RESEARCH ARTICLE

Modeling and impedance matching for radio frequency driven plasma lamp considering cold and hot states

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Abstract

A new dual-state impedance matching scheme for a microwave driven plasma lamp using a solid-state power amplifier (SSPA) is presented. The impedance of the plasma lamp depends on the amount of input radio frequency (RF) energy, and therefore has very different values for hot and cold states. First, a method for effectively modeling the electrical characteristics of a plasma lamp that depends on RF power has been proposed. Second, a new technique has been proposed to achieve dual-state impedance matching for two state impedances at two very close frequencies using a T-shaped matching network with two section shunt stub and additional transmission line. The proposed method can achieve dual state impedance matching in two frequency bands located very closely when compared to the conventional methods. The accuracy of the proposed model and the effectiveness of the proposed dual-state matching are verified via a plasma lamp system with a 2.45 GHz 300 W GaN SSPA.

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Introduction

Electrodeless plasma lamps have excellent optical performance in terms of their sun-like spectrum, long lifetime, and high efficacy (lumens per watt) [1, 2]. Plasma lamps operate by supplying radio frequency (RF) energy (such as 450 MHz or 2450 MHz) to the lamp using a solid-state power amplifier (SSPA). The plasma lamp consists of a resonator and a bulb comprising materials such as Ar and InBr. The bulb has two different states: the off and on states, which correspond to the cold and hot states, respectively. The impedance of plasma lamps varies according to the RF energy that is supplied to the bulb [3, 4]. It is impossible to match two very different impedances simultaneously at one frequency. In result, we need additional ignition process (such as contacting metal ignitor in the bulb or applying very high voltage signal to the bulb) to turn on the plasma lamp if the output matching for the hot-state is only considered. To solve this problem, we propose an advanced matching technique for two different load impedances at two very close frequencies (the ratio of the two frequencies is about 1.005). This letter describes the extraction of an accurate equivalent circuit model of a plasma lamp with electrical characteristics that vary depending on the state. A new dual-state impedance

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matching design technique using a transmission line (T-line) based on the proposed model for the plasma lamp has been suggested, and its effectiveness has been verified with a plasma lamp system with a 300 W Gallium nitride (GaN) SSPA.

Plasma lamp modeling

An electrodeless plasma lighting system (PLS) using an RF SSPA consists of a signal generation component (NI USRP-2901), power amplifier (RFHIC MEL-500), matching component, resonator, plasma bulb (RFHIC URF-SP22), and a controller, as shown in Fig 1. The plasma lamp is comprised of a resonator and plasma bulb [2, 4, 9]. The impedance of the lamp is expressed as Z_L in Fig 2; the bulb impedances Z_c and Z_h respectively differ for the cold and hot states. In addition, the impedance of the hot state varies according to the RF energy supplied to the bulb [3, 4]. The impedance of the resonator with the plasma bulb is calculated by measuring the magnitude and phase difference of the reflected power using a directional coupler (ZGBDC35-93HP). A circuit model that incorporates both Z_c and Z_h is developed in consideration of the plasma lamp structure. Z_h has a different resonance frequency from Z_c , and increasing the energy on hot-state reduces the resistance of the plasma bulb [5]. An equivalent circuit model of a plasma lamp is suggested in Fig 2a. The resonator in the lamp is modeled with series capacitor C_s , shunt parallel inductor L_p , and capacitor C_p . R_c is used to represent the bulb in the cold state; the bulb in the hot-state is represented by implementing a variable resistor R_h , inductor L_h , and capacitor C_h . The resulting impedances are as follows:

$$Z_h = \frac{1}{j\omega C_p + \frac{1}{j\omega L_p} + \frac{j\omega C_h(R_h + j\omega L_h)}{R_h + j\omega L_h(j\omega C_h R_h + 1)}} + \frac{1}{j\omega C_s},$$

$$Z_c = \frac{1}{j\omega C_p + \frac{1}{j\omega L_p} + \frac{1}{R_c}} + \frac{1}{j\omega C_s}$$
(1)

The measured Z_c and Z_h are shown in Fig 2b. We measure the impedance of the plasma lamp in the off state (Fig 2b; Point C) and operating state (Fig 2b; Point H_L to H_H). The parameters of the equivalent circuit model extracted from the measured data are shown in Table 1.

A comparison of the data simulated using AWR[®] and the data obtained via measurements is presented in the form of a Smith chart (Fig 2b; S1 File). The impedances simulated via the proposed model are in good agreement with the measured impedances as shown in Fig 2b.

Fig 1. System block diagram of the electrodeless plasma lighting system.

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Fig 2. Electrical model of plasma lamp (Fig 2a), and comparison of measured and simulated impedances (Fig 2b).

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Dual-state impedance transformer

Because the impedance of the plasma lamp in the hot-state is different from that in the cold-state, an impedance matching technique is required to reduce the cold- and hot-state return losses at frequencies of f_c and f_h , respectively. The narrow gap of the two frequencies is

Table 1. Parameters of the plasma bulb model.

C_s [nF]	C_p [nF]	L_p [μ H]	R_c [Ω]	C_h [nF]	L_h [nH]	R_h [Ω]
3.02	0.441	6.19	3.2	3.11	38.65	$34(H_H) - 226(H_L)$

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Fig 3. Proposed T-shaped dual-state matching design schematic.

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advantageous for the PLS, because the frequency range in which SSPA typically operates with high efficiency is narrow. The best configuration of PLS is possible when the gaps of both frequencies are zero, but it can not be implemented. There are several techniques to perform dual-impedance matching [6–8], but it is difficult to derive design parameters satisfying a narrow frequency gap. To design an implementable impedance transformer, we propose a method using T-shaped T-lines with double-section shunt stubs, as shown in Fig 3.

Z_n ($n = 1, 2, 3, 4$) and θ_n are the characteristic impedances and electrical length of the T-line, respectively, for a frequency f_h . The electrical length of the T-line is $m\theta_n$ for f_c , where $m = f_c/f_h$. If the impedances of the plasma lamp are not within a certain range, T-lines with characteristic impedances Z_1 and Z_2 are difficult to implement, because the values of Z_1 and Z_2 are either too high or low. To overcome this problem, we change the impedance from Z_h and Z_c to $Z_{h'} = R_{h'} + jX_{h'}$ and $Z_{c'} = R_{c'} + jX_{c'}$, respectively, using series T-lines with characteristic impedance Z_T and electrical length θ_T for the frequency f_h . $Z_{h'}$ and $Z_{c'}$ are determined as follows:

$$Z_{h'} = Z_T \frac{Z_h + jZ_T \tan \theta_T}{Z_T + jZ_h \tan \theta_T}, \quad Z_{c'} = Z_T \frac{Z_c + jZ_T \tan (m\theta_T)}{Z_T + jZ_c \tan (m\theta_T)}. \quad (2)$$

Input admittances $Y_{in,n}(f)$ ($n = 1, 2, 3, 4$) at multiple ports are defined in Fig 3; $Y_{in,n}(f_h)$ and $Y_{in,n}(f_c)$ are the input admittances of the hot and cold states, respectively. Conditions to achieve impedance matching for $Z_{h'}$ at a frequency of f_h and $Z_{c'}$ at a frequency of f_c are as follows:

$$Y_{in,1}(f_h) = Y_{in,1}^*(f_c), \quad Y_{in,2}(f_h) = Y_{in,2}^*(f_c), \quad Y_{in,3}(f_h) = Y_{in,3}^*(f_h) \quad (3)$$

where $*$ denotes the conjugate of a complex value. The $Y_{in,1}(f_h)$ and $Y_{in,1}(f_c)$ are as follows.

$$Y_{in,1}(f_h) = \frac{Z_1 + j(R_{h'} + jX_{h'})\tan\theta_1}{Z_1(R_{h'} + jX_{h'} + jZ_1\tan\theta_1)}, \quad Y_{in,1}(f_c) = Z_1 \frac{Z_1 + j(R_{c'} + jX_{c'})\tan(m\theta_1)}{(R_{c'} + jX_{c'} + jZ_1\tan(m\theta_1))} \quad (4)$$

The equation $Y_{in,1}(f_h) = Y_{in,1}^*(f_c)$ is divided into a real part and an imaginary part, and the equation $\tan(a+b) = (\tan a + \tan b)/(1 - \tan a \tan b)$ is applied to obtain the following

equations.

$$\tan(\theta_1 + m\theta_1) = \frac{Z_1(R_{h'} - R_{c'})}{R_{h'}X_{c'} - R_{c'}X_{h'}} \quad (5)$$

$$\tan(\theta_1 + m\theta_1) = \frac{Z_1(X_{h'} + X_{c'})}{R_{h'}R_{c'} + X_{h'}X_{c'} - Z_1^2} \quad (6)$$

Solving for Z_1 and θ_1 , the followings are obtained

$$Z_1 = \sqrt{\alpha R_{h'}^2 + \frac{(\beta^2 - \alpha)X_{h'}^2}{\alpha - 1}}, \theta_1 = \frac{\pi + \tan^{-1}\left(\frac{Z_1(1 - \alpha)}{X_{h'}(\beta - \alpha)}\right)}{m + 1} \quad (7)$$

where $\alpha = R_{c'} / R_{h'}$ and $\beta = X_{c'} / X_{h'}$.

$Y_{in,2}(f_h)$ can be written as

$$Y_{in,2}(f_h) = \frac{Z_2 + jZ_0 \tan \theta_2}{Z_2(Z_0 + jZ_2 \tan \theta_2)}. \quad (8)$$

To satisfy $Y_{in,2}(f_h) = Y_{in,2}^*(f_c)$ for real Z_2 and Z_0 , $\tan(\theta_2)$ must be the negative value of $\tan(m\theta_2)$ from Eq (8). The result can be written as $\tan(\theta_2) = -\tan(m\theta_2) = \tan(-m\theta_2 - \pi)$, we conclude that $\theta_2 = \pi/(m + 1)$. The condition $G_{1,h} = G_{2,h}$ must be satisfied where $Y_{in,n}(f_h) = G_{n,h} + jB_{n,h}$ and $Y_{in,n}(f_c) = G_{n,c} + jB_{n,c}$. $G_{1,h} = G_{2,h}$ can be written as $(Z_2Z_0 + Z_0Z_2 \tan^2 \theta_2)/(Z_2(Z_0^2 + Z_2^2 \tan^2 \theta_2)) = G_{1,h}$. By solving this equation, Z_2 can be acquired, as follows:

$$Z_2 = \sqrt{\frac{Z_0(1 - Z_0G_{1,h} + \tan^2 \theta_2)}{G_{1,h} \tan^2 \theta_2}} \text{ where } \theta_2 = \frac{\pi}{m + 1}. \quad (9)$$

The final step to realize impedance matching is to cancel susceptances $(B_{1,h} + B_{2,h})$ and $(B_{1,c} + B_{2,c})$ by using an open or shorted shunt stub. The susceptance of the stub must satisfy the following equation:

$$j(B_{1,h} + B_{2,h}) = -jB_{3,h}, \quad j(B_{1,c} + B_{2,c}) = -jB_{3,c}. \quad (10)$$

As the value of $|B_{1,h} + B_{2,h}|$ approaches zero, the characteristic impedance value of the single shunt stub becomes several thousand ohms; however, Nikravan and Atlasbaf demonstrated that using a single-section shunt stub under these conditions is problematic [6]. Alternatively, if we use a double-section stub, the characteristic impedances $Z_n (n = 3, 4)$ can be selected within the feasible impedance range. Using an open stub, $Y_{in,3}$ for cold and hot states are respectively given as follows [8]:

$$Y_{in,3}(f_h) = -j \frac{Z_3 + Z_4 \cot \theta_4 \tan \theta_3}{Z_3(Z_3 \tan \theta_3 - Z_4 \cot \theta_4)} \quad (11)$$

$$Y_{in,3}(f_c) = -j \frac{Z_3 + Z_4 \cot(m\theta_4) \tan(m\theta_3)}{Z_3(Z_3 \tan(m\theta_3) - Z_4 \cot(m\theta_4))}. \quad (12)$$

Table 2. Extracted component values of the dual impedance transformer.

Unit	Z_T	Z_1	Z_2	Z_3	Z_4
Ω	50	10.24	32.4	60	10
Unit	θ_T	θ_1	θ_2	θ_3	θ_4
degree	30.8	112.5	89.74	114.34	533.99

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When Z_3 and Z_4 are determined, θ_3 can be expressed as a function of θ_4 from Eqs (11) and (12) as follows:

$$\theta_3 = \begin{cases} \tan^{-1} \left(\frac{B_{3,h} Z_3 Z_4 \cot \theta_4 + Z_3}{B_{3,h} Z_3^2 - Z_4 \cot \theta_4} \right), & @ f_h \\ \frac{1}{m} \tan^{-1} \left(\frac{B_{3,c} Z_3 Z_4 \cot(m\theta_4) + Z_3}{B_{3,c} Z_3^2 - Z_4 \cot(m\theta_4)} \right), & @ f_c \end{cases} \quad (13)$$

From Eq (13), we can determine θ_3 and θ_4 via numerical analysis. It should be noted that, for a shorted stub, θ_3 and θ_4 can be acquired using similar derivations.

Verification of the proposed method

To validate a dual-state impedance matching method for plasma lamps, we have extracted the parameters of the dual-state matching circuit for the plasma lamp model (Table 1). The extracted impedance transformer component values corresponding to implementation of double-section open-circuited stub with $Z_3 = 60 \Omega$ and $Z_4 = 10 \Omega$ are listed in Table 2.

The impedances of the plasma lamp with the dual-state impedance transformer were 50Ω in the hot and the cold states at frequencies of 2456 and 2470 MHz, respectively. The dual impedance transformer is implemented using microstrip line ($\epsilon_r = 3.5$, height = 0.706 mm Taconic substrate). The overall PLS test setup with the implemented microstrip line (IML) matching circuit is shown in Fig 4. Fig 5a and 5b show the simulated return losses and impedances of the load as resulting from ideal transmission line matching circuit and IML matching

Fig 4. PLS test setup.

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Fig 5. Return losses (Fig 5a) and impedances (Fig 5b) of cold and hot states following impedance transformer implementation.

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circuit respectively. The result of Smith chart in Fig 5b shows that the impedances for both cold and hot states are well matched for 50 Ω .

The results show a significant decrease in the return loss for both states at the given frequencies. By applying the proposed method, the return losses are simultaneously improved to -31 dB and -22 dB from -6 dB and -0.8 dB for hot and cold states, respectively. The proposed scheme allows dual-state impedance matching for two bands with a frequency spacing of 14 MHz, which is a narrower frequency interval than the previous results [4, 6–9]. Table 3 compares the performance of the proposed method with those of the previous methods. There are works of impedance matching for a plasma bulb [2, 4, 9], but they use a single matching for an

Table 3. Comparison with the previous works.

	f_h [MHz]	f_c/f_h	$ f_c - f_h $	$ S_{11} @f_h$	$ S_{11} @f_c$	Matching
[2]	433	1.02	11.5 MHz	-	-2.6 dB	Single
[4]	2400	1.106	245 MHz	-10.4 dB	-32.8 dB	Single
[9]	2390	1.029	70 MHz	-40 dB	-10 dB	Single
[6]	6000	1.33	2000 MHz	-45 dB	-23 dB	Dual
[7]	1500	1.5	750 MHz	-23 dB	-35 dB	Dual
[8]	900	2.6	1500 MHz	-58 dB	-20 dB	Dual
This work	2457	1.005	14 MHz	-31 dB	-22 dB	Dual

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impedance of one plasma state. The dual impedance matching methods [6–8] are not designed for a narrow frequency band gap because they do not consider very different impedances within a narrow frequency band gap. The proposed method is a new attempt to match two very different impedances in the near frequency band as far as the authors know.

The plasma lamp used in the experiment is turned on by contacting with a metal without the proposed dual matching. Using the proposed method, the plasma lamp can be turned on without metal contact and the low return loss in the hot-state can be achieved simultaneously.

Conclusion

We propose an accurate plasma lamp circuit model and a corresponding dual-state impedance matching method that considers both cold and hot states. Subsequently, a plasma lighting system with 300 W GaN SSPA was implemented for model validation. Results showed that the dual matching between the SSPA and plasma lamp improved the power efficiency of the SSPA in the hot-state and facilitated bulb operation in the cold-state. Thus, the proposed model and matching method can be applied to improve the efficiency of an RF-driven energy system.

Supporting information

S1 File. Fig 2 Plasma bulb modeling and measurement. Comparison of measured and simulated impedances of plasma lamp.
(TXT)

S2 File. Fig 5 Ideal cold and hot state S₁₁ data set compare with IML.
(TXT)

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