}{128} \right) \right] \quad (3)$$

and

$$\sigma_0(\hat{\gamma}) \approx \gamma_0 \sqrt{\frac{1}{n} \left(\gamma_0^2 + \frac{1}{2} \right) + \frac{1}{n^2} \left(8\gamma_0^4 + \gamma_0^2 + \frac{3}{8} \right) + \frac{1}{n^3} \left(69\gamma_0^6 + \frac{7\gamma_0^4}{2} + \frac{3\gamma_0^2}{4} + \frac{3}{16} \right)}, \quad (4)$$

where γ_0 and n are the in-control γ and sample size, respectively.

A Markov chain approach is adopted to evaluate the *ARL*, *SDRL*, *EARL* and the percentiles of the run length distribution. The formulae to evaluate the *ARL*, *SDRL* and *EARL* is shown in Yeong et al. [24] and is reproduced in [S1 Appendix](#) for ease of reference.

To study the percentiles of the run length distribution, the probability mass function (p.m.f.) and cumulative distribution function (c.d.f.) of the run length needs to be obtained. The p.m.f. and c.d.f. of the run length for the side sensitive synthetic- γ chart can be computed as [37]

$$f_{RL}(l) = P(RL = l) = \mathbf{q}^T(\mathbf{Q}^{l-1})\mathbf{r}, \quad (5)$$

and

$$F_{RL}(l) = P(RL \leq l) = \mathbf{1} - \mathbf{q}^T(\mathbf{Q}^{l-1})\mathbf{1}, \quad (6)$$

where $l \in \{1, 2, 3, \dots\}$, $\mathbf{q}$ and $\mathbf{1}$ are as defined in Equations (A5) and (A6), while $\mathbf{Q}$ and $\mathbf{r}$ are as defined in Equation (A1).

The $(100\theta)^{\text{th}}$ percentile of the run length distribution of the side sensitive synthetic- γ chart can be obtained from Eq (6) by searching for l_θ such that [26]

$$P(RL \leq l_\theta - 1) \leq \theta \text{ and } P(RL \leq l_\theta) > \theta, \quad (7)$$

where $0 < \theta < 1$. For example, the *MRL*, which is the 50th percentile of the run length, can be obtained from Eq (7) by setting $\theta = 0.5$. The in-control percentiles can be obtained by setting $\gamma = \gamma_0$ when computing the transition probabilities of $\mathbf{Q}$ through Equations (A2) to (A4), while the out-of-control percentiles are obtained by setting $\gamma = \tau\gamma_0$, where τ is the shift size.

The percentiles of the run length can only be obtained if τ can be known in advance. However, practitioners usually find it difficult to specify τ due to insufficient data. Furthermore, τ may not be deterministic and may vary according to some unknown stochastic model [6]. Under such conditions, the expected percentiles of the run-length distribution, $E(l_\theta)$ are evaluated instead. $E(l_\theta)$ does not require τ to be specified as a specific value. Instead, τ only needs to be specified as a range of possible values, $(\tau_{\min}, \tau_{\max})$. $E(l_\theta)$ can be computed as follows:

$$E(l_\theta) = \int_{\tau_{\min}}^{\tau_{\max}} f_\tau(\tau) l_\theta(\tau) d\tau, \quad (8)$$

where $f_\tau(\tau)$ is the probability density function (p.d.f.) of τ . It is assumed that $f_\tau(\tau)$ is a uniformly distributed continuous random variable over the interval $(\tau_{\min}, \tau_{\max})$ [6]. The integral in Eq (8) is approximated using the Gauss-Legendre quadrature [38].

3. Analysis of the percentiles of the run length distribution

In this section, the percentiles of the run length distribution are analysed based on the optimal charting parameters by Yeong et al. [24]. Yeong et al. [24] adopted two types of designs. In the first design, the optimal charting parameters are obtained to minimize ARL_1 , subject to constraints in the ARL_0 , while the second design obtains the optimal charting parameters to minimize the $EARL$, subject to the same constraints in ARL_0 . For ease of reference, the first and second designs are referred to as ARL -based and $EARL$ -based designs, respectively. Both the in-control and out-of-control run length percentiles are analysed in this section.

Table 1 shows the optimal charting parameters (L , LCL and UCL), the ARL_1 and $SDRL_1$ of the ARL -based design of the side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$. Note that $\tau > 1$ indicates an upward shift in γ_0 , while $\tau < 1$ indicates a downward shift. Practitioners are often more interested in detecting an upward shift, since an upward shift results in a larger $\frac{\sigma}{\mu}$ ratio which shows that there is increased variability in the process due to a relatively large σ compared to μ . The increased variability is usually due to the presence of assignable cause(s). It is important to detect the presence of such assignable cause(s) so that they can be removed to reduce variability in the process. Comparatively, detecting a downward shift is less crucial as a downward shift shows decreased variability. Thus, this paper only considers $\tau > 1$.

Table 1. Optimal charting parameters and the corresponding ARL_1 and $SDRL_1$ values for the ARL -based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$.

τ	L	LCL	UCL	ARL_1	$SDRL_1$
$n = 5$					
1.1	42	0.0017	0.0924	64.74	84.69
1.2	23	0.0039	0.0902	21.35	27.11
1.3	15	0.0055	0.0885	10.18	12.25
1.5	8	0.0080	0.0860	4.18	4.44
2.0	4	0.0109	0.0832	1.72	1.27
τ	L	LCL	UCL	ARL_1	$SDRL_1$
$n = 7$					
1.1	37	0.0116	0.0843	52.13	67.97
1.2	19	0.0136	0.0824	15.75	19.66
1.3	12	0.0150	0.0810	7.30	8.47
1.5	7	0.0166	0.0794	3.02	2.89
2.0	3	0.0192	0.0767	1.38	0.82
τ	L	LCL	UCL	ARL_1	$SDRL_1$
$n = 10$					
1.1	35	0.0193	0.0780	41.27	53.44
1.2	16	0.0210	0.0762	11.43	13.91
1.3	10	0.0221	0.0752	5.19	5.68
1.5	5	0.0238	0.0735	2.22	1.91
2.0	3	0.0250	0.0723	1.17	0.47
τ	L	LCL	UCL	ARL_1	$SDRL_1$
$n = 15$					
1.1	31	0.0259	0.0724	31.15	39.93
1.2	13	0.0274	0.0709	7.84	9.14
1.3	7	0.0285	0.0697	3.53	3.61
1.5	4	0.0296	0.0687	1.63	1.13
2.0	2	0.0309	0.0673	1.05	0.24

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Table 2. In-control percentiles of the run length distribution for the ARL-based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$.

	5 th	10 th	20 th	30 th	40 th	50 th	60 th	70 th	80 th	90 th	95 th
τ	$n = 5$										
1.1	6	13	26	41	125	211	316	451	641	967	1293
1.2	5	10	20	74	143	225	325	454	636	947	1258
1.3	4	8	26	84	150	229	325	448	623	921	1220
1.5	3	6	41	97	161	237	329	449	618	906	1194
2.0	2	4	54	108	171	244	335	451	615	895	1176
τ	$n = 7$										
1.1	6	12	24	55	128	211	313	445	631	948	1266
1.2	5	9	18	80	148	228	326	453	631	936	1242
1.3	4	7	34	90	156	233	328	450	622	917	1211
1.5	3	6	46	102	166	242	336	456	625	914	1203
2.0	2	12	59	111	172	244	332	446	606	880	1154
τ	$n = 10$										
1.1	6	12	24	63	133	216	318	449	634	950	1266
1.2	4	8	27	84	150	228	323	446	619	915	1210
1.3	4	7	41	98	164	242	337	460	633	928	1224
1.5	3	5	53	107	170	243	334	450	614	895	1175
2.0	2	14	62	116	178	251	341	457	621	900	1180
τ	$n = 15$										
1.1	6	12	24	72	141	224	325	455	638	951	1265
1.2	4	8	37	94	160	238	334	457	630	927	1223
1.3	3	6	48	102	165	239	329	445	609	890	1170
1.5	3	11	58	111	173	246	335	450	612	888	1165
2.0	2	20	65	117	177	247	334	446	603	872	1141

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The optimal charting parameters in Table 1 are obtained to minimize the ARL_1 , subject to $ARL_0 = 370.4$. For example, for $n = 5$ and $\tau = 1.1$, adopting the charting parameters $(L, LCL, UCL) = (42, 0.0017, 0.0924)$ will result in the smallest ARL_1 , while maintaining the ARL_0 as 370.4. The $(ARL_1, SDRL_1) = (64.74, 84.69)$ when these optimal charting parameters are adopted.

To get a better picture on the actual run length performance of the ARL-based side sensitive synthetic- γ chart, the entire in-control and out-of-control run length distribution is analyzed. Table 2 shows the 5th until 95th in-control percentiles of the side sensitive synthetic- γ chart, while Table 3 shows the corresponding out-of-control percentiles. The percentiles are obtained by substituting the optimal charting parameters in Table 1, together with the corresponding γ_0 , n and τ , into Eq (7).

Analysing the percentiles of the run length distribution will give us a better idea on the actual run length performance of the chart. For example, by referring to Table 2, when the optimal charting parameters for the ARL-based design is adopted, the MRL_0 obtained is between 211 to 251, with smaller values obtained for smaller τ and n . This is significantly different from the ARL_0 of 370.4, which shows that for 50% of the time, the false alarm will happen much earlier than the 370th sample. When practitioners design the chart based on an ARL_0 of 370.4, they would expect that most of the time, false alarms only happen by the 370th sample. However, this is not the case from the MRL_0 values obtained in Table 2. In fact, 370 falls between the 60th and 70th percentile. This shows the in-control run length distribution is skewed to the right, and interpreting the frequency of false alarms based purely on the ARL_0 is not accurate. Furthermore, studying the difference between the extreme percentiles (for

Table 3. Out-of-control percentiles of the run length distribution for the ARL-based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$.

	5 th	10 th	20 th	30 th	40 th	50 th	60 th	70 th	80 th	90 th	95 th
τ	$n = 5$										
1.1	3	5	10	15	21	29	38	73	109	175	240
1.2	1	2	4	6	9	12	15	20	35	56	78
1.3	1	1	2	3	5	6	8	10	13	26	35
1.5	1	1	1	2	2	3	4	4	6	8	14
2.0	1	1	1	1	1	1	2	2	2	3	4
τ	$n = 7$										
1.1	2	4	8	13	18	24	32	57	86	139	191
1.2	1	2	3	5	7	9	12	15	23	41	57
1.3	1	1	2	3	3	5	6	8	10	18	25
1.5	1	1	1	1	2	2	3	3	4	6	9
2.0	1	1	1	1	1	1	1	1	2	2	3
τ	$n = 10$										
1.1	2	3	7	11	15	20	27	35	69	110	152
1.2	1	1	2	4	5	7	9	11	15	29	40
1.3	1	1	1	2	3	3	4	6	7	10	18
1.5	1	1	1	1	1	2	2	2	3	4	5
2.0	1	1	1	1	1	1	1	1	1	2	2
τ	$n = 15$										
1.1	2	3	6	9	12	16	21	28	52	83	115
1.2	1	1	2	3	4	5	6	8	11	20	27
1.3	1	1	1	1	2	2	3	4	5	7	11
1.5	1	1	1	1	1	1	1	2	2	3	4
2.0	1	1	1	1	1	1	1	1	1	1	1

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example, the difference between the 5th and 95th percentile) enables practitioners to have a better idea on the variation of the run length.

A similar trend is observed for the out-of-control percentiles, especially for small values of τ . In particular, there is a significant difference between the ARL_1 in Table 1 with the MRL_1 in Table 3. For example, for $n = 5$ and $\tau = 1.1$, Table 3 shows an MRL_1 of 29, but the ARL_1 is 64.74 from Table 1. This shows that for more than 50% of the time, the out-of-control condition will be detected before what is indicated by the ARL_1 . In most cases, the ARL_1 lies close to the 70th percentile. Hence, evaluating the side sensitive synthetic- γ chart based purely on the ARL_1 will result in an inaccurate representation of the actual performance of the chart when it is implemented since in most cases the shift is detected earlier than the ARL_1 .

From Table 3, the difference between the extreme percentiles reduces as τ and n increases. This shows that there is less variation in the out-of-control run lengths for larger τ and n . For example, for $n = 5$ and $\tau = 1.1$, the difference between the 5th and 95th percentile is 237, while for $n = 5$ and $\tau = 2.0$, the corresponding difference is only 3. Similarly, for $n = 15$ and $\tau = 1.1$, the difference between the extreme percentiles is 113 (compared to the corresponding difference of 237 for $n = 5$ and $\tau = 1.1$).

The percentiles in Tables 2 and 3 can only be obtained if τ can be specified in advance. Since τ cannot be specified in certain practical scenarios, this section also analyses the expected percentiles. In this paper, $(\tau_{\min}, \tau_{\max})$ is set as (1, 2]. Table 4 shows the optimal charting parameters (L , LCL and UCL) and the $EARL$ of the $EARL$ -based design of the side sensitive synthetic- γ chart for $\gamma_0 = 0.05$ and $n \in \{5, 7, 10, 15\}$. Tables 5 and 6 show the in-control percentiles and out-

Table 4. Optimal charting parameters and the corresponding EARL values for the EARL-based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $(\tau_{\min}, \tau_{\max}) = (1, 2]$.

n	L	LCL	UCL	$EARL$
5	25	0.0036	0.0905	16.90
7	25	0.0128	0.0832	13.73
10	27	0.0199	0.0774	11.16
15	29	0.0260	0.0723	8.85

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Table 5. In-control percentiles of the run length distribution for the EARL-based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $(\tau_{\min}, \tau_{\max}) = (1, 2]$.

n	5 th	10 th	20 th	30 th	40 th	50 th	60 th	70 th	80 th	90 th	95 th
5	5	10	21	71	140	222	323	452	635	947	1259
7	5	10	21	71	140	222	322	451	633	943	1254
10	5	10	22	70	139	219	318	446	625	932	1239
15	6	11	24	74	144	226	327	458	641	955	1268

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Table 6. Out-of-control expected percentiles of the run length distribution for the EARL-based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $(\tau_{\min}, \tau_{\max}) = (1, 2]$.

n	5 th	10 th	20 th	30 th	40 th	50 th	60 th	70 th	80 th	90 th	95 th
5	1.24	1.65	2.56	3.82	5.84	9.02	13.58	18.85	27.82	43.04	57.82
7	1.15	1.49	2.29	3.32	4.96	7.33	10.90	15.56	22.63	34.49	47.04
10	1.15	1.39	2.15	2.87	4.07	6.06	8.71	12.61	18.02	27.81	37.37
15	1.12	1.36	1.90	2.59	3.24	4.76	7.14	9.83	14.63	22.05	29.92

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of-control expected percentiles, respectively, when the charting parameters in Table 4 are adopted. The expected percentiles are obtained by substituting the optimal charting parameters in Table 4, together with the corresponding γ_0 , n and $(\tau_{\min}, \tau_{\max}) = (1, 2]$, into Eq (8).

The in-control percentiles in Table 5 show a similar trend as the ARL-based design. The MRL_0 ranges from 219 to 226, which is significantly smaller than the ARL_0 of 370.4. From Table 6, the $EMRL$ is also significantly smaller than the $EARL_1$. For example, for $n = 5$, the $EMRL$ is 9.02, while the corresponding $EARL$ is 16.90 from Table 4. The expected percentiles in Table 6 decreases for larger n . Similarly, the difference between the extreme expected percentiles decreases for larger n , which shows less variation in the expected percentiles for larger n .

4. MRL and EMRL-based design of the side sensitive synthetic- γ chart

As the MRL and $EMRL$ provide more accurate results than the ARL and $EARL$ when the run length distribution is skewed, this section proposes an alternative design where the optimal charting parameters which minimize the MRL_1 and $EMRL$ are obtained, subject to constraints in the MRL_0 .

The following are the algorithms to obtain the optimal charting parameters based on the MRL -based design.

1. Determine the values for γ_0 , n and τ .
2. Initialize $L = 1$.

3. Solve Eq (7) for K by setting $l_{0.5} = \xi$ and $\tau = 1$. Then, calculate LCL and UCL from Eqs (1) and (2), respectively. This combination of (LCL, UCL) will result in $MRL_0 = \xi$.
4. By using the (LCL, UCL) in Step 3, numerically search for $l_{0.5}$ that satisfies Eq (7) for the γ_0 , n and τ determined in Step 1. The $MRL_1 = l_{0.5}$.
5. Increase L by 1.
6. Repeat Steps 3 to 5 until the MRL_1 for $L+1$ is larger than the MRL_1 for L . This (L, LCL, UCL) combination will be the optimal charting parameters for the MRL -based side sensitive synthetic- γ chart. If there is more than one combination of (L, LCL, UCL) with the smallest MRL_1 , the combination with the smallest value for the difference between $l_{0.95}$ and $l_{0.05}$ is chosen as the optimal charting parameters, where $l_{0.05}$ and $l_{0.95}$ are the 5th and 95th percentiles of the out-of-control run length distribution, respectively.

To ensure a fair comparison between the ARL -based and MRL -based design, ξ is set to be equivalent to the MRL_0 in Table 2. For example, by referring to the MRL_0 for $n = 5$ and $\tau = 1.1$ in Table 2, $\xi = 211$. Table 7 shows the optimal charting parameters for the MRL -based design, and the 5th percentile (Q05), MRL_1 , 95th percentile (Q95), ARL_1 and ARL_0 when these optimal charting parameters are adopted are also shown. For example, for $n = 5$ and $\tau = 1.1$, the optimal charting parameters for the MRL -based design are $(L, LCL, UCL) = (22.0043, 0.0898)$, and adopting these optimal charting parameters result in $(Q05, MRL_1, Q95, ARL_1, ARL_0) = (2.22, 231, 63.38, 350.42)$.

By comparing the optimal charting parameters in Table 1 with that in Table 7, it can be observed that the optimal L for the ARL -based design is generally larger for small values of τ and n compared to the MRL -based design. For example, for $n = 5$ and $\tau = 1.1$, the optimal $L = 42$ for the ARL -based design, while the optimal $L = 22$ for the MRL -based design. The smaller optimal L for the MRL -based design is also associated with a smaller conforming region, as shown by larger values of LCL and smaller values of UCL . For example, for $n = 5$ and $\tau = 1.1$, the optimal $(LCL, UCL) = (0.0017, 0.0924)$ for the ARL -based design, while the optimal $(LCL, UCL) = (0.0043, 0.0898)$ for the MRL -based design.

Next, comparing the MRL_1 in Table 3 with that in Table 7 shows that the MRL -based design results in smaller MRL_1 compared with that in the ARL -based design, especially for small values of τ . For example, for $n = 5$ and $\tau = 1.1$, the $MRL_1 = 29$ for the ARL -based design in Table 3, while the $MRL_1 = 22$ for the ARL -based design in Table 7. This shows that the MRL -based design results in better MRL_1 performance compared with the ARL -based design. Note that both designs have the same MRL_0 .

Both the MRL -based and ARL -based designs show similar ARL_1 . For example, for $n = 5$ and $\tau = 1.1$, the $ARL_1 = 64.74$ for the ARL -based design in Table 1, while the $ARL_1 = 63.38$ for the MRL -based design in Table 7. This shows that the MRL -based design results in a smaller median number of samples to detect the shift, but with a similar average number of samples to detect the shift.

The ARL_0 is set as 370.4 for the ARL -based design, while the ARL_0 for the MRL -based design in Table 7 is between 344.30 to 375.27. Note that the MRL -based design does not fix the value for ARL_0 . The ARL_0 for the MRL -based design is generally smaller than that of the ARL -based design, however, the difference is not large.

The MRL -based design in Table 7 can only be implemented if τ can be specified in advance. This paper also considers the $EMRL$ -based design for cases where τ is unknown. Similar steps as shown in paragraph 2 for the MRL -based design are adopted, but Steps 1, 4 and 6 are replaced with the following.

Table 7. Optimal charting parameters and the corresponding Q05, MRL_1 , Q95, ARL_1 and ARL_0 for the MRL -based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$.

τ	$n = 5$							
	L	LCL	UCL	Q05	MRL_1	Q95	ARL_1	ARL_0
1.1	22	0.0043	0.0898	2	22	231	63.38	350.42
1.2	8	0.0083	0.0858	1	8	81	22.58	350.87
1.3	4	0.0111	0.0829	1	4	42	11.78	348.18
1.5	4	0.0110	0.0831	1	2	15	4.44	360.02
2.0	4	0.0109	0.0832	1	1	4	1.72	370.36
τ	$n = 7$							
	L	LCL	UCL	Q05	MRL_1	Q95	ARL_1	ARL_0
1.1	18	0.0140	0.0820	2	18	186	51.10	344.30
1.2	6	0.0173	0.0787	1	6	62	17.42	350.38
1.3	3	0.0194	0.0766	1	3	31	8.93	350.55
1.5	8	0.0161	0.0798	1	2	8	3.04	375.27
2.0	3	0.0193	0.0767	1	1	3	1.38	366.77
τ	$n = 10$							
	L	LCL	UCL	Q05	MRL_1	Q95	ARL_1	ARL_0
1.1	16	0.0212	0.0761	2	15	151	41.38	347.30
1.2	7	0.0231	0.0742	1	5	43	12.04	351.20
1.3	11	0.0219	0.0754	1	3	17	5.23	379.19
1.5	3	0.0251	0.0722	1	1	7	2.29	364.56
2.0	2	0.0260	0.0713	1	1	2	1.17	373.35
τ	$n = 15$							
	L	LCL	UCL	Q05	MRL_1	Q95	ARL_1	ARL_0
1.1	13	0.0275	0.0708	1	12	117	32.12	353.27
1.2	3	0.0302	0.0681	1	3	34	9.64	356.54
1.3	9	0.0280	0.0702	1	2	9	3.55	369.80
1.5	3	0.0301	0.0681	1	1	3	1.63	368.30
2.0	1	0.0324	0.0659	1	1	1	1.08	363.63

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1. Determine the values for γ_0 , n , $\tau_{\min}$ and $\tau_{\max}$.
4. By using the (LCL, UCL) in Step 3, evaluate $E(l_{0.5})$ from Eq (8). The $EMRL = E(l_{0.5})$.
6. Repeat Steps 3 to 5 until the $EMRL$ for $L+1$ is larger than the $EMRL$ for L . This (L, LCL, UCL) combination will be the optimal charting parameters for the $EMRL$ -based side sensitive synthetic- γ chart.

Similar to the $EARL$ -based design, $(\tau_{\min}, \tau_{\max})$ is set as $(1, 2]$. To ensure a fair comparison between the $EARL$ -based and $EMRL$ -based design, ξ is set to be equivalent to the MRL_0 in Table 5. Table 8 shows the optimal charting parameters for $n \in \{5, 7, 10, 15\}$ and its

Table 8. Optimal charting parameters and the corresponding EQ05, $EMRL$, EQ95, $EARL$ and ARL_0 for the $EMRL$ -based side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $(\tau_{\min}, \tau_{\max}) = (1, 2]$.

n	L	LCL	UCL	EQ05	$EMRL$	EQ95	$EARL$	ARL_0
5	7	0.0089	0.0852	1.10	9.90	57.99	17.20	344.46
7	15	0.0144	0.0816	1.12	7.49	46.17	13.58	356.84
10	14	0.0215	0.0758	1.12	5.90	37.05	11.02	349.06
15	12	0.0276	0.0706	1.05	5.03	29.72	8.97	354.87

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corresponding 5th expected percentile (EQ05), *EMRL*, 95th expected percentile (EQ95), *EARL* and ARL_0 when these optimal charting parameters are adopted.

By comparing the optimal charting parameters for the *EMRL*-based design in Table 8 with the corresponding optimal charting parameters for the *EARL*-based design in Table 4, it can be observed that the optimal L in Table 8 is smaller than the optimal L in Table 4. Furthermore, the conforming region for the *EMRL*-based design is smaller than that of the *EARL*-based design, as shown by the larger LCL and smaller UCL in Table 8, compared to the LCL and UCL in Table 4. Minimal differences are shown between the EQ05, *EMRL* and EQ95 values of the *EARL* and *EMRL*-based designs, by comparing Tables 6 and 8. The *EARL* values in Tables 4 and 8 are also similar. Hence, the *EARL* and *EMRL*-based designs show similar performance. The ARL_0 of the *EMRL*-based design is slightly smaller than the ARL_0 of the *EARL*-based design, but the difference is not very large. Note that the ARL_0 of the *EARL*-based design is fixed as 370.4.

5. Comparisons

In this section, the *MRL* and *EMRL*-based designs of the side sensitive synthetic- γ chart is compared with the corresponding designs for the synthetic- γ chart without side sensitivity. To obtain the *MRL* and *EMRL*-based designs for the synthetic- γ chart without side sensitivity, a similar procedure as shown in Section 4 is adopted, but modified for the synthetic- γ chart without side sensitivity. Table 9 shows the Q05, MRL_1 and Q95 for the *MRL*-based designs of these two charts, while Table 10 shows the EQ05, $EMRL_1$ and EQ95 for the *EMRL*-based designs.

From Table 9, the side sensitive synthetic- γ chart shows smaller MRL_1 and Q95 compared to the synthetic- γ chart without side sensitivity, especially for small values of τ . For example, for $n = 5$ and $\tau = 1.1$, $(MRL_1, Q95) = (32, 343)$ for the synthetic- γ chart without side sensitivity, whereas $(MRL_1, Q95) = (22, 231)$ for the side sensitive synthetic- γ chart. This shows that incorporating side sensitivity reduces the median number of samples required to detect the shift, and at the same time reduces the variability in the run length due to a smaller difference between Q95 and Q05. This is consistent with the results by Yeong et al. (2021), which shows that the side sensitive synthetic- γ chart shows better *ARL* performance than the synthetic- γ chart without side sensitivity. From Table 10, a similar conclusion is reached for the *EMRL*-based design. Thus, it can be concluded that the side sensitive synthetic- γ chart shows better performance than the synthetic- γ chart without side sensitivity.

6. Illustrative example

This section shows the implementation of the *MRL* and *EMRL*-based design on an actual industrial example. The example was also adopted by Yeong et al. [24] who proposed the side sensitive synthetic- γ chart. The example is from a sintering process where compressed metal powder is heated to a temperature that allows bonding of the individual particles. The strength of the bond between particles is influenced by pore shrinkage [39]. One of the characteristics that is related to pore shrinkage is the pressure test drop time (T_{pd}) from 2 bar to 1.5 bar, which must be larger than 30 seconds.

Sintering steel with a heterogeneous microstructure and an irregular grain size will lead to an anomalous increase in the standard deviation of T_{pd} (σ_{pd}), which will result in a change in the correlation structure between the mean of T_{pd} (μ_{pd}) and σ_{pd} and subsequently results in shifts in the coefficient of variation of T_{pd} (γ_{pd}). Hence, the special cause can be detected by monitoring γ_{pd} . Furthermore, Castagliola et al. [6] through a regression study showed that

Table 9. Comparison of the Q05, MRL_1 and Q95 of the MRL -based synthetic- γ chart (without side sensitivity) and side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $\tau \in \{1.1, 1.2, 1.3, 1.5, 2.0\}$.

τ	$n = 5$					
	Synthetic- γ Chart			Side Sensitive Synthetic- γ Chart		
	Q05	MRL_1	Q95	Q05	MRL_1	Q95
1.1	3	32	343	2	22	231
1.2	1	10	106	1	8	81
1.3	1	5	52	1	4	42
1.5	1	2	23	1	2	15
2.0	1	1	4	1	1	4
τ	$n = 7$					
	Synthetic- γ Chart			Side Sensitive Synthetic- γ Chart		
	Q05	MRL_1	Q95	Q05	MRL_1	Q95
1.1	2	27	279	2	18	186
1.2	1	8	85	1	6	62
1.3	1	4	40	1	3	31
1.5	1	2	15	1	2	8
2.0	1	1	3	1	1	3
τ	$n = 10$					
	Synthetic- γ Chart			Side Sensitive Synthetic- γ Chart		
	Q05	MRL_1	Q95	Q05	MRL_1	Q95
1.1	2	23	236	2	15	151
1.2	1	7	64	1	5	43
1.3	1	3	27	1	3	17
1.5	1	1	13	1	1	7
2.0	1	1	2	1	1	2
τ	$n = 15$					
	Synthetic- γ Chart			Side Sensitive Synthetic- γ Chart		
	Q05	MRL_1	Q95	Q05	MRL_1	Q95
1.1	2	18	189	1	12	117
1.2	1	5	45	1	3	34
1.3	1	2	18	1	2	9
1.5	1	1	4	1	1	3
2.0	1	1	1	1	1	1

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$\sigma_{pd} = \gamma_{pd} \times \mu_{pd}$, which provides additional evidence that the process can be monitored by detecting changes in γ_{pd} .

Table 11 (left) shows a Phase I dataset of $m = 20$ samples, each with a sample size $n = 5$. $\bar{X}_k$, S_k and $\hat{\gamma}_k$ denotes the sample mean, standard deviation and coefficient of variation,

Table 10. Comparison of the EQ05, $EMRL_1$ and EQ95 of the $EMRL$ -based synthetic- γ chart (without side sensitivity) and side sensitive synthetic- γ chart for $\gamma_0 = 0.05$, $n \in \{5, 7, 10, 15\}$ and $(\tau_{\min}, \tau_{\max}) = (1, 2]$.

n	Synthetic- γ Chart			Side Sensitive Synthetic- γ Chart		
	EQ05	$EMRL_1$	EQ95	EQ05	$EMRL_1$	EQ95
5	1.24	12.98	79.49	1.10	9.90	57.99
7	1.17	11.05	67.49	1.12	7.49	46.17
10	1.15	10.07	58.92	1.12	5.90	37.05
15	1.17	7.71	47.52	1.05	5.03	29.72

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Table 11. Phase I and Phase II datasets from a sintering process.

Phase I				Phase II			
k	$\bar{X}_k$	S_k	$\hat{\gamma}_k$	k	$\bar{X}_k$	S_k	$\hat{\gamma}_k$
1	664.2	268.9	0.405	1	906.4	476.0	0.525
2	705.6	308.6	0.437	2	805.1	493.9	0.613
3	1051.5	539.9	0.513	3	1187.2	1105.9	0.932
4	1047.3	359.0	0.343	4	663.4	304.8	0.459
5	618.2	136.3	0.220	5	1012.1	367.4	0.363
6	781.4	446.4	0.571	6	863.2	350.4	0.406
7	797.8	342.5	0.429	7	1561.0	1652.2	1.058
8	678.9	275.4	0.406	8	697.1	253.2	0.363
9	848.3	320.5	0.378	9	1024.6	120.9	0.118
10	1015.3	453.7	0.447	10	355.3	235.2	0.662
11	777.4	276.4	0.356	11	485.6	106.5	0.219
12	813.9	170.7	0.210	12	1224.3	915.4	0.748
13	716.9	397.4	0.554	13	1365.0	1051.6	0.770
14	937.6	421.2	0.449	14	704.0	449.7	0.639
15	915.1	331.9	0.363	15	1584.7	1050.8	0.663
16	873.2	285.0	0.326	16	1130.0	680.6	0.602
17	984.3	573.7	0.583	17	824.7	393.5	0.477
18	819.3	156.2	0.191	18	921.2	391.0	0.424
19	839.0	244.0	0.291	19	870.3	730.0	0.839
20	585.8	322.3	0.550	20	1068.3	150.8	0.141

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respectively, of the k^{th} sample, $k = 1, 2, \dots, 20$. The data is taken from Castagliola et al. [6]. Castagliola et al. [6] showed that the Phase I data were in-control, with an estimated in-control γ ($\hat{\gamma}_0$) of 0.417 based on a root-mean-square computation.

Table 11 (right) shows the Phase II data after the occurrence of a special cause that increased process variability. A shift of 25% in the coefficient of variation shows that something is wrong in the production of the parts. Hence, the side-sensitive synthetic- γ chart is designed to detect a shift of $\tau = 1.25$. The *MRL*-based design in Section 4 is adopted to obtain the optimal charting parameters, where $\xi = 250$. The optimal charting parameters are $(L, LCL, UCL) = (7, 0, 0.8418)$, which results in $(Q05, MRL_1, Q95) = (1, 7, 76)$. By comparison, the *ARL*-based design by Yeong et al. [24] results in $(Q05, MRL_1, Q95) = (1, 10, 68)$. Fig 2 shows the γ sub-chart of the side-sensitive synthetic- γ chart by adopting the *MRL*-based design.

From Fig 2, two non-conforming samples are detected, i.e. Samples 3 and 7, with *CRL* of 3 and 4, respectively. Since both *CRL*s are less than 7, out-of-control signals are produced at Samples 3 and 7. This shows that the side-sensitive synthetic- γ chart can show a quick detection of the special cause. The *ARL*-based design by Yeong et al. [24] also detected the out-of-control condition at Samples 3 and 7.

The *MRL*-based design can only be adopted if τ can be specified. As the exact size of shift caused by the special cause may not be known, this section also considers the *EMRL*-based design with $(\tau_{\min}, \tau_{\max}) = (1, 2]$. From the *EMRL*-based design in Section 4, the optimal charting parameters are $(L, LCL, UCL) = (8, 0, 0.8504)$, which results in $(EQ05, EMRL_1, EQ95) = (1.12, 12.43, 74.94)$. Note that as in the *MRL*-based design, $\xi = 250$. Fig 3 shows the γ sub-chart of the side-sensitive synthetic- γ chart by adopting the *EMRL*-based design. Similar to the *MRL*-based design, Samples 3 and 7 are detected as out-of-control samples.

Fig 2. The γ sub-chart of the MRL-based side-sensitive synthetic- γ chart applied to the sintering process (Phase II).

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7. Conclusions

This paper evaluates the performance of the side-sensitive synthetic- γ chart by studying its' entire run length distribution. An analysis of the run length distribution shows that false alarms frequently happens much earlier than that indicated by the ARL_0 . In addition, small shifts are often detected earlier compared to the ARL_1 . This will reduce the confidence towards the chart when the actual performance is frequently different from that indicated by the ARL .

Fig 3. The γ sub-chart of the EMRL-based side-sensitive synthetic- γ chart applied to the sintering process (Phase II).

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Hence, the performance of the side-sensitive synthetic- γ chart should not be evaluated only in terms of the *ARL*, but should be accompanied by the entire run length distribution. This paper also proposes alternative designs for the side-sensitive synthetic- γ chart based on the *MRL* and *EMRL*. Compared to designs based on the *ARL*, the proposed design based on the *MRL* shows better *MRL*₁ performance, i.e. it requires a smaller median number of samples to detect shifts in γ . For the proposed design based on the *EMRL*, the performance is similar to that based on the *EARL*. Comparisons with the synthetic- γ chart without side sensitivity shows that side sensitivity reduces the median number of samples required to detect a shift and reduces the variability in the run length. Implementation of the proposed designs on an actual industrial example shows that they are efficient in detecting out-of-control conditions.

Supporting information

S1 Appendix.
(DOCX)

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References

1. Aslam M, Shafqat A, Albassam M, Malela-Majika JC, Shongwe SC. A new control chart under uncertainty with applications in petroleum and meteorology. *PLoS One* 2021; 16(2): e0246185. <https://doi.org/10.1371/journal.pone.0246185> PMID: 33539442
2. Riaz M, Abid M, Nazir HZ, Abbasi SA. An enhanced nonparametric EWMA sign control chart using sequential mechanism. *PLoS One* 2019; 14(11): e0225330. <https://doi.org/10.1371/journal.pone.0225330> PMID: 31751403
3. Abujija MR, Riaz M, Lee MH. Enhanced cumulative sum charts for monitoring process dispersion. *PLoS One* 2015; 10(4): e0124520. <https://doi.org/10.1371/journal.pone.0124520> PMID: 25901356
4. Kang CW, Lee MS, Seong YJ, Hawkins DM. A control chart for the coefficient of variation. *J Qual Technol* 2007; 39(2): 151–158.
5. Hong EP, Kang CW, Baek JW, Kang HW. Development of cv control chart using EWMA technique. *J Soc Korea Ind Syst Eng* 2008; 31(4): 114–120.
6. Castagliola P, Celano G, Psarakis S. Monitoring the coefficient of variation using EWMA charts. *J Qual Technol* 2011; 43(3): 249–265.
7. Zhang J, Li Z, Chen B, Wang Z. A new exponentially weighted moving average control chart for monitoring the coefficient of variation. *Comp Ind Eng* 2014; 78: 205–212.
8. Zhang J, Li Z, Wang Z. Control chart for monitoring the coefficient of variation with an exponentially weighted moving average procedure. *Qual Reliab Eng Int* 2018; 34(2): 188–202.
9. Castagliola P, Achouri A, Taleb H, Celano G, Psarakis S. Monitoring the coefficient of variation using control charts with run rules. *Qual Technol Quant M* 2013; 10(1): 75–94.
10. You HW, Khoo MBC, Castagliola P, Haq A. Monitoring the coefficient of variation using the side sensitive group runs chart. *Qual Reliab Eng Int* 2016; 32(5): 1913–1927.
11. Teoh WL, Khoo MBC, Castagliola P, Yeong WC, Teh SY. Run-sum control charts for monitoring the coefficient of variation. *Eur J Oper Res* 2017; 257(1): 144–158.

12. Castagliola P, Achouri A, Taleb H, Celano G, Psarakis S. Monitoring the coefficient of variation using a variable sampling interval control chart. *Qual Reliab Eng Int* 2013; 29(8): 1135–1149.
13. Castagliola P, Achouri A, Taleb H, Celano G, Psarakis S. Monitoring the coefficient of variation using a variable sample size control chart. *Int J Adv Manuf Tech* 2015; 80: 1561–1576.
14. Yeong WC, Khoo MBC, Lim SL, Lee MH. A direct procedure for monitoring the coefficient of variation using a variable sample size scheme. *Commun Stat Simulat* 2017; 46(6): 4210–4225.
15. Khaw KW, Khoo MBC, Yeong WC, Wu Z. Monitoring the coefficient of variation using a variable sample size and sampling interval control chart. *Commun Stat Simulat* 2017; 46(7): 5772–5794.
16. Yeong WC, Khoo MBC, Tham LK, Teoh WL, Rahim MA. Monitoring the coefficient of variation using a variable sampling interval EWMA chart. *J Qual Tech* 2017; 49: 380–401.
17. Yeong WC, Lim SL, Khoo MBC, Castagliola P. Monitoring the coefficient of variation using a variable parameters chart. *Qual Eng* 2018; 30(2): 212–235.
18. Anis NBM, Yeong WC, Chong ZL, Lim SL, Khoo MBC. Monitoring the coefficient of variation using a variable sample size EWMA chart. *Comp Ind Eng* 2018; 126: 378–398.
19. Yeong WC, Khoo MBC, Lim SL, Teoh WL. The coefficient of variation chart with measurement error. *Qual Technol Quant M* 2017; 14(4): 353–377.
20. Tran KP, Heuchenne C, Balakrishnan N. On the performance of coefficient of variation charts in the presence of measurement errors. *Qual Reliab Eng Int* 2019; 35(1): 329–350.
21. Tran KP, Nguyen HD, Tran PH, Heuchenne C. On the performance of CUSUM control charts for monitoring the coefficient of variation with measurement errors. *Int J Adv Manuf Tech* 2019; 104: 1903–1917.
22. Saha S, Khoo MBC, Castagliola P, Haq A. Side sensitive modified group runs charts with and without measurement errors for monitoring the coefficient of variation. *Qual Reliab Eng Int* 2021; 37(2): 598–617.
23. Calzada ME, Scariano SM. A synthetic control chart for the coefficient of variation. *J Stat Comput Sim* 2013; 83(5): 853–867.
24. Yeong WC, Lee PY, Lim SL, Khaw KW, Khoo MBC. A side-sensitive synthetic coefficient of variation chart. *Qual Reliab Eng Int* 2021; 37(5): 2014–2033.
25. Montgomery DC. *Introduction to statistical quality control*. 8th ed. USA: Wiley; 2019.
26. Gan FF. An optimal design of EWMA control charts based on median run length. *J Stat Comput Sim* 1993; 45(3–4): 169–184.
27. Hu XL, Castagliola P, Tang AA, Zhou XJ, Zou. Conditional median run length performance of the synthetic $\bar{x}$ chart with unknown process parameters. *Qual Reliab Eng Int* 2020; 36(3): 1111–1131.
28. Qiao YL, Hu XL, Sun JS, Xu Q. Optimal design of one-sided exponential cumulative sum charts with known and estimated parameters based on the median run length. *Qual Reliab Eng Int* 2021; 37: 123–144.
29. Qiao YL, Hu XL, Sun JS, Xu Q. Optimal design of one-sided exponential EWMA charts with estimated parameters based on the median run length. *IEEE Access* 2019; 7: 76645–76658.
30. Teoh WL, Lim JY, Khoo MBC, Yeong WC. Optimal design of the EWMA charts for monitoring the coefficient of variation based on median run length and expected median run length. *J Test Eval* 2019; 47(1): 459–479.
31. Hu XL, Castagliola P, Tang A, Zhong JL. Guaranteed conditional performance of the median run length based EWMA $\bar{x}$ chart with unknown process parameters. *Commun Stat Simulat* 2019; forthcoming. <https://doi.org/10.1080/03610918.2019.1642485>
32. Teoh WL, Chong JK, Khoo MBC, Castagliola P, Yeong WC. Optimal designs of the variable sample size $\bar{x}$ chart based on median run length and expected median run length. *Qual Reliab Eng Int* 2017; 33(1): 121–134.
33. Lee MH, Khoo MBC. Optimal designs of synthetic np control chart based on median run length. *Commun Stat–Theor M* 2017; 46(17): 3034–3053.
34. Lee MH, Khoo MBC. Optimal designs of multivariate synthetic |S| control chart based on median run length. *Commun Stat–Theor M* 2017; 46(6): 3034–3053.
35. Teoh WL, Khoo MBC, Teh SY. Optimal designs of the median run length based double sampling $\bar{x}$ chart for minimizing the average sample size. *PLoS One* 2013; 8(7): e68580. <https://doi.org/10.1371/journal.pone.0068580> PMID: 23935873
36. Reh W, Scheffler B. Significance tests and confidence intervals for coefficients of variation. *Comput Stat Data An* 1996; 22(4): 449–452.

37. Latouche G, Ramaswami V. *Introduction to matrix analytic methods in stochastic modelling*. USA: ASA/Siam Series on Statistics and Applied Probability; 1999.
38. Kovvali N. *Theory and applications of Gaussian quadrature methods*. USA: Morgan & Claypool Publishers; 2011.
39. Kalpakjian S. *Manufacturing engineering and technology*. 2nd ed. New York: Addison Wesley; 1992.