RESEARCH ARTICLE

A New Exponential Ratio-Type Estimator with Linear Combination of Two Auxiliary Variables

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Abstract

In sample surveys, it is usual to make use of auxiliary information to increase the precision of estimators. We propose a new exponential ratio-type estimator of a finite population mean using linear combination of two auxiliary variables and obtain mean square error (MSE) equation for proposed estimator. We find theoretical conditions that make proposed estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh *et al.* In addition, we support these theoretical results with the aid of two numerical examples.

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Citation: Lu J, Yan Z, Peng X (2014) A New Exponential Ratio-Type Estimator with Linear Combination of Two Auxiliary Variables. PLoS ONE 9(12): e116124. doi:10.1371/journal.pone.0116124

Editor: Gang Han, Yale School of Public Health, United States of America

Received: July 10, 2014

Accepted: December 2, 2014

Published: December 26, 2014

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Data Availability: The authors confirm that all data underlying the findings are fully available without restriction. All relevant data are within the paper.

Funding: This work was supported by the National Natural Science Foundation of China (No. 11361036 and No. 11461051). The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing Interests: The authors have declared that no competing interests exist.

Introduction

In the sampling theory, the use of supplementary information provided by auxiliary variables in survey sampling was extensively discussed. Such information is generally used in ratio, product and regression type estimators for the estimation of population mean of study variable. In literature, number of authors introduced many ratio and regression type estimators by using general linear transformation of the auxiliary variable [1–5]. For recent development, exponential estimators have been widely studied by several authors. Singh *et al.* [6] suggested the modified exponential ratio and product estimators in two phase sampling and analyzes their properties, these estimators were compared for their precision with simple mean per unit, usual double sampling ratio and product estimators. On base of the estimator of Singh *et al.*, Ozgul and Cingi [7] suggested a class of exponential regression cum ratio estimator in two phase sampling, MSE of the proposed estimator were obtained. However, these estimators were considered using one auxiliary variate.

In this study, a new exponential ratio-type estimator using linear combination of two auxiliary variates is considered to estimate a finite population mean for the variable of interest. And we obtain mean square error (MSE) equation for the proposed estimator. We find theoretical conditions that make proposed exponential ratio-type estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh *et al.* We compared the traditional ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator proposed by Abu-Dayeh *et al.* and proposed exponential ratio-type estimator using two statistic data sets. And we obtained the satisfactory results.

Materials and Methods

The existed estimators

The traditional multivariate ratio estimator using information of two auxiliary variables x_1 and x_2 to estimate the population mean, $\bar{Y}$ [8], as follows:

$$\bar{y}_{MR} = \varepsilon_1 \bar{y} \frac{\bar{X}_1}{\bar{x}_1} + \varepsilon_2 \bar{y} \frac{\bar{X}_2}{\bar{x}_2} \quad (1)$$

where $\bar{y}$ denote the sample means of the variable y , $\bar{x}_i$ and $\bar{X}_i (i=1,2)$ denote respectively the sample and the population means of the variable $x_i (i=1,2)$; $\varepsilon_1, \varepsilon_2$ are the weights that satisfy the condition, $\varepsilon_1 + \varepsilon_2 = 1$

The MSE of this traditional multivariate ratio estimator is given by

$$MSE(\bar{y}_{MR}) \cong \frac{1-f}{n} (S_y^2 + \varepsilon_1^2 R_1^2 S_{x_1}^2 + \varepsilon_2^2 R_2^2 S_{x_2}^2 - 2\varepsilon_1 R_1 S_{yx_1} - 2\varepsilon_2 R_2 S_{yx_2} + 2\varepsilon_1 \varepsilon_2 R_1 R_2 S_{x_1 x_2}) \quad (2)$$

where $f = \frac{n}{N}$; n and N are respectively the number of units in the sample and the population; $S_y^2, S_{x_1}^2$ and $S_{x_2}^2$ are the population variances of Y, X_1 and X_2 , respectively; $S_{x_1 x_2}, S_{yx_1}$ and S_{yx_2} are the population covariances between X_1 and X_2 , Y and X_1, Y and X_2 , respectively; $R_1 = \frac{\bar{Y}}{\bar{X}_1}, R_2 = \frac{\bar{Y}}{\bar{X}_2}$.

The optimum values of ε_1 and ε_2 are given by

$$\varepsilon_1^* = \frac{S_{x_2}^2 R_2^2 - S_{yx_2} R_2 + S_{yx_1} R_1 - S_{x_1 x_2} R_1 R_2}{S_{x_1}^2 R_1^2 + S_{x_2}^2 R_2^2 - 2S_{x_1 x_2} R_1 R_2}, \varepsilon_2^* = 1 - \varepsilon_1^*$$

The minimum MSE of $\bar{y}_{MR}$ can be shown to be:

$$MSE_{\min}(\bar{y}_{MR}) \cong \frac{1-f}{n} (S_y^2 + \varepsilon_1^* 2R_1^2 S_{x_1}^2 + \varepsilon_2^* 2R_2^2 S_{x_2}^2 - 2\varepsilon_1^* R_1 S_{yx_1} - 2\varepsilon_2^* R_2 S_{yx_2} + 2\varepsilon_1^* \varepsilon_2^* R_1 R_2 S_{x_1 x_2}) \quad (3)$$

Bahl and Tuteja [9] proposed an exponential ratio-type estimator which is given by

$$\bar{y}_{BT} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (4)$$

The MSE of $\bar{y}_{BT}$ is given by

$$MSE(\bar{y}_{BT}) \cong \frac{1-f}{n} (S_y^2 + \frac{1}{4} R^2 S_x^2 - R S_{yx}) \quad (5)$$

Abu-Dayeh *et al.* [10] proposed the estimator using two auxiliary variables given by

$$\bar{y}_{AD} = k_1 \bar{y} \left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\gamma_1} + k_2 \bar{y} \left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \quad (6)$$

where $k_1 + k_2 = 1$.

MSE of this estimator is given as follows:

$$MSE(\bar{y}_{AD}) \cong \frac{1-f}{n} [S_y^2 + k_1^2 \gamma_1^2 R_1^2 S_{x_1}^2 + k_2^2 \gamma_2^2 R_2^2 S_{x_2}^2 + 2k_1 \gamma_1 R_1 S_{yx_1} + 2k_2 \gamma_2 R_2 S_{yx_2} + 2k_1 k_2 \gamma_1 \gamma_2 R_1 R_2 S_{x_1 x_2}] \quad (7)$$

The optimum values of k_1 and k_2 are given by

$$k_1^* = \frac{\gamma_2^2 R_2^2 S_{x_2}^2 - \gamma_1 R_1 S_{yx_1} + \gamma_2 R_2 S_{yx_2} - \gamma_1 \gamma_2 R_1 R_2 S_{x_1 x_2}}{\gamma_1^2 R_1^2 S_{x_1}^2 - 2\gamma_1 \gamma_2 R_1 R_2 S_{x_1 x_2} + \gamma_2^2 R_2^2 S_{x_2}^2}, \quad k_2^* = 1 - k_1^*$$

$$MSE_{\min}(\bar{y}_{AD}) \cong \frac{1-f}{n} [S_y^2 + k_1^{*2} \gamma_1^2 R_1^2 S_{x_1}^2 + k_2^{*2} \gamma_2^2 R_2^2 S_{x_2}^2 + 2k_1^* \gamma_1 R_1 S_{yx_1} + 2k_2^* \gamma_2 R_2 S_{yx_2} + 2k_1^* k_2^* \gamma_1 \gamma_2 R_1 R_2 S_{x_1 x_2}] \quad (8)$$

The proposed family of ratio estimators

We propose a new exponential ratio-type estimator using linear combination of two auxiliary variables as follows:

$$\bar{y}_{lcr} = \bar{y} \exp\left(\frac{\bar{X}_{lc} - \bar{x}_{lc}}{\bar{X}_{lc} + \bar{x}_{lc}}\right) \quad (9)$$

where $\bar{X}_{lc} = w_1 \bar{X}_1 + w_2 \bar{X}_2$, $\bar{x}_{lc} = w_1 \bar{x}_1 + w_2 \bar{x}_2$; w_1, w_2 are weights that satisfy the condition: $w_1 + w_2 = 1$.

MSE of this estimator can be found using Taylor series method defined as

$$\begin{aligned} f(\bar{y}, \bar{x}_1, \bar{x}_2) &\cong f(\bar{Y}, \bar{X}_1, \bar{X}_2) + \frac{\partial f}{\partial \bar{y}} \bigg|_{(\bar{Y}, \bar{X}_1, \bar{X}_2)} (\bar{y} - \bar{Y}) \\ &+ \frac{\partial f}{\partial \bar{x}_1} \bigg|_{(\bar{Y}, \bar{X}_1, \bar{X}_2)} (\bar{x}_1 - \bar{X}_1) + \frac{\partial f}{\partial \bar{x}_2} \bigg|_{(\bar{Y}, \bar{X}_1, \bar{X}_2)} (\bar{x}_2 - \bar{X}_2) \end{aligned} \quad (10)$$

where $f(\bar{y}, \bar{x}_1, \bar{x}_2) = \bar{y}_{lcr}$

$$\bar{y}_{lcr} - \bar{Y} \cong (\bar{y} - \bar{Y}) - \frac{1}{2} w_1 R_{lc} (\bar{x}_1 - \bar{X}_1) - \frac{1}{2} w_2 R_{lc} (\bar{x}_2 - \bar{X}_2)$$

where $\frac{\bar{Y}}{w_1 \bar{X}_1 + w_2 \bar{X}_2} = R_{lc}$.

The MSE of this new multivariate exponential ratio-type estimator is given by

$$\begin{aligned} MSE(\bar{y}_{lcr}) &= E(\bar{y}_{lcr} - \bar{Y})^2 \\ &\cong \frac{1-f}{n} \\ &(S_y^2 + \frac{1}{4} w_1^2 R_{lc}^2 S_{x_1}^2 + \frac{1}{4} w_2^2 R_{lc}^2 S_{x_2}^2 - w_1 R_{lc} S_{yx_1} - w_2 R_{lc} S_{yx_2} + \frac{1}{2} w_1 w_2 R_{lc}^2 S_{x_1 x_2}) \end{aligned} \quad (11)$$

The optimum values of w_1 and w_2 are given by

$$w_1^* = \frac{B}{A+B}, \quad w_2^* = 1 - w_1^*$$

$$\begin{aligned} \text{where } A &= 2S_{yx_2} \bar{X}_1^2 - S_{x_1 x_2} \bar{X}_1 \bar{Y} - 2S_{yx_1} \bar{X}_1 \bar{X}_2 + S_{x_1}^2 \bar{X}_2 \bar{Y}, \\ B &= S_{x_2}^2 \bar{X}_1 \bar{Y} - 2S_{yx_2} \bar{X}_1 \bar{X}_2 - S_{x_1 x_2} \bar{Y} \bar{X}_2 + 2S_{yx_1} \bar{X}_2^2 \end{aligned}$$

The minimum MSE of $\bar{y}_{lcr}$ can be shown to be:

$$MSE_{\min}(\bar{y}_{lcr}) \cong \frac{1-f}{n} (S_y^2 + \frac{1}{4} w_1^{*2} R_{lc}^{*2} S_{x_1}^2 + \frac{1}{4} w_2^{*2} R_{lc}^{*2} S_{x_2}^2 - w_1^* R_{lc}^* S_{yx_1} - w_2^* R_{lc}^* S_{yx_2} + \frac{1}{2} w_1^* w_2^* R_{lc}^{*2} S_{x_1 x_2}) \quad (12)$$

$$\text{where } \frac{\bar{Y}}{w_1^* \bar{X}_1 + w_2^* \bar{X}_2} = R_{lc}^*$$

Efficiency comparisons

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of traditional multivariate ratio estimator using information of two auxiliary variables given in Eq. (3) as follows:

$$MSE_{\min}(\bar{y}_{lcr}) < MSE_{\min}(\bar{y}_{MR})$$

if

$$\begin{aligned} & (\frac{1}{4} w_1^{*2} R_{lc}^{*2} - \varepsilon_1^{*2} R_1^2) S_{x_1}^2 + (\frac{1}{4} w_2^{*2} R_{lc}^{*2} - \varepsilon_2^{*2} R_2^2) S_{x_2}^2 \\ & - (w_1^* R_{lc}^* - 2\varepsilon_1^* R_1) S_{yx_1} - (w_2^* R_{lc}^* - 2\varepsilon_2^* R_2) S_{yx_2} \\ & + (\frac{1}{2} w_1^* w_2^* R_{lc}^{*2} - 2\varepsilon_1^* \varepsilon_2^* R_1 R_2) S_{x_1 x_2} < 0 \end{aligned} \quad (13)$$

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of the estimator of Bahl and Tuteja given in Eq. (5) as follows:

$$MSE_{\min}(\bar{y}_{lcr}) < MSE(\bar{y}_{BTi})$$

if

$$\begin{aligned} & \frac{1}{4} (w_1^{*2} R_{lc}^{*2} S_{x_1}^2 + w_2^{*2} R_{lc}^{*2} S_{x_2}^2 - R_i^2 S_{x_i}^2) - (w_1^* R_{lc}^* S_{yx_1} + w_2^* R_{lc}^* S_{yx_2} - R_i S_{yx_i}) \\ & + \frac{1}{2} w_1^* w_2^* R_{lc}^{*2} S_{x_1 x_2} < 0 \quad (i=1,2) \end{aligned} \quad (14)$$

where $\bar{y}_{BTi}$ denote $\bar{y}_{BT}$ using auxiliary variable x_i ($i=1,2$).

We compare the MSE of the proposed exponential ratio-type estimator using linear combination of two auxiliary variables given in Eq. (12) with the MSE of the estimator of Abu-Dayeh *et al* given in Eq.(8) as follows:

Table 1. Data Statistics of example 1.

$N = 18$	$\bar{X}_2 = 38.444$	$S_{x_1}^2 = 74.679$	$S_{yx_2} = 46.512$
$n = 8$	$\bar{Y} = 13.797$	$S_{x_2}^2 = 174.967$	$S_{x_1x_2} = 11.546$
$\bar{X}_1 = 2.444$	$S_y^2 = 35.486$	$S_{yx_1} = 42.262$	

doi:10.1371/journal.pone.0116124.t001

$$MSE_{\min}(\bar{y}_{lcr}) < MSE_{\min}(\bar{y}_{AD})$$

if

$$\begin{aligned} & \left(\frac{1}{4} w_1^{*2} R_{lc}^{*2} - k_1^{*2} \gamma_1^2 R_1^2 \right) S_{x_1}^2 + \left(\frac{1}{4} w_2^{*2} R_{lc}^{*2} - k_2^{*2} \gamma_2^2 R_2^2 \right) S_{x_2}^2 \\ & - (w_1^* R_{lc}^* - 2k_1^* \gamma_1 R_1) S_{yx_1} - (w_2^* R_{lc}^* - 2k_2^* \gamma_2 R_2) S_{yx_2} \\ & + \left(\frac{1}{2} w_1^* w_2^* R_{lc}^{*2} - 2k_1^* k_2^* \gamma_1 \gamma_2 R_1 R_2 \right) S_{x_1x_2} < 0 \end{aligned} \quad (15)$$

Numerical illustration

To examine the merits of the proposed estimator, we have considered two natural population data sets. we apply the traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator of Abu-Dayeh *et al*, given in Eqs. (1), (4), (6) and proposed exponential ratio-type estimator of a finite population mean using linear combination of two auxiliary variables, given in Eq. (9). The MSE of these estimators are computed as given in Eqs. (3), (5), (8), (12).

Example 1. In order to precisely estimate cotton output in one region, the sample size $n=8$ villages were taken out from $N=18$ villages using SRSWOR[8].

Y: Cotton output.

X_1 : The area of the plant.

X_2 : The proportion of good seed.

The statistics of example 1 are given in [table 1](#)

Example 2. The data set of this example can be seen in the reference [3].

Y: Number of ‘placebo’ children.

X_1 : Number of paralytic polio cases in the placebo group.

Table 2. Data Statistics of example 2.

$N = 34$	$\bar{X}_2 = 2.91$	$S_{x_1}^2 = 10.180$	$S_{yx_2} = 9.990$
$n = 10$	$\bar{Y} = 4.92$	$S_{x_2}^2 = 9.732$	$S_{x_1x_2} = 6.805$
$\bar{X}_1 = 2.59$	$S_y^2 = 24.807$	$S_{yx_1} = 11.642$	

doi:10.1371/journal.pone.0116124.t002

Table 3. MSE Values of Ratio Estimators of example 1.

Estimators	MSE
$\bar{y}_{MR}$	0.2707
$\bar{y}_{BT1}$	27.2133
$\bar{y}_{BT2}$	1.6963
$\bar{y}_{AD}$	2.7847
$\bar{y}_{lcr}$	0.2506

doi:10.1371/journal.pone.0116124.t003

X_2 : Number of paralytic polio cases in the ‘not inoculated’ group.
The statistics of example 2 are given in [table 2](#)

Results and Discussion

MSE values of the traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja, the estimator of Abu-Dayeh *et al* and proposed exponential ratio-type estimator using linear combination of two auxiliary variables can be seen in [Table 3](#) and [Table 4](#).

From [Table 3](#) and [4](#), we notice that our proposed exponential ratio-type estimator using linear combination of two auxiliary variables is more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator of Abu-Dayeh *et al*. We examine the conditions, determined in paper, for two data sets,

The examining of condition (13), (14) and (15) about example 1 can be seen as follows.

(i) condition (13)

$$\begin{aligned} & \left(\frac{1}{4} w_1^{*2} R_{lc}^{*2} - \varepsilon_1^{*2} R_1^2 \right) S_{x_1}^2 + \left(\frac{1}{4} w_2^{*2} R_{lc}^{*2} - \varepsilon_2^{*2} R_2^2 \right) S_{x_2}^2 \\ & - (w_1^* R_{lc}^* - 2\varepsilon_1^* R_1) S_{yx_1} - (w_2^* R_{lc}^* - 2\varepsilon_2^* R_2) S_{yx_2} \\ & + \left(\frac{1}{2} w_1^* w_2^* R_{lc}^{*2} - 2\varepsilon_1^* \varepsilon_2^* R_1 R_2 \right) S_{x_1 x_2} = -0.289 < 0 \end{aligned}$$

(ii) condition (14)

$$\begin{aligned} & \frac{1}{4} (w_1^{*2} R_{lc}^{*2} S_{x_1}^2 + w_2^{*2} R_{lc}^{*2} S_{x_2}^2 - R_1^2 S_{x_1}^2) - (w_1^* R_{lc}^* S_{yx_1} + w_2^* R_{lc}^* S_{yx_2} - R_1 S_{yx_1}) \\ & + \frac{1}{2} w_1^* w_2^* R_{lc}^{*2} S_{x_1 x_2} = -388.263 < 0 \end{aligned}$$

Table 4. MSE Values of Ratio Estimators of example 2.

Estimators	MSE
$\bar{y}_{MR}$	0.9062
$\bar{y}_{BT1}$	0.8383
$\bar{y}_{BT2}$	1.0497
$\bar{y}_{AD}$	0.9804
$\bar{y}_{lcr}$	0.8216

doi:10.1371/journal.pone.0116124.t004

$$\begin{aligned} & \frac{1}{4}(w_1^{*2}R_{lc}^{*2}S_{x_1}^2 + w_2^{*2}R_{lc}^{*2}S_{x_2}^2 - R_2^2S_{x_2}^2) - (w_1^*R_{lc}^*S_{yx_1} + w_2^*R_{lc}^*S_{yx_2} - R_2S_{yx_2}) \\ & + \frac{1}{2}w_1^*w_2^*R_{lc}^{*2}S_{x_1x_2} = -20.819 < 0 \end{aligned}$$

(iii) condition (15)

$$\begin{aligned} & (\frac{1}{4}w_1^{*2}R_{lc}^{*2} - k_1^{*2}\gamma_1^2R_1^2)S_{x_1}^2 + (\frac{1}{4}w_2^{*2}R_{lc}^{*2} - k_2^{*2}\gamma_2^2R_2^2)S_{x_2}^2 \\ & - (w_1^*R_{lc}^* - 2k_1^*\gamma_1R_1)S_{yx_1} - (w_2^*R_{lc}^* - 2k_2^*\gamma_2R_2)S_{yx_2} \\ & + (\frac{1}{2}w_1^*w_2^*R_{lc}^{*2} - 2k_1^*k_2^*\gamma_1\gamma_2R_1R_2)S_{x_1x_2} = -36.492 < 0 \end{aligned}$$

The examining of condition (13), (14) and (15) about example 2 can be seen as follows.

(iv) condition (13)

$$\begin{aligned} & (\frac{1}{4}w_1^{*2}R_{lc}^{*2} - \varepsilon_1^{*2}R_1^2)S_{x_1}^2 + (\frac{1}{4}w_2^{*2}R_{lc}^{*2} - \varepsilon_2^{*2}R_2^2)S_{x_2}^2 \\ & - (w_1^*R_{lc}^* - 2\varepsilon_1^*R_1)S_{yx_1} - (w_2^*R_{lc}^* - 2\varepsilon_2^*R_2)S_{yx_2} \\ & + (\frac{1}{2}w_1^*w_2^*R_{lc}^{*2} - 2\varepsilon_1^*\varepsilon_2^*R_1R_2)S_{x_1x_2} = -1.198 < 0 \end{aligned}$$

(v) condition (14)

$$\begin{aligned} & \frac{1}{4}(w_1^{*2}R_{lc}^{*2}S_{x_1}^2 + w_2^{*2}R_{lc}^{*2}S_{x_2}^2 - R_1^2S_{x_1}^2) - (w_1^*R_{lc}^*S_{yx_1} + w_2^*R_{lc}^*S_{yx_2} - R_1S_{yx_1}) \\ & + \frac{1}{2}w_1^*w_2^*R_{lc}^{*2}S_{x_1x_2} = -0.235 < 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{4}(w_1^{*2}R_{lc}^{*2}S_{x_1}^2 + w_2^{*2}R_{lc}^{*2}S_{x_2}^2 - R_2^2S_{x_2}^2) - (w_1^*R_{lc}^*S_{yx_1} + w_2^*R_{lc}^*S_{yx_2} - R_2S_{yx_2}) \\ & + \frac{1}{2}w_1^*w_2^*R_{lc}^{*2}S_{x_1x_2}) = -3.230 < 0 \end{aligned}$$

(vi) condition (15)

$$\begin{aligned} & (\frac{1}{4}w_1^{*2}R_{lc}^{*2} - k_1^{*2}\gamma_1^2R_1^2)S_{x_1}^2 + (\frac{1}{4}w_2^{*2}R_{lc}^{*2} - k_2^{*2}\gamma_2^2R_2^2)S_{x_2}^2 \\ & - (w_1^*R_{lc}^* - 2k_1^*\gamma_1R_1)S_{yx_1} - (w_2^*R_{lc}^* - 2k_2^*\gamma_2R_2)S_{yx_2} \\ & + (\frac{1}{2}w_1^*w_2^*R_{lc}^{*2} - 2k_1^*k_2^*\gamma_1\gamma_2R_1R_2)S_{x_1x_2} = -2.249 < 0 \end{aligned}$$

The result shows that the condition (13), (14) and condition (15) are satisfied. Therefore, we suggest that we should apply the proposed estimator to example 1 and 2.

Conclusions

We develop a new exponential ratio-type estimator of a finite population mean using two auxiliary variables and find theoretical conditions that make proposed estimator more efficient than traditional multivariate ratio estimator using information of two auxiliary variables, the estimator of Bahl and Tuteja and the estimator proposed by Abu-Dayeh et al. These theoretical conditions are also satisfied by the results of two numerical examples.

Author Contributions

Conceived and designed the experiments: JL. Performed the experiments: JL. Analyzed the data: JL ZY XP. Contributed reagents/materials/analysis tools: JL ZY XP. Wrote the paper: JL.

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