RESEARCH ARTICLE

A study of the car-to-train assignment problem for rail express cargos in the scheduled and unscheduled train services network

Boliang Lin^{1*}, Jingsong Duan¹, Jiaxi Wang¹, Min Sun¹, Wengao Peng², Chang Liu¹, Jie Xiao¹, Siqi Liu¹, Jianping Wu¹

1 School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China, **2** Department of Transportation, China Railway Guangzhou Group Co., Ltd., Guangzhou, Guangdong, China

* blin@bjtu.edu.cn

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Abstract

A freight train service network generally involves two categories of trains: unscheduled trains, whose operating frequencies fluctuate with the freight demand, and scheduled trains, which are operated based on regular timetables similar to passenger trains. The timetables for scheduled trains are released to the public once determined, and they are not influenced by the freight demand. Typically, the total capacity of scheduled trains can satisfy the predicted average demand of express cargos. However, in practice, the demand always changes. Therefore, a method to assign the shipments to scheduled and unscheduled train services has become an important issue faced in railway transportation. This paper focuses on the coordinated optimization of rail express cargo assignment in a hybrid train services network. On the premise of fully utilizing the capacity of scheduled train services, we propose a car-to-train assignment model to reasonably assign rail express cargos to scheduled and unscheduled trains. The objective aims to maximize the profit of transporting the rail express cargos. The constraints include the capacity restriction on the service arcs, flow balance constraints, transportation due date constraints and logical relationship constraints among the decision variables. Furthermore, we discuss a linearization technique to convert the nonlinear transportation due date constraint into a linear constraint, making it possible to solve by a standard optimization solver. Finally, an illustrative case study based on the Beijing-Guangzhou Railway Line is carried out to demonstrate the effectiveness and efficiency of the proposed solution approach.

Introduction

In recent years, the competition between railway and highway transportation has become increasingly intense, especially for the long-distance transportation of high-value freight. The governments expect that more freight flow transported by highways should be diverted to the

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railways in order to reduce carbon emissions. For example, in Europe, 30% of road freight over 300 km is expected to shift to railways or waterways by 2030. The Ministry of Transport of China also recommends that the railways and waterways should support more freight transportation loads [1]. However, road freight transport is convenient and efficient currently, and many shippers tend to choose the road transportation mode. Therefore, as one of the largest and busiest railway systems, the China Railway has focused on the development of scheduled train services in past years in an effort to obtain a higher market share in high-value freight transportation. For example, on the official website of the China Railway, we can find 156 scheduled train services, including trains that are bound for West Asia and Europe. The scheduled trains in China can be divided into the following three levels according to the operating speed: 160 km/h, 120 km/h and 80 km/h (the speed of an ordinary freight train is usually less than 50 km/h). The operating speed of scheduled trains is fast, which is beneficial to guarantee the delivery period of the shipments. Nevertheless, the high cost of new express train may exceed the income of the attracted cargos and may be not cost-effective. In addition, according to the current practice, some train schedule plan is usually made on the basis of historical data and the experience of the staff. However, some of the train services attract fewer freight flows than expected. In practice, an unreasonable plan can be improved only when a failure has occurred. Based on our previous estimation, the transportation demand of high-value freight is very large and is far beyond the supply of the 156 train services. Therefore, in this situation, it is necessary to improve the railway freight transport service to attract more freight flow from the road.

Currently, the trial and error based scheduled train service plan provided adopted by the railway company is myopic and is far from optimal. Therefore, it is an important task to evaluate whether the train service plan is reasonable enough as well as globally optimize the car-to-train plan using state-of-the-art methods. These important tasks were traditionally done manually, but there is an increasing move toward automated software based on mathematical models and algorithms. If mathematical models and algorithms can be conducted before the application of the train service plan, the decision failure can be reduced significantly. Therefore, optimizing the car-to-train assignment for the rail express cargo is an important problem that needs to be addressed both theoretically and practically.

Related work

Generally, the core of the car-to-train assignment problem involves the two consolidation processes: the car-to-block assignment and the block-to-train assignment.

The objective of the blocking problem is to determine which blocks should be built at each yard and the assignment of cars to these blocks. The objective is to minimize the total cost. One of the first models was proposed by Bodin et al. [2], who developed a nonlinear model to determine a classification plan for all the classification yards, which can be viewed as a multi-commodity flow problem with capacity constraints in terms of the maximum number of blocks and the maximum car volume that can be handled. Newton et al. [3] modeled the railway blocking problem as a network design problem with the objective of minimizing the total mileage, handling and delay costs. They developed a column generation, branch-and-bound algorithm to solve the model. In recent studies, the blocking problem is regarded as a very large-scale, multicommodity, service network design problem with billions of decision variables. Different heuristic optimization algorithms have been proposed to solve the problem. Barnhart [4] considered the flow constraints on the nodes and arcs and proposed a Lagrangian relaxation heuristic algorithm to solve the problem. The model was tested on a major railroad, and the validity of the model and algorithm were verified. Yaghini [5] presented an algorithm

based on ant colony optimization to solve the blocking problem and compared the results with the solution generated with CPLEX software. The model was tested in Iran Railways, which reduced the operational cost considerably and saved time in shipping the freight. Gorman [6] addressed the joint train-scheduling and demand-flow problem for a major US freight railroad. A tabu-enhanced genetic search was used to find acceptable solutions, which consistently achieves better approximations to determine the optimum solution and maintains its performance as the problem size grows. Yaghini et al. [7] proposed a hybrid algorithm that integrated the Simplex method and simulated annealing to solve this problem. Ahuja et al. [8] developed the very large-scale neighborhood (VLSN) search algorithm to solve the blocking problem that can find the nearly optimal solution in one to two hours of computing time on a standard workstation computer. As the problem of air pollution is becoming increasingly severe compared to that of highway transport, the railway is a better choice with low energy consumption and carbon emissions. Lin et al. [1] and Liu et al. [9] considered the factor of carbon emissions in the railway network design problem and flow assignment model on railway and highway networks.

The block-to-train problem involves determining which train services need to be supplied at what frequency and assigning blocks to train services. Thomet [10] developed a cancellation procedure that gradually replaces direct trains with a series of intermediate train connections in order to minimize operation and delay costs. Kwona et al. [11] used a time-space network technique to improve a given blocking plan and the block-to-train assignment. The problem was formulated as a linear multicommodity flow problem, and the column generation technique was used as a solution approach. Jha et al. [12] provided two formulations for the block-to-train problem, including both an arc-based and a path-based time-space network; the path-based model was generally smaller than the arc-based model, and it can better handle practical constraints. The heuristic algorithm integrating a Lagrangian relaxation-based method and a greedy construction method was proposed to solve the path-based model, which can obtain an optimal solution within a few minutes of computational time. Xiao and Lin et al. [13] presented a comprehensive optimization model for the Train Formation Plan (TFP) network problem using both single-block trains and two-block trains. Compared to the single-block train service plan, they aimed to maximize the car-hour consumption savings at the yards. Based on the study of reference [13], Xiao et al. [14] defined an integer programming model to solve the block-to-train assignment problem aiming to maximize the total cost savings of the whole railroad network. The freight shipment demand for the railroad is increasing significantly and will cause severe congestion and inefficiency. Taesung and Ouyang [15] proposed a customized network assignment model that could capture traffic delays, and they adopted a convex combinations algorithm to determine the shipment routing equilibrium. Borndörfer et al. [16] also established a model aimed to minimize the sum of all expected delays and all running times, and they considered passenger trains.

Since the above two subproblems are interrelated, some researchers consider these two issues as an integrated problem and establish a car-to-train model directly to solve the train connection service problem. Keaton [17, 18] formulated the combined problem as a 0–1 mixed integer programming model that minimizes the sum of the train costs, car time costs, and classification yard costs, while not exceeding the limits on train size and yard volume. A heuristic approach based on Lagrangian relaxation was presented to solve the problem. Crainic et al. [19] formulated a general optimization model that takes into account the problem of routing traffic and scheduling train services. A heuristic algorithm was developed to solve this mixed-integer multicommodity problem. Haghani [20, 21] established a comprehensive model considering train routing and makeup and empty car routing in a space-time network. A heuristic decomposition technique was developed, which decomposes the problem into

smaller subproblems based on the type of decision variables. Lin et al. [22] combined the blocking problem and the train makeup problem to build a bi-level linear integer model to solve the train service network problem of the China railway system and the simulated annealing algorithm is applied to real-life instances. Zhu et al. [23] addressed the scheduled service network design problem by proposing a model that combines service selection and scheduling, car classification and blocking, train makeup, and the routing of time-dependent customer shipments based on a cyclic three-layer space-time network representation of the associated operations and decisions and their relations and time dimensions. A methodology integrating slope scaling, a dynamic block generation mechanism, long-term memory-based perturbation strategies, and an ellipsoidal search was proposed, which is efficient and robust and yields high-quality solutions. Fügenschuh et al. [24] presented an integer programming formulation for the freight car-to-train routing problem that arises at Deutsche Bahn. The model aims at finding the car routes with the most economical cost, which considers the train and car travel kilometers and the number of used sorting tracks. Based on the high-speed railway network, Su et al. [25] put forward a multiobjective optimization model including the average travel time of trains, energy consumption and user satisfaction. An improved GA was employed to solve the train routing problem, and an improved GA based on train control was applied to a large-scale network. Moreover, Zhu [26, 27] proposed an efficient algorithm based on the Kuhn-Munkres (K-M) algorithm to solve role assignment problem, which is similar to the car-to-train assignment problem to some extent.

In this paper, we study the car-to-train assignment problem of rail express cargos on the mixed scheduled and unscheduled train services network and conduct an illustrative case study based on the Beijing-Guangzhou Railway Line. The contributions of this work to the existing research literature can be summarized as follows:

1. The operating frequencies of the unscheduled trains fluctuate with the freight demand, while the scheduled trains are operated based on regular timetables similar to the passenger trains; this paper coordinately considers the train service network that involves both the unscheduled trains and the scheduled trains.
2. Based on the mixed scheduled and unscheduled train service network, a car-to-train assignment model is formulated to assign the rail express cargos. Taking into account the characteristics of high value-added goods, the transportation due date constraint is added to the model, which includes the transportation time on the arcs and the transfer time between the arcs.
3. Due to the limited capacity of the scheduled train arcs and the relatively sufficient capacity of the unscheduled train arcs, when considering the capacity constraint of the arcs, we mainly aim at making full use of the scheduled trains' capacities.

The remainder of this paper is organized as follows. Section 3 describes the car-to-train assignment problem of rail express cargos in detail. Section 4 provides a model formulation and its linearization techniques. A numerical example is conducted in Section 5. Finally, the conclusions of this paper are presented in Section 6.

Problem descriptions

In this paper, a car is a rail vehicle or a wagon used as loading units for measuring the volume of shipments, while the train is consisted of multiple cars with the same or different destinations. A car-to-train assignment problem is to determine how to assign the shipments (cars) into the corresponding train services.

Fig 1. The railway service network.

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A railway service network may consist of multiple types of train services, such as the local train services, the through train services and the express train services. Therefore, to transport a given shipment from its origin to its destination, we can use either a single type of train service or a combination of multiple train services. To better explain the overall transportation process, we provide more details using the example depicted in Fig 1.

In Fig 1, the red circles 1'~6' represent railway classification yards, the green circles 1'', 3'', 5'' and 6'' are railway logistics centers, and the golden diamonds s , s' and t denote big enterprises that have direct access (i.e., the enterprises' special rail lines) to railways. The blue arcs Arc 2 and Arcs 5~8 represent shuttle train services (formed at one classification yard and broken up at the adjacent yard), and the green arcs Arc 1 and Arc 3 are through train services (i.e., long-distance ordinary train services that are formed at one classification yard, pass through one or more yards, and are finally broken up at a relatively far yard). Moreover, the yellow dotted arcs Arc 4, Arc 9, Arc 10, and Arcs 14~23 are local train services (or pickup and delivery train services between adjacent yards that are powered by shunting engines), and the red arcs Arc 11, Arc 12 and Arc 13 represent express train service arcs, which belong to the same express train service 1''→6''. Since the express train service 1''→6'' has car block swap operations at logistics centers 3'' and 5'', we decompose the train services into three connected service arcs.

Two shipments $s \rightarrow t$ and $s' \rightarrow t$ are considered, and their required transportation due dates are T_{st} and $T_{s't}$ respectively. The possible transportation strategies for the first shipment $s \rightarrow t$ were analyzed. One possible strategy is that the shipment is first transported to yard 1' by the local train service Arc 4 from its origin s and is then transported to yard 3' carried by the through train service Arc 1. After the classification operation, the shipment is grouped into the through train service Arc 3 and transported to the last yard 6' on its itinerary. Finally, the shipment is sent to its destination t by the local train service Arc 9 to complete its entire itinerary. During the overall transportation process, the shipment is reclassified three times (at yards 1', 3' and 6'). The train service chain used by the train service is as follows:

1. $s \rightarrow \text{Arc4} \rightarrow \text{Arc1} \rightarrow \text{Arc3} \rightarrow \text{Arc9} \rightarrow t$.

Other possible transportation strategies that can be adopted by the shipment include (but are not limited to) the following:

2. $s \rightarrow \text{Arc4} \rightarrow \text{Arc1} \rightarrow \text{Arc7} \rightarrow \text{Arc8} \rightarrow \text{Arc9} \rightarrow t$;
3. $s \rightarrow \text{Arc4} \rightarrow \text{Arc5} \rightarrow \text{Arc6} \rightarrow \text{Arc7} \rightarrow \text{Arc8} \rightarrow \text{Arc9} \rightarrow t$; and
4. $s \rightarrow \text{Arc10} \rightarrow \text{Arc11} \rightarrow \text{Arc12} \rightarrow \text{Arc13} \rightarrow \text{Arc14} \rightarrow t$.

In Strategy (2), the shipment needs to be classified four times (at yards 1', 3', 5' and 6'). Whereas in Strategy (3), the shipment needs to be classified five times (at yards 1', 2', 3', 5' and 6'). In contrast, only two classification operations (at yards 1' and 6') are needed in Strategy (4). However, delays occurred due to the car block swap operations at logistics centers 3'' and 5'' in this strategy, which may cause the transport time be more than the due date.

Similarly, for another shipment $s' \rightarrow t$, the following transportation strategies can be adopted:

1. $s' \rightarrow \text{Arc}23 \rightarrow \text{Arc}2 \rightarrow \text{Arc}3 \rightarrow \text{Arc}9 \rightarrow t$;
2. $s' \rightarrow \text{Arc}23 \rightarrow \text{Arc}2 \rightarrow \text{Arc}7 \rightarrow \text{Arc}8 \rightarrow \text{Arc}9 \rightarrow t$; and
3. $s' \rightarrow \text{Arc}23 \rightarrow \text{Arc}2 \rightarrow \text{Arc}18 \rightarrow \text{Arc}12 \rightarrow \text{Arc}13 \rightarrow \text{Arc}14 \rightarrow t$.

In Strategies (1) and (2), the shipment is reclassified three times and four times, respectively. In contrast, in Strategy (3), the shipment is reclassified three times and has one instance of a car block swap operation.

The train services are characterized by cost, operating speed, capacity and service frequency. For example, the express train service is usually of high cost, fast speed and low frequency, while the district train service has a lower speed at a lower cost and is always provided more frequently. As a result, some of the transportation strategies could violate the due date restrictions. Furthermore, some other strategies possibly result in overloads on the links through which the considered shipments and other shipments pass together. For instance, Strategy (1) of transporting shipment $s \rightarrow t$ and Strategy (1) of transporting shipment $s' \rightarrow t$ use the common through train service, i.e., Arc 3. If the volumes of these two shipments are relatively high, due to the limited capacity on Arc 3, its capacity constraint may be violated when the two abovementioned strategies are adopted simultaneously. Therefore, selecting the optimal transportation strategies for all the shipments while respecting the due date constraints and capacity constraints is typically a complicated combinatorial optimization problem.

Mathematical model

This section aims to provide a mathematical description for the rail express cargo car-to-train assignment problem. The model aims at determining: (1) the portion of the freight volume of the high-value cargo that can be transported by the scheduled express trains (i.e. to evaluate whether the current train service plan is reasonable enough); (2) which train service arcs are used by each shipment (i.e. the train service chain of each shipment); (3) the workload (or capacity utilization rate) of the service arcs especially involved in the scheduled express trains. Our goal is to design an optimal car-to-train assignment plan that maximizes the net income of the railway company to deliver the rail express cargo. While the main constraints considered in our model include: transportation due date, train service arc capacity, flow balance at each station and logical relationship between decision variables.

To facilitate the model formulation, we make the following assumptions throughout this paper:

1. Each city has at least one railway station, and different stations (including marshalling stations and logistics centers) in a city can be connected by local train services.
2. Arcs between any two nodes are regarded as different arcs even if they belong to the same train services. For example, in Fig 1, though Arc 11 and Arc 12 belong to the same express train services, they are treated as two different arcs. Note that these two arcs are connected in Logistics Center 3'', where car block swap operations may occur.

3. Each shipment should not be split during the transportation process. This means that each shipment can only choose a single route (a train service chain). However, railway operators can decide whether to transport the whole volume of a shipment or just a portion of it.
4. The frequency of ordinary freight trains fluctuates with the traffic volume (once the volume reaches the predefined size of a train, an ordinary train will be dispatched. Thus, the number of trains dispatched per day fluctuates.). Because the traffic volume of each ordinary train arc is much larger than the express train arc, it is assumed that the traffic flows exceeding the capacity of an express train arc can definitely be transported by an ordinary train arc.

Notations

The notations used in the mathematical model are described as follows:

1. Sets
 - V : Set of all nodes, yards and logistics centers;
 - E : Set of train service arcs;
 - E^{express} : Set of express train service arcs;
 - G : Set of shipments, $g \in G$;
2. Parameters
 - s_m : Start node of an arc m , $s_m \in V$, $m \in E$;
 - t_m : Tail node of an arc m , $t_m \in V$, $m \in E$;
 - C_m : Capacity of an arc m ;
 - L_m : Length of an arc m ;
 - τ_m : Transportation time on an arc m ;
 - τ_{mn} : Transfer time from arc m to arc n for a car;
 - N_g : Volume of the g th shipment that originates at i and is destined for j , $i \in V_{\text{supply}}$, $j \in V_{\text{demand}}$;
 - o_g : The origin of the g th shipment;
 - d_g : The destination of the g th shipment;
 - R_g : Per car cost of transporting the g th shipment paid by the shippers;
 - T_g : Transportation due date of the g th shipment;
 - λ : The cost of the express train service per car kilometer;
 - M : A sufficiently large number.
3. (3) Decision variables
 - ξ_g : Continuous variables. Proportion of the volume of the g th shipment that is able to be transported;
 - x_g^m : Binary variables. $x_g^m = 1$ if the g th shipment uses the m th service arc; otherwise, $x_g^m = 0$.

Model formulation

According to the notations above, the car-to-train assignment problem for the rail express cargo can be written as follows:

Objective function:

$$\max \sum_{g \in G} R_g N_g \xi_g - \lambda \sum_{m \in E^{\text{express}}} C_m L_m \quad (1)$$

Subject to

$$\sum_{m \in E} x_g^m \tau_m + \sum_{m \in E} \sum_{n: s_n = t_m} \tau_{mn} x_g^m x_g^n \leq T_g \quad \forall g \in G \quad (2)$$

$$\sum_{g \in G} N_g \zeta_g x_g^m \leq C_m \quad \forall m \in E \quad (3)$$

$$M \zeta_g \geq x_g^m \quad \forall g \in G, m \in E \quad (4)$$

$$\zeta_g \sum_{m: s_m = o_g} x_g^m = \zeta_g \quad \forall g \in G, m \in E \quad (5)$$

$$\zeta_g \sum_{m: t_m = d_g} x_g^m = \zeta_g \quad \forall g \in G, m \in E \quad (6)$$

$$\sum_{m: t_m = k} x_g^m = \sum_{n: s_n = k} x_g^n \quad \forall g \in G, k \in V, k \neq o_g, k \neq d_g, m, n \in E \quad (7)$$

$$x_g^m \in \{0, 1\} \quad \forall g \in G, m \in E \quad (8)$$

$$\zeta_g \in [0, 1] \quad \forall g \in G \quad (9)$$

In the model above, Eq (1) is the objective function, which maximizes the net income of transporting the rail express cargo. The net income is equal to the total transportation fees paid by the shippers minus the total operating costs of organizing the express train services by the railway company.

The second term in the objective function is the total operation cost of all express trains provided by Railway Company. The fixed operation cost of express train i is C_i^{fix} , the per mile traveling cost is C_i^{TrainRun} and the transport distance is ℓ_i . These parameters form the following equation:

$$\lambda \sum_{m \in E^{\text{express}}} C_m L_m = \sum_{i \in S^{\text{express}}} (C_i^{\text{fix}} + C_i^{\text{TrainRun}} \ell_i) \quad (10)$$

In formula (10), S_i^{express} is the set of express trains.

Express trains are usually operated based on regular timetables similar to passenger trains, they are not influenced by the freight demand. Even if the freight demand is very small, which rarely happens in practice because the express train schedule is made based on historical data, the train also has to be operated based on timetable and the railway company has to pay the operation cost. Therefore, the total cost of express trains can be viewed as a constant. The share of express cargo in the railway freight volume is small and very little of them is transported by the ordinary trains. Therefore, although the ordinary trains are operated based on freight demand, the express cargo volume has very limited influence on the frequencies of ordinary trains. As a result, the cost can be viewed as independent with the express cargo.

Since the service network is given in advance, the latter costs are a constant, which means it does not affect the optimal solution. Therefore, in the computational experiments, we will remove the constant express train service organization costs from the objective function.

Constraint (2) ensures that the total transportation time of a shipment should not be longer than its predefined due date. In this study, the total transportation time consists of the transportation

time on the arcs and the transfer time between the arcs. Constraint (3) is the capacity restriction on the service arcs. Constraint (4) guarantees the logical relationship between the two groups of decision variables. Constraint (5)~(7) are the well-known flow balance constraints. Specifically, Constraint (5) is a flow balance constraint at the departure nodes, Constraint (6) is a flow balance constraint at the arrival nodes, and Constraint (7) is a flow balance constraint at the intermediate (passing) nodes. Finally, decision variable domains are specified by Constraints (8) and (9).

Linearization

There exist several nonlinear constraints in the original model, including constraints (2), (3), (5) and (6). Clearly, due date constraint (2) is a nonlinear constraint because it involves the production of two decision variables. In general, a linear model is easier to solve than a nonlinear model. We are hence motivated to linearize original constraint (2) to a linear constraint. Therefore, we need to first introduce an auxiliary binary decision variable y_g^{mn} , and it is defined as follows:

$$y_g^{mn} = x_g^m \cdot x_g^n \quad \forall g \in G, m \in E, n \in E, t_m = s_n \quad (11)$$

Hence, constraint (2) can be converted to:

$$\sum_{m \in E} x_g^m t_m + \sum_{m \in E} \sum_{n: s_n = t_m} \tau_{mn} y_g^{mn} \leq T_g \quad \forall g \in G \quad (12)$$

In addition, we add the following constraints into the original model:

$$x_g^m + x_g^n - 1 \leq y_g^{mn} \leq \frac{1}{2} \cdot (x_g^m + x_g^n) \quad \forall g \in G, m \in E, n \in E, t_m = s_n \quad (13)$$

$$y_g^{mn} \in \{0, 1\} \quad \forall g \in G, m \in E, n \in E, t_m = s_n \quad (14)$$

By replacing $x_g^m \cdot x_g^n$ with y_g^{mn} in constraint (2) and adding constraints (13) and (14) to the original model, we transform constraint (2) to a linear constraint.

Then, we need to linearize constraints (3), (5) and (6). We introduce a continuous binary decision variable $\bar{x}_g^m$, and it is defined as follows:

$$\bar{x}_g^m = \zeta_g \cdot x_g^m \quad \forall g \in G, m \in E \quad (15)$$

Hence, constraints (3), (5) and (6) can be converted to constraints (16), (17) and (18) respectively.

$$\sum_{g \in G} N_g \bar{x}_g^m \leq C_m \quad \forall m \in E \quad (16)$$

$$\sum_{m: s_m = o_g} \bar{x}_g^m = \zeta_g \quad \forall g \in G, m \in E \quad (17)$$

$$\sum_{m: t_m = d_g} \bar{x}_g^m = \zeta_g \quad \forall g \in G, m \in E \quad (18)$$

In addition, we add the following constraints (19), (20) and (21) into the original model:

$$\zeta_g - (1 - x_g^m)M \leq \bar{x}_g^m \leq \zeta_g + (1 - x_g^m)M \quad \forall g \in G, m \in E \quad (19)$$

$$-x_g^m M \leq \bar{x}_g^m \leq x_g^m M \quad \forall g \in G, m \in E \quad (20)$$

$$\bar{x}_g \in [0, 1] \quad \forall g \in G \quad (21)$$

By replacing $\xi_g \cdot x_g^m$ with $\bar{x}_g^m$ in constraint (3), (5) and (6) and adding constraints (19), (20) and (21) to the original model, we transform constraint (3), (5) and (6) to linear constraints. Eventually, the original model is converted to a simpler mixed integer linear programming (MILP) model.

The car-to-train assignment problem can be viewed a generalization of the well-known Multicommodity Flow Problem (MFP). The MFP has already been proved to be NP-complete in existing literature. Therefore, the studied problem also belongs to the NP-complete class. Note that the studied problem is more complicated and computationally more challenging due to the additional transportation due date constraints. The solution method for our MILP model are branch-and-cut, and the commercially available solver Gurobi is employed in our computational study of this paper using its built-in branch-and-cut algorithm framework.

Numerical results

In this section, we perform computational analysis on a numerical example in the context of the railway corridor called “Beijing-Guangzhou Railway Line”, one of the most important north-south rail lines in China. The route, from Beijing to Guangzhou, has a total length of 2,324 kilometers. It connects 6 provincial capitals or municipalities and many large- and medium-sized cities, and it is one of the busiest major railways in China (see Fig 2). The goods transported to the south on the Beijing-Guangzhou railway mainly include coal, steel,

Fig 2. Beijing-Guangzhou railway line.

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petroleum, timber and export materials. Relatively speaking, the proportion of high value-added goods to the north is high. Therefore, this paper uses the northward direction of the Beijing-Guangzhou railway as an example for analysis.

This route passes through many important marshalling stations, including Fengtai West Station (Y007), Shijiazhuang South Station (Y006), Zhengzhou North Station (Y005), Wuhan North Station (Y004), Zhuzhou North Station (Y003), Hengyang North Station (Y002) and Jiangcun Station (Y001). We also consider some logistics centers related to this corridor, such as Dalang Station (B001) and Tangxi Station (B002) in Guangzhou, Wujiashan Station (B003) in Wuhan, Shigong Station (B004) at Shijiazhuang Hub, and Huangcun Station (B005) at Beijing Hub. In addition, the Nancang Station (B006) at Tianjin Hub is both an important marshalling station and a freight station. Although it does not belong to the Beijing-Guangzhou railway, due to its close relationship with Fengtai West Station, we also put it into the case system.

According to the data provided by the Railway Customer Service Center of China (12306 website), there are five express service trains (shown in Fig 3 as yellow, cyan, red, green and purple) related to the above marshalling stations and logistics centers. The curves of the same color belong to the same train, the point indicated by the arrow arc represents the destination of the train, and the point indicated by the curve without an arrow indicates the middle station of the train. For example, the red curve indicates the express train originating from B002, passing through Y003 and Y004 and terminating at Y005.

In addition, there are local trains running between the marshalling station and the logistics center and ordinary trains running from one marshalling station to another marshalling station (shown as blue arcs in Fig 3).

Parameters of the model and information on the shipments

The freight demand, cost, and transportation due date for each station are shown in Table 1 for a total of 58 flows, with the flow ID from FLOW_01 to FLOW_58 corresponding to $g = 1$ to 58. The freight cost R_g is based on the Chinese Railway Tariff (see the data published on the website of 12306) and the transportation distance of shipment g . The average car volume in a day and freight transportation due date are analyzed and speculated based on historical data. Column o_g in Table 1 shows the origin station of the flow and column d_g indicates the destination station of the flow. The unit of car volume N_g is car, the unit of freight transportation due date T_g is hour, and the unit of freight cost R_g is CNY / car.

We classify the train service arcs in Fig 3, where ARC_01~ARC_16 are the service arcs of the express trains corresponding to $m = 1 \sim 16$ and ARC_17~ARC_43 are the service arcs of the

Fig 3. Service network for testing the model.

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Table 1. Information on the shipments.

g	o_g	d_g	N_g	T_g	R_g	g	o_g	d_g	N_g	T_g	R_g
FLOW_01	B001	Y003	4.72	20	5598	FLOW_30	Y001	B006	5.28	72	19892
FLOW_02	B001	B005	14.64	72	19770	FLOW_31	Y001	B005	4.48	72	19808
FLOW_03	B001	Y004	4.48	26	9201	FLOW_32	Y002	Y004	3.76	24	5211
FLOW_04	B001	B003	29.44	36	9567	FLOW_33	Y002	B003	4.56	24	5577
FLOW_05	B001	Y005	13.76	34	13146	FLOW_34	Y002	Y005	4.24	29	9157
FLOW_06	B001	Y006	7.76	60	16363	FLOW_35	Y002	Y006	3.76	48	12373
FLOW_07	B001	B004	9.04	60	16603	FLOW_36	Y002	B004	13.68	60	12614
FLOW_08	B001	Y007	2.36	72	28534	FLOW_37	Y002	Y007	7.76	45	14434
FLOW_09	B001	Y007	7.68	72	19929	FLOW_38	Y002	B006	5.2	72	16645
FLOW_10	B001	B006	17.52	72	19854	FLOW_39	Y002	B005	5.04	60	16561
FLOW_11	B001	B006	2.15	72	28428	FLOW_40	Y003	Y005	25.92	24	19892
FLOW_12	B002	Y003	11.28	24	5673	FLOW_41	Y003	Y006	19.6	43	19808
FLOW_13	B002	Y004	8.56	36	9276	FLOW_42	Y003	B004	10.4	46	5211
FLOW_14	B002	B003	5.36	48	9642	FLOW_43	Y003	Y007	7.28	60	5577
FLOW_15	B002	Y005	5.12	36	13222	FLOW_44	Y003	B006	4.96	65	9157
FLOW_16	B002	Y006	3.76	60	16439	FLOW_45	Y003	B005	3.68	56	12373
FLOW_17	B002	B004	3.84	60	16679	FLOW_46	Y004	Y006	26.96	24	12614
FLOW_18	B002	B005	7.28	72	19694	FLOW_47	Y004	B004	4.8	32	14434
FLOW_19	B002	Y007	4.16	72	19853	FLOW_48	Y004	Y007	5.76	26	16645
FLOW_20	B002	Y007	8.72	72	28427	FLOW_49	Y004	B006	18.4	48	16561
FLOW_21	B002	B006	3.64	72	28321	FLOW_50	Y004	B005	12.24	48	8119
FLOW_22	B002	B006	17.44	72	19779	FLOW_51	B003	Y006	7.6	36	11336
FLOW_23	Y001	Y003	37.68	24	5560	FLOW_52	B003	B004	6.64	24	11576
FLOW_24	Y001	Y004	25.84	36	9163	FLOW_53	B003	Y007	5.92	36	13396
FLOW_25	Y001	B003	19.36	65	9529	FLOW_54	B003	B006	5.92	48	13322
FLOW_26	Y001	Y005	18.88	48	13108	FLOW_55	B003	B005	4.72	36	13237
FLOW_27	Y001	Y006	14.72	58	16325	FLOW_56	Y005	Y007	4.72	36	7732
FLOW_28	Y001	B004	8.72	48	16565	FLOW_57	Y005	B006	4.48	36	7973
FLOW_29	Y001	Y007	5.52	72	19966	FLOW_58	Y005	B005	3.84	36	9793

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ordinary trains corresponding to $m = 17 \sim 43$. The parameters of all the arcs are shown in Table 2.

The columns s_m and t_m in Table 2 represent the origin and destination of the train service arc m , respectively, column τ_m represents the running time on arc m of the train, and column C_m represents the arc's capacity. Because the operating frequency of ordinary freight trains in China's rail system fluctuates with freight demands, we set the C_m value of the ordinary trains as 200 cars per day, which is equivalent to no capacity constraint. The column "Train_No." is the arc belonging to the train trip.

The transfer time of Wuhan North Station and Hengyang North Station from 2005 to 2014 is listed in Table 3 (at this moment, the data after 2015 is not available). Wuhan North Station and Hengyang North Station are important marshalling stations on the Beijing-Guangzhou Railway Line. According to Table 3, the average transfer time is between 7.55 h and 8.30 h. The detailed data of all marshalling stations in the China railway system is not publically available. Therefore, we set the average transfer time 8 hours. Note that although this value may slightly change over different years, it has no essential influence on the mathematical modeling framework.

Table 2. Information on the train services.

m	s_m	t_m	τ_m	C_m	Train_No.	m	s_m	t_m	τ_m	C_m	Train_No.
ARC_01	B001	Y003	7.3	20	X001	ARC_23	Y001	Y004	27.4	200	H007
ARC_02	B001	Y003	7.3	13	X002	ARC_24	Y002	Y003	3.3	200	H008
ARC_03	Y003	B003	7.9	27	X002	ARC_25	Y002	Y004	14.6	200	H009
ARC_04	B003	Y005	9.2	24	X002	ARC_26	Y002	Y005	27.1	200	H010
ARC_05	Y005	B004	6.7	19	X002	ARC_27	Y003	Y004	11.3	200	H011
ARC_06	B004	B006	5.4	21	X002	ARC_28	Y003	Y005	23.8	200	H012
ARC_07	Y005	B004	8.0	15	X003	ARC_29	Y003	Y007	40.5	200	H013
ARC_08	B004	B005	4.0	13	X003	ARC_30	Y004	B003	1.2	200	H014
ARC_09	B002	Y003	7.6	7	X004	ARC_31	B003	Y004	1.2	200	H015
ARC_10	Y003	Y004	7.0	12	X004	ARC_32	Y004	Y005	12.5	200	H016
ARC_11	Y004	Y005	10.0	13	X004	ARC_33	Y004	Y006	22.5	200	H017
ARC_12	Y001	Y003	7.5	25	X005	ARC_34	Y004	Y007	29.1	200	H018
ARC_13	Y003	Y004	12	25	X005	ARC_35	Y005	Y006	10.2	200	H019
ARC_14	Y004	Y005	7.5	25	X005	ARC_36	Y005	Y007	16.9	200	H020
ARC_15	Y005	Y007	9.4	25	X005	ARC_37	Y006	B004	1.4	200	H021
ARC_16	Y007	B006	4.0	25	X005	ARC_38	B004	Y006	1.4	200	H022
ARC_17	B001	Y001	0.5	200	H001	ARC_39	Y006	Y007	6.8	200	H023
ARC_18	Y001	B001	0.5	200	H002	ARC_40	Y007	B005	0.5	200	H024
ARC_19	B002	Y001	0.5	200	H003	ARC_41	Y007	B006	3.0	200	H025
ARC_20	Y001	B002	0.5	200	H004	ARC_42	B005	Y007	0.5	200	H026
ARC_21	Y001	Y002	12.8	200	H005	ARC_43	B006	Y007	3.0	200	H027
ARC_22	Y001	Y003	16.1	200	H006						

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In fact, the transfer time value depends on all the shipments rather than one shipment. It is used to estimate the delivery time of cargos at the strategical level. In actual transportation, the transfer time of specific cargo may be influenced by many factors and it is complex to be accurately predicted and calculated. The problem in this paper falls into the strategical planning class, rather than detailed operational-level design of train timetables (schedules). The accurate value of transfer time for every shipment will be optimized at the operational level.

For different arcs belonging to the same train, the transfer time can be obtained from the express train timetable published on the 12306 website. Table 4 shows the values of the time delay parameter τ_{mn} of the transfer between two arcs, which have the same train number based on the train schedules. The unit of parameter τ_{mn} in Table 4 is hour.

Computational results and analysis

We solve the model of the car-to-train assignment problem by Gurobi 7.5.2 implemented by Python 2.7.14. The parameters of the computer are Intel Core i5-3337U CPU and 4 GB RAM. The above instance based on the Beijing-Guangzhou railway line can be solved to optimality after running for 1747s. A total of 565.03 cars were involved in the calculation of the 58 flows, and the results show that 457.21 cars could be delivered within the prescribed due date, accounting for

Table 3. The average transfer time of Wuhan North Station and Hengyang North Station from 2005 to 2014.

	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Ave.
Wuhan North (hr)	8.70	8.50	8.60	8.50	8.80	8.00	8.30	7.80	8.00	7.80	8.30
Hengyang North (hr)	5.90	5.50	6.80	8.00	7.40	6.90	6.70	9.30	10.00	9.00	7.55

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Table 4. Transfer time between arc m and arc n .

Arc_m	Arc_n	τ_{mn}	Arc_m	Arc_n	τ_{mn}	Arc_m	Arc_n	τ_{mn}
ARC_02	ARC_03	3.4	ARC_07	ARC_08	4.3	ARC_13	ARC_14	1.5
ARC_03	ARC_04	2.0	ARC_09	ARC_10	2.8	ARC_14	ARC_15	5.0
ARC_04	ARC_05	5.6	ARC_10	ARC_11	2.5	ARC_15	ARC_16	4.0
ARC_05	ARC_06	2.9	ARC_12	ARC_13	3.0			

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79.16% of the total transportation demand, which is equivalent to $\bar{R}_g = 79.16\%$. The corresponding objective function is 6140871.34 CNY. The detailed car-to-train assignment plan is shown in Table 5.

The column “Time” in Table 5 indicates the actual transportation time for the shipments. $\Delta T = T_g - T'_g$ indicates the gap between the due date and the actual transportation time. According to Table 5, among the 58 flows, there are 42 flows that fully satisfy the demand, 8 flows partially meet the demand, and the remaining 8 flows cannot meet the demand. The reason can be analyzed as follows. For those paths satisfying the transportation due date constraints, the capacity of the arcs on these paths may be insufficient; for the other paths, the transportation due date constraints may be violated if they are used.

The car volume and workload of each arc are shown in Table 6. In the table, f_m indicates the actual car volume through arc m ; $\beta_m = f_m/C_m$ is the workload of arc m . The average load of 16 express train arcs in the system is 75.54%. Among them, the workloads of 8 arcs reached the capacity values, and the workloads of ARC_03, ARC_06, ARC_08 and ARC_16 are all less than 50%. For ordinary train arcs, because they also transport other bulk non-value-added car flows, their workloads are not discussed here.

The flow pattern of each arc (i.e., which shipments are carried by the train) is shown in Fig 4. In Fig 4, we select two express trains and two ordinary trains for illustration. The express train X003 (shown by red arcs in Fig 4) originates at station B002 and ends at station Y005. The flow it loads in the first segment of the trip (B002→Y003) is FLOW_22, the flows in the second segment are FLOW_17 and FLOW_21, and those in the third segment are FLOW_08, FLOW_17, FLOW_19 and FLOW_21. Similarly, the flows carried by express train X003 (shown by purple arcs in Fig 4) originate at station Y005 and end at station B005; the flows in the first segment of the trip (Y005→B004) are FLOW_7, FLOW_17, FLOW_31 and FLOW_42; the second segment is FLOW_31. The ordinary train (Y001→Y004) transports FLOW_13, FLOW_14, FLOW_24, FLOW_25 and FLOW_27. The ordinary train (Y003→Y007) transports FLOW_27, FLOW_35, FLOW_36, FLOW_41, FLOW_46, FLOW_47 and FLOW_51.

Conclusions

In this paper, a car-to-train assignment model for rail express cargos is proposed. According to the characteristics of high value-added goods, the transportation due date constraint is added to the model, considering both the transportation time on the arcs and the transfer time between the arcs. In addition, the arc capacity constraint and flow balance constraint are considered. Moreover, linearization techniques are used to make the model linear so that it can be directly solved by a standard optimization solver. Finally, an illustrative case study is carried out based on the Beijing-Guangzhou Railway Line, which is one of the busiest railway lines in China. The example consists of 13 nodes and 43 arcs, and we consider the northward direction. The detailed car-to-train assignment plan is obtained based on the analysis of the optimal solution. The results indicate that the model has a certain significance in real-life application to the rail express cargos assignment problem.

Table 5. Results of the car-to-train assignment.

g	T_g	T'_g	ΔT	ξ_g	Car-to-Train Assignment Plan
FLOW_01	20	—	—	0.00%	—
FLOW_02	72	64.3	7.7	88.80%	B001→ARC_02→ARC_29→ARC_40→B005
FLOW_03	26	—	—	0.00%	—
FLOW_04	36	—	—	0.00%	—
FLOW_05	34	—	—	0.00%	—
FLOW_06	60	—	—	0.00%	—
FLOW_07	60	55.1	4.9	3.65%	B001→ARC_01→ARC_28→ARC_07→B004
FLOW_08	72	71.3	0.7	100.00%	B001→ARC_17→ARC_21→ARC_25→ARC_11→ARC_15→Y007
FLOW_09	72	68.1	3.9	100.00%	B001→ARC_17→ARC_12→ARC_13→ARC_34→Y007
FLOW_10	72	66.8	5.2	100.00%	B001→ARC_01→ARC_29→ARC_41→B006
FLOW_11	72	67.8	4.2	100.00%	B001→ARC_01→ARC_29→ARC_16→B006
FLOW_12	24	—	—	0.00%	—
FLOW_13	36	35.9	0.1	100.00%	B002→ARC_19→ARC_23→Y004
FLOW_14	48	45.1	2.9	100.00%	B002→ARC_19→ARC_23→ARC_30→B003
FLOW_15	36	—	—	0.00%	—
FLOW_16	60	58.2	1.8	18.35%	B002→ARC_19→ARC_12→ARC_13→ARC_14→ARC_35→Y006
FLOW_17	60	59.5	0.5	71.61%	B002→ARC_19→ARC_12→ARC_10→ARC_11→ARC_07→B004
FLOW_18	72	—	—	0.00%	—
FLOW_19	72	71.3	0.7	100.00%	B002→ARC_19→ARC_21→ARC_25→ARC_11→ARC_15→Y007
FLOW_20	72	65.8	6.2	100.00%	B002→ARC_19→ARC_21→ARC_25→ARC_14→ARC_15→Y007
FLOW_21	72	66.5	5.5	100.00%	B002→ARC_19→ARC_12→ARC_10→ARC_11→ARC_05 →ARC_06→B006
FLOW_22	72	67.1	4.9	40.14%	B002→ARC_09→ARC_29→ARC_41→B006
FLOW_23	24	16.1	7.9	100.00%	Y001→ARC_22→Y004
FLOW_24	36	27.4	8.6	100.00%	Y001→ARC_23→Y004
FLOW_25	65	36.6	28.4	100.00%	Y001→ARC_23→ARC_30→B003
FLOW_26	48	47.9	0.1	100.00%	Y001→ARC_21→ARC_25→ARC_32→B003
FLOW_27	58	57.9	0.1	100.00%	Y001→ARC_23→ARC_33→Y006
FLOW_28	48	46.2	1.8	39.45%	Y001→ARC_12→ARC_13→ARC_14→ARC_05→B004
FLOW_29	72	64.5	7.5	100.00%	Y001→ARC_21→ARC_25→ARC_34→Y007
FLOW_30	72	54.5	17.5	100.00%	Y001→ARC_12→ARC_13→ARC_14→ARC_05→ARC_06→B006
FLOW_31	72	58.9	13.1	33.93%	Y001→ARC_12→ARC_03→ARC_04→ARC_07→ARC_08→B006
FLOW_32	24	14.6	9.4	100.00%	Y001→ARC_25→Y004
FLOW_33	24	15.8	8.2	100.00%	Y002→ARC_25→ARC_30→Y003
FLOW_34	29	27.1	1.9	100.00%	Y002→ARC_25→ARC_32→Y005
FLOW_35	48	37.1	10.9	100.00%	Y002→ARC_25→ARC_33→Y006
FLOW_36	60	46.5	13.5	100.00%	Y002→ARC_25→ARC_33→ARC_37→B004
FLOW_37	45	43.7	1.3	100.00%	Y002→ARC_25→ARC_34→Y007
FLOW_38	72	54.7	17.3	100.00%	Y002→ARC_25→ARC_34→ARC_41→ARC_41→B006
FLOW_39	60	52.2	7.8	100.00%	Y002→ARC_25→ARC_34→ARC_40→B005
FLOW_40	24	23.8	0.2	100.00%	Y003→ARC_28→Y005
FLOW_41	43	41.8	1.2	100.00%	Y003→ARC_27→ARC_33→Y006
FLOW_42	46	39.8	6.2	100.00%	Y003→ARC_07→ARC_28→B004
FLOW_43	60	48.7	11.3	100.00%	Y003→ARC_28→ARC_36→Y007
FLOW_44	65	59.7	5.3	100.00%	Y003→ARC_28→ARC_36→ARC_41→B006
FLOW_45	56	49.0	7.0	100.00%	Y003→ARC_29→ARC_40→B005
FLOW_46	24	22.5	1.5	100.00%	Y004→ARC_33→Y006

(Continued)

Table 5. (Continued)

g	T_g	T'_g	ΔT	ξ_g	Car-to-Train Assignment Plan
FLOW_47	32	31.9	0.1	100.00%	Y004→ARC_33→ARC_37→B004
FLOW_48	26	21.9	4.1	100.00%	Y004→ARC_14→ARC_15→Y007
FLOW_49	48	40.1	7.9	100.00%	Y004→ARC_34→ARC_41→B006
FLOW_50	48	37.6	10.4	100.00%	Y004→ARC_34→ARC_40→B005
FLOW_51	36	31.7	4.3	100.00%	B003→ARC_31→ARC_33→Y006
FLOW_52	24	21.5	2.5	100.00%	B003→ARC_04→ARC_05→B004
FLOW_53	36	34.1	1.9	100.00%	B003→ARC_04→ARC_36→Y007
FLOW_54	48	46.1	1.9	100.00%	B003→ARC_04→ARC_36→ARC_16→B006
FLOW_55	36	33.5	2.5	84.75%	B003→ARC_04→ARC_15→ARC_40→B005
FLOW_56	36	16.9	19.1	100.00%	Y005→ARC_36→Y007
FLOW_57	36	27.9	8.1	100.00%	Y005→ARC_36→ARC_41→B006
FLOW_58	36	25.4	10.6	100.00%	Y005→ARC_36→ARC_40→B005

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Table 6. Workloads of the express train arcs.

m	f_m	C_m	β_m	m	f_m	C_m	β_m
ARC_01	20.00	20	100.00%	ARC_09	7.00	7	100.00%
ARC_02	13.00	13	100.00%	ARC_10	6.39	12	53.25%
ARC_03	1.52	27	5.63%	ARC_11	12.91	13	99.31%
ARC_04	24.00	24	100.00%	ARC_12	25.00	25	100.00%
ARC_05	19.00	19	100.00%	ARC_13	17.09	25	68.36%
ARC_06	8.92	21	42.48%	ARC_14	23.89	25	95.56%
ARC_07	15.00	15	100.00%	ARC_15	25.00	25	100.00%
ARC_08	1.52	13	11.69%	ARC_16	8.07	25	32.28%

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Fig 4. Assignment results of a part of the arcs.

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Note that it is assumed that the train operation plan, mainly the scheduled train services network, is given, which is developed manually in practice without any optimization-based approach. In this case, the resulting car-to-train assignment plan can serve as a solid aid in improving the quality of train operation planning. In the long term, researchers can focus on

the joint optimization of the train operation plan and the car-to-train assignment plan. In addition, the railway network and the cargo volume in China are huge. For such a large-scale network and data, the efficiency of the algorithm in this paper is far from enough for practical application, so the solution method of the model needs to be further improved.

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Author Contributions

Conceptualization: Boliang Lin.

Data curation: Boliang Lin, Jingsong Duan, Min Sun, Wengao Peng.

Formal analysis: Boliang Lin, Chang Liu.

Funding acquisition: Boliang Lin, Wengao Peng.

Investigation: Boliang Lin, Min Sun, Wengao Peng, Siqi Liu.

Methodology: Boliang Lin, Jingsong Duan, Jiayi Wang.

Project administration: Boliang Lin, Wengao Peng.

Resources: Boliang Lin.

Software: Boliang Lin, Jingsong Duan, Jiayi Wang, Jianping Wu.

Supervision: Boliang Lin.

Validation: Boliang Lin.

Visualization: Boliang Lin, Jingsong Duan, Chang Liu, Jie Xiao.

Writing – original draft: Boliang Lin, Jingsong Duan, Jiayi Wang.

Writing – review & editing: Boliang Lin, Jingsong Duan, Jiayi Wang, Min Sun, Jie Xiao, Siqi Liu, Jianping Wu.

References

1. Liu C, Lin BL, Wang JX, Xiao J, Liu SQ. Flow assignment model for quantitative analysis of diverting bulk freight from road to railway. *Plos One*. 2017; 12(8):e0182179. <https://doi.org/10.1371/journal.pone.0182179> PMID: 28771536
2. Bodin LD, Golden BL, Schuster AD. A model for the blocking of trains. *Transportation Research Part B: Methodological*. 1980; 14(1–2):115–120. [https://doi.org/10.1016/0191-2615\(80\)90037-5](https://doi.org/10.1016/0191-2615(80)90037-5)
3. Newton HN, Barnhart C, Vance PH. Constructing railroad blocking plans to minimize handling costs. *Transportation Science*. 1998; 32(4):330–345. <https://doi.org/10.1287/trsc.32.4.330>
4. Barnhart C, Jin H, Vance PH. Railroad blocking a network design application. *Operations Research*. 2000; 48(4):603–614. <http://doi.org/10.1287/opre.48.4.603.12416>
5. Yaghini M, Foroughi A, Nadjar B. Solving railroad blocking problem using ant colony optimization algorithm. *Applied Mathematical Modelling*. 2011; 35(12):5579–5591. <https://doi.org/10.1016/j.apm.2011.05.018>
6. Gorman MF. An application of genetic and tabu searches to the freight railroad operating plan problem. *Annals of Operations Research*. 1998; 78(0):51–69. <http://hdl.handle.net/10.1023/A:1018906301828>
7. Yaghini M, Momeni M, Sarmadi M. Solving train formation problem using simulated annealing algorithm in a simplex framework. *Journal of Advanced Transportation*. 2014; 48(5):402–416. <http://doi.org/10.1002/at.1183>

8. Ahuja RK, Jha KC, Liu J. Solving real-life railroad blocking problems. *Interfaces*. 2007; 37(5): 404–419. <http://doi.org/10.1287/inte.1070.0295>
9. Lin BL, Liu C, Wang HJ, Lin RX. Modeling the railway network design problem: A novel approach to considering carbon emissions reduction. *Transportation Research Part D: Transport and Environment*. 2017; 56:95–109. <https://doi.org/10.1016/j.trd.2017.07.008>
10. Thomet MA. A User-Oriented Freight Railroad Operating Policy. *IEEE Transactions on Systems, Man, and Cybernetics*. 1971; SMC-1(4):349–356. <https://doi.org/10.1109/TSMC.1971.4308318>
11. Kwona OK, Martlandb CD, Sussmanb JM. Routing and scheduling temporal and heterogeneous freight car traffic on rail networks. *Transportation Research Part E: Logistics and Transportation Review*. 1998; 34 (2):101–115. [https://doi.org/10.1016/S1366-5545\(97\)00022-7](https://doi.org/10.1016/S1366-5545(97)00022-7)
12. Jha KC, Ahuja RK, Sahin G. New approaches for solving the block-to-train assignment problem. *Networks*. 2008; 51(1):48–62. <https://doi.org/10.1002/net.20195>
13. Xiao J, Lin BL. Comprehensive optimization of the one-block and two-block train formation plan. *Journal of Rail Transport Planning & Management*. 2016; 6(3):218–236. <https://doi.org/10.1016/j.jrtpm.2016.09.002>
14. Xiao J, Joern P, Lin BL, Wang JX. Solving the block-to-train assignment problem using the heuristic approach based on the genetic algorithm and tabu search. *Transportation Research Part B: Methodological*. 2018; 108:148–171. <https://doi.org/10.1016/j.trb.2017.12.014>
15. Hwang T, Ouyang Y. Assignment of Freight Shipment Demand in Congested Rail Networks. *Transportation Research Record: Journal of the Transportation Research Board*. 2014; 2448(-1):37–44. <http://doi.org/10.3141/2448-05>
16. Borndörfer R, Klug T, Schlechte T, Fügenschuh A, Schang T, Schülldorf H. The Freight Train Routing Problem for Congested Railway Networks with Mixed Traffic. *Transportation Science*. Published online in Articles in Advance 31 Mar 2016. <https://doi.org/10.1287/trsc.2015.0656>
17. Keaton MH. Designing Optimal Railroad Operating Plans: Lagrangian Relaxation and Heuristic Approaches. *Transportation Research Part B: Methodological*. 1989; 23(6):415–431. [https://doi.org/10.1016/0191-2615\(89\)90042-8](https://doi.org/10.1016/0191-2615(89)90042-8)
18. Keaton MH. Designing Railroad Operating Plans: A Dual Adjustment Method for Implementing Lagrangian Relaxation. *Transportation Science*. 1992; 26(4):263–279. <http://doi.org/10.1287/trsc.26.4.263>
19. Crainic T, Ferland JA, Rousseau JM. A Tactical Planning Model for Rail Freight Transportation. *Transportation Science*. 1984; 18 (2):165–184. <http://doi.org/10.1287/trsc.18.2.165>
20. Haghani AE. Rail freight transportation: A review of recent optimization models for train routing and empty car distribution. *Journal of Advanced Transportation*. 1987; 21(2):147–172. <http://doi.org/10.1002/atr.5670210205>
21. Haghani AE. Formulation and solution of a combined train routing and makeup, and empty car distribution model. *Transportation Research Part B: Methodological*. 1989; 23(6):433–452. [https://doi.org/10.1016/0191-2615\(89\)90043-X](https://doi.org/10.1016/0191-2615(89)90043-X)
22. Lin BL, Wang ZM, Ji LJ, Tian YM, Zhou GQ. Optimizing the freight train connection service network of a large-scale rail system. *Transportation Research Part B: Methodological*. 2012; 46(5):649–667. <http://doi.org/10.1016/j.trb.2011.12.003>
23. Zhu E, Cranic TG, Gendreau M. Scheduled Service Network Design for Freight Rail Transportation. *Operations Research*. 2014; 62(2):383–400. <https://doi.org/10.1287/opre.2013.1254>
24. Fügenschuh A, Homfeld H, Schülldorf H. Single-Car Routing in Rail Freight Transport. *Transportation Science*. 2015; 49(1):130–148. <https://doi.org/10.1287/trsc.2013.0486>
25. Sun Y, Cao C, Wu C. Multi-objective optimization of train routing problem combined with train scheduling on a high-speed railway network. *Transportation Research Part C: Emerging Technologies*. 2014; 44:1–20. <http://doi.org/10.1016/j.trc.2014.02.023>
26. Zhu HB. Avoiding Conflicts by Group Role Assignment. *IEEE Trans. on Systems, Man, and Cybernetics: Systems*. 2016; 46(4):535–547. doi: <https://doi.org/10.1109/tsmc.2015.2438690>
27. Zhu HB, Zhou MC, Alkins R. Group Role Assignment via a Kuhn-Munkres Algorithm-based Solution. *IEEE Trans. on Systems, Man, and Cybernetics, Part A: Systems and Humans*. 2012; 42(3):739–750. doi: <https://doi.org/10.1109/tsmca.2011.2170414>