Estimating Seasonal Nitrogen Removal and Biomass Yield by Annuals with the Extended Logistic Model

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Abstract

The Extended Logistic Model (ELM) has been previously shown to adequately describe seasonal biomass production and N removal with respect to applied N for several types of annuals and perennials. In this analysis, data from a corn (*Zea mays* L.) study with variable applied N were analyzed to test hypotheses that certain parameters in the ELM are invariant with respect to site specific attributes, like environmental conditions and soil type. Invariance to environmental conditions suggests such parameters may be functions of the crop characteristics and certain other management practices alone (like plant population, planting date, harvest date). The first parameter analyzed was Δb , the difference between the N uptake shifting parameter and the biomass shifting parameter. The second parameter tested was N_{cm} , the maximum N concentration. Both parameters were shown to be statistically invariant, despite soil and site differences. This was determined using analysis of variance with normalized nonlinear regression of the ELM on the data from the study. This analysis lends further evidence that there are common parameters involved in the ELM that do not rely on site-specific or situation-specific factors. More insight into the derivation of, definition of, and logic behind the various parameters involved in the model are also given in this paper.

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Introduction

Effective water and nutrient management plays an essential role in future attempts at sustainable agricultural production. As the world's population continues to grow, the potable water supply is limited and must be guarded from unnecessary withdrawals and contamination from excessive nutrient loads. Strict monitoring and exercises in groundwater modeling of all agricultural operations is cost prohibitive. It therefore becomes necessary to investigate crop nutrient removal from their environment, and to adopt management procedures and rules that are based on a sound scientific foundation.

Overman et al. first proposed the logistic model as a nutrient management tool to describe seasonal biomass yield dependence of forage grasses on applied N [1]. The original application of the logistic model to plant biomass production was based on inductive reasoning, a process where inferences are made from "real world" observations [2]. While inductive reasoning is innately a more empirical method of model development, all models, no matter the complexity, have some element of empiricism [3]. It is because of the application to the "real world," that engineering and the applied sciences are, for practical reasons, inherently more empirical. The logistic model was extended to include seasonal plant N uptake (removal from the environment) dependence on applied N by forage grasses [4] and then for annuals, like corn [5]. The ELM is a five parameter, non-linear, parametric model that is capable of describing the seasonal biomass yields, N uptake, and N concentration with respect to applied N. Work conducted over the years has indicated that the ELM can effectively describe both

annual [6–11] and perennial [8,12–20] crops, for a wide range of nutrient inputs.

The ELM, begins with the simple logistic expression that relates N uptake, N_u , to the N applied, N , which is given by the following relationship:

$$N_u = \frac{A_N}{1 + \exp(b_N - cN)} \quad (1)$$

where A_N is the relative maximum N uptake in kg ha^{-1} , b_N is the dimensionless N uptake shifting parameter, and c is the applied N response parameter given in ha kg^{-1} . The phase relationship between biomass production, Y , and N uptake, N_u , is given by

$$Y = \frac{Y_m N_u}{k_N + N_u} \quad (2)$$

where Y_m is the maximum potential biomass production in Mg ha^{-1} , and k_N is the N uptake response parameter in kg ha^{-1} . The transformation of Eq. (1) by using Eq. (2) yields the following logistic expression:

$$Y = \frac{A}{1 + \exp(b - cN)} \quad (3)$$

that relates biomass production to applied N, where A is the relative maximum biomass production in Mg ha^{-1} , b is the biomass yield shifting parameter, and c is the same applied N

response parameter that applies to the N uptake logistic equation. The parameters A_N , A , b_N , b , and c are currently key parameters used with the ELM, and are the easiest to determine from regression analysis.

From the transformation of Eq. (1) into Eq. (3), the relative maximum biomass yield parameter can be written in terms of the maximum potential biomass yield parameter, the relative maximum N uptake parameter, and the N uptake response parameter.

$$A = \frac{Y_m A_N}{k_N + A_N} \quad (4)$$

The biomass shifting parameter can be written in terms of the N uptake shifting parameter, the relative maximum N uptake parameter, and the N uptake response coefficient.

$$b = b_N + \ln\left(\frac{k_N}{k_N + A_N}\right) \quad (5)$$

The difference between the shifting parameter for N uptake and the biomass yield can be written as the following:

$$\Delta b = b_N - b = -\ln\left(\frac{k_N}{k_N + A_N}\right) \quad (6)$$

N concentration is simply defined as the ratio of N uptake to biomass production. This leads to the following relationship:

$$N_c = \frac{N_u}{Y} = N_{cm} \frac{1 + \exp(b - cN)}{1 + \exp(b_N - cN)} \quad (7)$$

The N concentration model suggests that as N applied is increased to exorbitant levels, there is a maximum limit to the N concentration, N_{cm} . This maximum limit is simply the ratio of the relative maximum N uptake with respect to applied N, A_N , to the relative maximum biomass production with respect to applied N, A .

$$\lim_{N \rightarrow +\infty} N_c = \frac{A_N}{A} = N_{cm} \quad (8)$$

It seems logical to suggest that as the background amount of N present in the soil is decreased, the N concentration would be reduced to some prescribed lower limit, as there would be lower limit on the percent of proteins present in a given crop to sustain any growth. Mathematically, the model suggests that this lower limit of N concentration, N_{cb} , is a function of the maximum concentration and the difference between the N uptake shifting parameter and the biomass shifting parameter.

$$\lim_{N \rightarrow -\infty} N_c = N_{cm} \exp(-\Delta b) = N_{cl} \quad (9)$$

Also, this lower limit of N concentration, N_{cb} , can be found by taking the ratio of the N uptake response coefficient to the maximum potential biomass production parameter.

$$N_{cl} = \frac{k_N}{Y_m} \quad (10)$$

From the phase relationship between biomass production and N uptake, Eq. (2), N concentration can be found from the following equation with respect to N uptake:

$$N_c = \frac{k_N}{Y_m} + \frac{1}{Y_m} N_u \quad (11)$$

This predicts a linear relationship between N concentration, N_c , and N uptake, N_u . The line should have a slope equal to the inverse of the maximum potential biomass production and an intercept that equals the ratio of the N uptake response parameter to maximum potential biomass production. As this is a phase relationship, this is a functional segment that is bounded between N uptake values from 0 to the peak of A_N , and between N concentration values between N_{cl} and N_{cm} .

From the earlier work of Overman et al. [5], it has been shown that for a given site the applied N response parameter, c , and the N uptake intercept parameter, b_N , and the biomass intercept parameter, b , are not unique to the ELM when applied to grain or the whole plant. Meaning that the harvest index is constant for a given site. Their analysis showed that the only differences between the grain and the whole plant appear in the relative maximum N uptake parameter, A_N , and the relative maximum biomass production parameter, A [5]. Because the b_N , b , and c parameters were shown to be constant for both grain fraction and total biomass production, all the differences in both grain N uptake and biomass production, and the differences in the total plant N uptake and biomass production can be estimated with seven model parameters and a value for the seasonal amount of N applied. This is a comparative reduction of three parameters when b_N , b , and c are not held constant between grain and total plant biomass production.

The goal of this work is to continue to elevate the Extended Logistic Model (ELM) beyond the empiricism of it nascent beginnings and achieve a balance between what can be measured and what should be modeled, as called for by Montieth [21]. The intent is to shed new light on the significance of parameters used in the ELM and to contribute to the search for commonality among parameters. Normalized non-linear regression and analysis of variance (ANOVA) were used to show the invariance of two model parameters with respect to environmental differences, namely soil type and water availability.

Methods

Data Set

This analysis uses data collected by Eugene Kamprath from a corn (Pioneer 3320) N-rate field study that was conducted at three regional research stations in North Carolina from 1981 to 1984. A detailed explanation of the field experiment has been previously reported [22]. Supplemental irrigation was provided at the Clayton experiment station for the well-drained Dothan loamy fine sand (*fine-loamy, siliceous, thermic Plinthic Kandiudults*), at a rate of 10 to 12 cm a season, except for 1982 when no additional water was supplied. No irrigation was provided at the Kinston station for the well-drained Goldsboro sandy loam (*fine-loamy, siliceous, thermic Aquic Paleudults*). At the Plymouth experiment station, no irrigation was provided for the poorly-drained Portsmouth very fine sandy

Table 1. Analysis of variance for model parameters for corn grain and total plant biomass production and for corn grain and total plant N uptake, grown on three different soils.

Mode	Parameters Estimated	Degrees of Freedom	Normalized Residual Sum of Squares	Normalized Mean Sum of Squares	F Value
I	21	39	0.0247	0.000633	----
II, Common Δb .	19	41	0.0259	0.000631	----
II-I	--	2	0.00119	0.000595	0.940
III, Common Δb , & N_{cm} .	15	45	0.0267	0.000594	----
III-I	--	6	0.00202	0.000337	0.533

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loam (*fine-loamy over sandy or sandy-skeletal mixed, thermic Typic Umbraquills*). The experiments at each station were set up as a RCB design, with four replications. Both total plant and grain fraction biomass were sampled, and every year the experiment was conducted at a new location within the same soil type at each station. This was to limit the impact on the experiment of any residual N in the soil from the previous year. The fertilizer treatments were applied in the form of NH_4NO_3 at rates of 0, 56, 112, 168, and 224 kg ha^{-1} of N. Average values over the four year period were combined for each of the different treatments and the model parameters were evaluated based on those combined averages.

Normalization

Parameters A_N and A for grain, A_N and A for total plant, and b_N , b and c for each site are determined simultaneously, using Newton-Raphson non-linear regression. A detailed description of Newton-Raphson non-linear regression of logistic equation can be found in Overman and Scholtz [8]. The attempt of this methodology is to consistently distribute the standard error amongst all those parameters for further analysis. Because of the unit and an order of magnitude difference between biomass and N uptake parameter values, as well as a subsequent order of magnitude difference between grain and total plant parameter values, a normalization routine is also employed. The error sum of squares for each individual site is initially written as

$$ESS_{NORM} = \underbrace{\sum \left(\frac{Y_g}{A_g} - \frac{1}{1 + \exp(b - cN)} \right)^2 + \sum \left(\frac{N_{ug}}{A_{Ng}} - \frac{1}{1 + \exp(b_N - cN)} \right)^2}_{\text{grain fraction}} + \underbrace{\sum \left(\frac{Y_t}{A_t} - \frac{1}{1 + \exp(b - cN)} \right)^2 + \sum \left(\frac{N_{ut}}{A_{Nt}} - \frac{1}{1 + \exp(b_N - cN)} \right)^2}_{\text{total plant}} \quad (12)$$

where the total normalized error is resultant from the sum of the normalized error from the three sites. For this study the initial Hessian matrix is 21 by 21 elements and paired with a 21 element Jacobian vector. As a result of the normalization procedure, performing the Newton-Raphson procedure can diverge more readily than a non-normalized procedure. Because of this, it is important to establish a reasonable initial guess for each parameter. First, individual logistic response are evaluated for each data set from all sites, for grain biomass, grain N uptake, for total plant biomass, and total plant N uptake. This results in six A values, six A_N values, six b values, six b_N values, and 12 values for the c parameter. Using the fact that it can be shown for two

straight lines

$$y_1 = m_1x + b_1 \quad (13)$$

and

$$y_2 = m_2x + b_2, \quad (14)$$

Figure 1. Dependence of grain and total plant N uptake on applied N for corn grown at the Plymouth (A), Kinston (B), and Clayton (C) experiment stations in North Carolina. Curves are constructed from Eq. 1 and from parameters listed in Table 2. doi:10.1371/journal.pone.0095934.g001

Table 2. Standard logistic model parameters invariant to corn grain and total plant biomass production and for corn grain and total plant N uptake, grown on three different soils.

Soil	Component	Parameter	Estimate
Dothan	Biomass	b	0.319
	N Uptake	b_N	0.978
	Both	$c, \text{ ha kg}^{-1}$	0.0161
Goldsboro	Biomass	b	0.282
	N Uptake	b_N	0.941
	Both	$c, \text{ ha kg}^{-1}$	0.0209
Portsmouth	Biomass	b	-0.105
	N Uptake	b_N	0.555
	Both	$c, \text{ ha kg}^{-1}$	0.0111

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who share the same sampling of the independent variable, the best fit single slope shared between them is given by

$$\hat{m} = \frac{m_1 + m_2}{2} \quad (15)$$

and the corresponding intercepts become

$$\hat{b}_1 = b_1 + \frac{1}{2}(m_1 - m_2)\bar{x} \quad (16)$$

and

$$\hat{b}_2 = b_2 + \frac{1}{2}(m_2 - m_1)\bar{x}, \quad (17)$$

the initial guess for the c parameter is the average of all 12 values, and the initial guess for each value of b_N can be found from

$$\hat{b}_N = b_N + (c - \bar{c})\bar{N} \quad (18)$$

and for each value of b can be found from

$$\hat{b} = b + (c - \bar{c})\bar{N}. \quad (19)$$

The problem is bounded between the maximum and minimum values of the c parameter and each value of b_N is bounded between

$$\hat{b}_{Nmin} = b_N + (c - c_{max})\bar{N} \quad (20)$$

and

$$\hat{b}_{Nmax} = b_N + (c - c_{min})\bar{N} \quad (21)$$

and each value of b is bounded between

$$\hat{b}_{min} = b + (c - c_{max})\bar{N} \quad (22)$$

and

$$\hat{b}_{max} = b + (c - c_{min})\bar{N}. \quad (23)$$

Analysis

The first hypothesis of this analysis is that the difference between N uptake intercept parameter and the biomass intercept parameter, Δb , is invariant with respect to the differences in soil type and water availability for a given variety of an annual crop. Note that there is no attempt in this work to identify the effects of water availability or site characteristics on the ELM parameters, but to determine which are invariant to those characteristics. For this analysis, the same genetic line of corn is propagated by seeding and harvested at the same relative age. The second hypothesis is that maximum N concentration, N_{cm} , is also invariant with respect to the differences in soil type and water availability. A consequence of both hypotheses being affirmed is that the lower limit to the N concentration, N_{cb} in the same annual crop is also invariant with respect to soil type and water availability. Parameters were estimated by minimization of the normalized error sum of squares, and analysis of variance (ANOVA) was used to determine the validity of both hypotheses.

For the analysis of variance, three scenarios or modes were used, each with a targeted reduction in the number of parameters used in the ELM to describe the corn data in the Kamprath study. Mode I had 21 separate parameters that were estimated by minimization of the normalized error sum of squares. In Mode I, there are individual values for A , and A_N , for both grain and for total plant, and corresponding values for b , b_N , and c at each of the three sites. For Mode II, the number of parameters estimated dropped to 19, because the Δb parameter was held constant across the three sites. For Mode III, the Δb and the N_{cm} parameters were both held constant across the three sites, reducing the number of estimated parameters to 15.

Nonlinear Coefficients of Determination [23] (Nash-Sutcliffe Model Efficiency Coefficient [24]) will be provided for grain and total plant N uptake and for grain and total plant biomass production just as a relative comparison of fit.

Results

Table 1 contains the summary of the analysis of variance test. The comparison between Modes I and II leads to an increase the *degrees of freedom* to 41 and results in a variance ratio of 0.940. Because the critical $F(2,39,95\%)$ value is 3.24, it is concluded that there is no significant difference between the two modes. Thus in

Figure 2. Dependence of grain and total plant biomass production on applied N for corn grown at the Plymouth (A), Kinston (B), and Clayton (C) experiment stations in North Carolina. Curves are constructed from Eq. 3 and from parameters listed in Table 2.

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this study the Δb parameter is invariant to all soil and site differences.

Also from Table 1, the comparison between Modes I and III results in an increase the *degrees of freedom* to 45, and in a variance ratio of 0.533. With a critical $F(6,39,95\%)$ value of 2.34, not only is the Δb parameter is invariant, but so are the total plant and grain N_{cm} parameters. Thus, the soil, the field conditions, the environmental constraints, and even water availability play no role in either is the Δb or the two N_{cm} parameters. This leads to an invariance in the total plant and grain N_{cl} parameters, by virtue of Eq. (9).

The dependence of grain and whole plant N uptake on applied N at harvest is represented by Figure 1 for the three soil types. In general there is good agreement between the model line and the data. The resulting N uptake model lines (depicted in Figure 1), are generated from Eq. (1), using parameter values for b_N and c from Table 2 and values for A_{Ng} and A_{Nt} found in Table 3. Equation specific Non-linear Coefficient of Determination values and Error Sum of Squares are provided in Table 4.

Grain and whole plant biomass production versus applied N is shown in Figure 2 for all three soil types. In general there is good agreement between the model line and the data. The resulting biomass model lines (depicted in Figure 2), are generated from Eq. (3), using parameter values for b and c from Table 2 and values for A_g and A_t found in both Table 3. Equation specific Non-linear Coefficient of Determination values and Error Sum of Squares are provided in Table 4.

N concentration dependence on applied N is shown in Figure 3 for all three soil types. The resulting N concentration model lines (depicted in Figure 3), are generated from Eq. (7), using parameter values for b , b_N and c from Table 2 and values for N_{cmg} and N_{cmt} from Table 6.

The phase relationship between biomass production and N uptake for the corn grain and the whole plant is represented by Figure 4 for each of the three soils. The resulting biomass – N uptake phase model lines (depicted in Figure 4), are generated from Eq. (2), using parameter values for k_{Ng} , k_{Nt} , Y_{mg} and Y_{mt} found in Table 5.

The phase relationship between N Concentration and N uptake for the grain and the whole plant is represented by Figure 4 for each of the three soils. The resulting between N Concentration –

Table 3. Standard logistic model parameters specific to corn grain and total plant biomass production and for corn grain and total plant N uptake, grown on three different soils.

Soil	Component	Parameter	Plant Fraction	Estimate
Dothan	Biomass	A_g , Mg ha ⁻¹	Grain	11.5
		A_t , Mg ha ⁻¹	Total Plant	21.5
	N Uptake	A_{Ng} , kg ha ⁻¹	Grain	161
		A_{Nt} , kg ha ⁻¹	Total Plant	198
Goldsboro	Biomass	A_g , Mg ha ⁻¹	Grain	7.76
		A_t , Mg ha ⁻¹	Total Plant	14.4
	N Uptake	A_{Ng} , kg ha ⁻¹	Grain	108
		A_{Nt} , kg ha ⁻¹	Total Plant	132
Portsmouth	Biomass	A_g , Mg ha ⁻¹	Grain	9.50
		A_t , Mg ha ⁻¹	Total Plant	16.7
	N Uptake	A_{Ng} , kg ha ⁻¹	Grain	133
		A_{Nt} , kg ha ⁻¹	Total Plant	154

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Table 4. Standard statistical measures of fit, based on specific component and plant fraction data.

Soil	Component	Plant Fraction	Coefficient of Determination	Specific Error Sum of Squares
Dothan	Biomass	Grain	0.984	0.523
		Total Plant	0.969	2.361
	N Uptake	Grain	0.997	24.5
		Total Plant	0.993	83.9
Goldsboro	Biomass	Grain	0.992	0.145
		Total Plant	0.983	0.468
	N Uptake	Grain	0.998	7.81
		Total Plant	0.993	35.2
Portsmouth	Biomass	Grain	0.977	0.169
		Total Plant	0.985	0.592
	N Uptake	Grain	0.999	5.01
		Total Plant	0.991	19.4

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Figure 3. Dependence of grain and total plant N concentration on applied N for corn grown at the Plymouth (A), Kinston (B), and Clayton (C) experiment stations in North Carolina. Curves are constructed from Eq. 7 and from parameters listed in Table 2.
doi:10.1371/journal.pone.0095934.g003 **Figure 4.** Dependence of grain and total plant biomass production on N uptake for corn grown at the Plymouth (A), Kinston (B), and Clayton (C) experiment stations in North Carolina. Curves are constructed from Eq. 2 and from parameters listed in Table 3.
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Table 5. Standard phase model parameters for corn grain and total plant biomass production and for corn grain and total plant N uptake.

Soil	Parameter	Plant Fraction	Estimate
Dothan	k_{Ngr} kg ha ⁻¹	Grain	157
	k_{Ntr} kg ha ⁻¹	Total Plant	193
	Y_{mgr} Mg ha ⁻¹	Grain	22.7
	Y_{mtr} Mg ha ⁻¹	Total Plant	42.6
Goldsboro	k_{Ngr} kg ha ⁻¹	Grain	104
	k_{Ntr} kg ha ⁻¹	Total Plant	125
	Y_{mgr} Mg ha ⁻¹	Grain	15.1
	Y_{mtr} Mg ha ⁻¹	Total Plant	27.6
Portsmouth	k_{Ngr} kg ha ⁻¹	Grain	122
	k_{Ntr} kg ha ⁻¹	Total Plant	142
	Y_{mgr} Mg ha ⁻¹	Grain	17.7
	Y_{mtr} Mg ha ⁻¹	Total Plant	31.4

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N uptake phase model lines (depicted in Figure 5), are generated from Eq. (11), using parameter values for k_{Ngr} , k_{Ntr} , Y_{mgr} and Y_{mtr} found in Table 5.

Discussion

From this analysis it is concluded that there are aspects of the ELM that are invariant with respect to both soil type and water availability for a given variety of annual crop propagated by seeding and harvested at the same relative age. This analysis has shown, for the Kamprath N-rate study conducted on corn in North Carolina [22] that both the difference between N uptake intercept parameter and the biomass intercept parameter, Δb , and the maximum N concentration, N_{cm} , are in fact invariant with respect to the crop's surrounding environmental conditions. From the model, these facts lead to the conclusion that both the upper limit N concentration, N_{cm} , and the lower limit N concentration, N_{cl} , are both invariant with respect to soil type and water availability in the study analyzed. This further suggests that the N_{cm} and N_{cl} parameters are of more importance to the model. While other parameters, such as

$$\Delta b = \ln \left(\frac{N_{cm}}{N_{cl}} \right) \quad (25)$$

$$b = b_N - \Delta b = b_N - \ln \left(\frac{N_{cm}}{N_{cl}} \right) \quad (26)$$

$$Y_m = \frac{A_N}{N_{cm} - N_{cl}} \quad (27)$$

and

$$k_N = N_{cl} Y_m = \frac{N_{cl} A_N}{N_{cm} - N_{cl}} \quad (28)$$

$$A = \frac{A_N}{N_{cm}} \quad (24)$$

can be rewritten, to show the significance that upper and lower limit concentrations have in each parameter and ultimately seasonal plant response to nutrient application. Having upper

Table 6. Parametric factors invariant to site attributes, including soil type.

Plant Fraction	Parameter	Estimate
Both	Δb , kg ha ⁻¹	0.660
Grain	$N_{cm \text{ gr}}$ g kg ⁻¹	14.0
Total Plant	$N_{cm \text{ tr}}$ g kg ⁻¹	9.18
Grain	$N_{cl \text{ gr}}$ g kg ⁻¹	7.23
Total Plant	$N_{cl \text{ tr}}$ g kg ⁻¹	4.75
Grain	$N_{cm \text{ tr}} - N_{cl \text{ tr}} = A_{Ntr}/Y_{mtr}$ g kg ⁻¹	6.75
Total Plant	$N_{cm \text{ gr}} - N_{cl \text{ gr}} = A_{Ngr}/Y_{mgr}$ g kg ⁻¹	4.43

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Figure 5. Dependence of grain and total plant N concentration on N uptake for corn grown at the Plymouth (A), Kinston (B), and Clayton (C) experiment stations in North Carolina. Curves are constructed from Eq. 11 and from parameters listed in Table 3. doi:10.1371/journal.pone.0095934.g005

and lower limits to plant nutrient concentration corresponds with plant physiology. Without a minimum level of a given required nutrient, there can be no yield, seasonal or otherwise. There should also be a maximum concentration that can be approached, as there should be diminishing yield increases as higher agronomic rates are applied, or there would be unbounded growth.

If this invariance with respect to soil type and water availability holds for all crops propagated by seeding, the model could be written in terms of parameters that have measurable physiological significance and could give further insight into relationships that govern plant development and nutrient removal. Initial evidence appears promising that perennial crops, such as ryegrass (*Lolium perenne* L.), when held to comparably the same seasonal management practices also exhibit very nearly the same conclusions with regard to both constant values of the N_{cm} and N_{cl} parameters [25].

Given that N uptake and biomass production can be described by five parameters, and if two are invariant to all but crop type

and season length, the model reduces to three parameters (A_N , b_N , and c) when a crop and season length are chosen. From Overman & Scholtz [26] the logistic response originates within the soil's buffering capacity for P and for K, and the c parameter remains the same from the plant extractable logistic response to nutrient uptake logistic response, and to biomass production. It is here assumed that the c parameter for applied N also originates as the rate response parameter for the soil's buffering capacity of N. The c parameter can be modified by plant population [11]. A future step should be to analyze various field studies to catalogue soil physical and chemical characteristics and the resulting impact on the c parameter. Mathematically b_N represents shifting parameter which in conjunction with the c parameter as

$$N_{0.5} = \frac{b_N}{c} \quad (29)$$

$N_{0.5}$ represents the effective level of N necessary to achieve peak N uptake efficiency [10,16,17]. Ultimately, for environmental considerations, setting applied rate of N to the peak uptake level will result in the most N removed per unit N applied. Plus, provided the difference between b and b_N is greater than 0, then the yield will be on the upper portion of the logistic biomass curve to yield

$$Y = \frac{A_N}{N_{cm}[1 + \exp(-\Delta b)]} \quad (30)$$

The b_N parameter is affected by changes in plant population [11] and is also influenced by crop type [8,25]. The remaining parameter, A_N , is a linear parameter that is affected by the various environmental conditions, the crop type, the soil type, and various management practices [5–20]. Given Eq. (30), knowing the invariant Δb value for a given crop, and having a reasonable estimate for the background level of N already present in the soil, represented by Eq. (29), exists the beginning of a framework for a more reasonable and more sustainable nutrient management guide. Further analyses are being conducted to verify these findings with other annual propagated by seeding and with perennial crops.

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Author Contributions

Conceived and designed the experiments: RVS ARO. Performed the experiments: RVS. Analyzed the data: RVS. Contributed reagents/materials/analysis tools: RVS. Wrote the paper: RVS. Mentorship and review: ARO.

References

- Overman AR, Martin FG, Wilkinson SR (1990) A logistic equation for yield response of forage grass to nitrogen. *Commun Soil Sci Plan* 21: 595–609.
- Ferris T (1989) *Coming of age in the milky way*. New York: Bantam Doubleday Dell Publishing Group. 495 p.
- France J, Thornley JHM (1984) *Mathematical models in agriculture*. London: Butterworth and Company. 352 p.
- Overman AR, Wilkinson SR, Wilson DM (1994) An extended model of forage grass response to applied nitrogen. *Agron J* 86: 617–620.
- Overman AR, Wilson DM, Kamprath EJ (1994) Estimation of yield and nitrogen removal by corn. *Agron J* 86: 1012–1016.
- Reck WR, Overman AR (1996) Estimation of corn response to water and applied nitrogen. *J Plant Nutr* 19: 201–214.

7. Overman AR (1999) Model for accumulation of dry matter and plant nutrient elements by tobacco. *J Plant Nutr* 22: 1, 81–92.
8. Overman AR, Scholtz RV (2002) Mathematical models of crop growth and yield. New York: Marcel Decker. 342 p.
9. Overman AR, Scholtz RV (2002) Corn response to irrigation and applied nitrogen. *Commun Soil Sci Plan* 33: 3609–3619.
10. Overman AR, Brock KH (2003) Model analysis of corn response to applied nitrogen and tillage. *Commun Soil Sci Plan* 34: 2177–2191.
11. Overman AR, Scholtz RV, Brock KH (2006) Model analysis of corn response to applied nitrogen and plant population density. *Commun Soil Sci Plan* 37: 1157–1172.
12. Overman AR, Howard JC (2012) Model analysis of response to applied nitrogen by cotton. *J Plant Nutr* 35: 2118–2123.
13. Overman AR, Wilkinson SR (1995) Extended logistic model of forage grass response to applied nitrogen, phosphorus, and potassium. *T ASAE* 38: 103–108.
14. Overman AR, Stanley RL (1998) Bahiagrass responses to applied nitrogen and harvest interval. *Commun Soil Sci Plan* 29: 237–244.
15. Overman AR, Wilkinson SR (2003) Extended logistic model of forage grass response to applied nitrogen as affected by soil erosion. *T ASAE* 46: 1375–1380.
16. Overman AR, Brock KH (2003) Model comparison of coastal bermudagrass and pensacola bahiagrass response to applied nitrogen. *Commun Soil Sci Plan* 34: 2163–2176.
17. Overman AR, Scholtz RV, Taliaferro CM (2003) Model analysis of response of bermudagrass to applied nitrogen. *Commun Soil Sci Plan* 34: 1303–1310.
18. Overman AR, Rhoads FM, Brock KH (2005) Model analysis of response of pensacola bahiagrass to applied nitrogen, phosphorus, and potassium. I. Seasonal dry matter. *J Plant Nutr* 27: 1747–1756.
19. Overman AR, Rhoads FM, Brock KH (2005) Model analysis of response of pensacola bahiagrass to applied nitrogen, phosphorus, and potassium. II. Seasonal P and K uptake. *J Plant Nutr* 27: 1757–1777.
20. Overman AR, Scholtz RV (2005) Model analysis for response of dwarf elephantgrass to applied nitrogen and rainfall. *Commun Soil Sci Plan* 35: 2485–2494.
21. Monteith JL (1996) The quest for balance in crop modeling. *Agron J* 88: 695–697.
22. Kamprath EJ (1986) Nitrogen studies with corn on coastal plain soils, Technical Bulletin 282. Raleigh: North Carolina Agricultural Research Service, North Carolina State University. 15 p.
23. Draper NR, Smith H (1998) Applied regression analysis. 3rd ed. New York: John Wiley and Sons. 736 p.
24. Nash JE, Sutcliffe JV (1970) River flow forecasting through conceptual models part I - A discussion of principles. *J Hydrol* 10: 282–290.
25. Scholtz RV (2002) Mathematical modeling of agronomic crops: analysis of nutrient removal and dry matter accumulation. Doctor of Philosophy Dissertation. Gainesville: University of Florida. 139 p.
26. Overman AR, Scholtz RV (2012) A memoir on: A model of yield and phosphorus uptake in response to applied phosphorus by potato. Gainesville: University of Florida. 16p. Available: <http://ufdc.ufl.edu/IR00001234/00001> Accessed 11 December 2.