**Determinants of Human African Trypanosomiasis Elimination via Paratransgenesis**

**Supporting Information Text S1**

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***Wolbachia* and *Sodalis* Tsetse Population Dynamics and Equations**

Here we describe the population dynamics of tsetse, including vertical transmission of *Wolbachia* and *Sodalis.* Trypanosome transmission between tsetse, humans, and animals is described separately in another section below.

Let *V*pr be the number of tsetse positive for both *Wolbachia* and recombinant *Sodalis*, *V*pn be the number positive for *Wolbachia* and negative for *Sodalis*, *V*nw be the number negative for *Wolbachia* and positive for wild type *Sodalis*, *V*nr be the number negative for *Wolbachia* and positive for recombinant *Sodalis*, and *V*nn be the number negative for both *Wolbachia* and *Sodalis.* Then let *V*po = *V*pr + *V*pn be the number of *Wolbachia*-positive tsetse, *V*no = *V*nr + *V*nw + *V*nn be the number of *Wolbachia*-negative tsetse, *V*or = *V*pr + *V*nr be the number of recombinant *Sodalis*-positive tsetse, *V*ow = *V*nw be the number of wild type *Sodalis-*positive tsetse, *V*on = *V*pn + *V*nn be the number of *Sodalis-*negative tsetse, and *V* = *V*pr + *V*pn + *V*nr + *V*nw + *V*nn be the total number of tsetse.

Let $r\_{V}\left(V\right)$ be the fecundity of *Wolbachia*-negative, wild type *Sodalis*-positive tsetse ($V\_{nw}$) as a non-increasing function of tsetse population size. Let $1-r\_{W}$ be the relative fecundity of *Wolbachia*-positive tsetse, $1-r\_{Sr}$ be the relative fecundity of recombinant *Sodalis*-positive tsetse, $1-r\_{Sn}$ be the relative fecundity of *Sodalis*-negative tsetse, and the three factors be multiplicative and independent of tsetse population size. Let $μ\_{V}\left(V\right)$ be the death rate of *Wolbachia*-negative, wild type *Sodalis*-positive tsetse as a non-decreasing function of tsetse population size. Let $1+μ\_{Sw}$ be the relative death rate of *Wolbachia*-positive tsetse, $1+μ\_{Sr}$ be the relative death rate of recombinant *Sodalis*-positive tsetse, $1+μ\_{Sn}$1+muSn be the relative death rate of *Sodalis*-negative tsetse, and the three factors be multiplicative and independent of tsetse population size. Let $φ\_{W}$ be the proportion of eggs of *Wolbachia*-positive mothers that are *Wolbachia*-negative, and $φ\_{Sr}$ be the proportion of eggs of recombinant *Sodalis*-positive mothers that are *Sodalis*-negative. Finally, let $h\_{W}$ be the proportion of *Wolbachia*-negative eggs fertilized by *Wolbachia*-positive males that do not mature and develop.

 The following system of Ordinary Differential Equations describes the population dynamics:

$\frac{dV\_{pr}}{dt}=\left[r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)\left(1-ϕ\_{W}\right)\left(1-ϕ\_{Sr}\right)\right.$ (1) $\left.-μ\_{V}\left(V\right)\left(1+μ\_{W}\right)\left(1+μ\_{Sr}\right)+c\_{V}\right]V\_{pr}$,

$$\frac{dV\_{pn}}{dt}=r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)\left(1-ϕ\_{W}\right)ϕ\_{Sr}V\_{pr}$$

 $+\left[r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sn}\right)\left(1-ϕ\_{W}\right)-μ\_{V}\left(V\right)\left(1+μ\_{W}\right)\left(1+μ\_{Sn}\right)-c\_{V}\right]V\_{pn}$, (2)

$$\frac{dV\_{nr}}{dt}=r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)ϕ\_{W}\left(1-ϕ\_{Sr}\right)\left(1-h\_{W}\frac{V\_{po}}{V}\right)V\_{pr}$$

 $+\left[r\_{V}\left(V\right)\left(1-r\_{Sr}\right)\left(1-ϕ\_{Sr}\right)\left(1-h\_{W}\frac{V\_{po}}{V}\right)-μ\_{V}\left(V\right)\left(1+μ\_{Sr}\right)-c\_{V}\right]V\_{nr}$, (3)

$\frac{dV\_{nw}}{dt}=c\_{V}\left(V\right)+\left[r\_{V}\left(V\right)\left(1-ϕ\_{Sw}\right)\left(1-h\_{W}\frac{V\_{po}}{V}\right)-μ\_{V}\left(V\right)-c\_{V}\right]V\_{nw}$, (4)

$$\frac{dV\_{nn}}{dt}=r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)ϕ\_{W}ϕ\_{Sr}\left(1-h\_{W}\frac{V\_{po}}{V}\right)V\_{pr}$$

 $+ r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sn}\right)ϕ\_{W}\left(1-h\_{W}\frac{V\_{po}}{V}\right)V\_{pn}$

$$+ r\_{V}\left(V\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)ϕ\_{Sr}\left(1-h\_{W}\frac{V\_{po}}{V}\right)V\_{nr}+r\_{V}\left(V\right)ϕ\_{Sw}\left(1-h\_{W}\frac{V\_{po}}{V}\right)V\_{nw}$$

 $+\left[r\_{V}\left(V\right)\left(1-r\_{Sn}\right)\left(1-h\_{W}\frac{V\_{po}}{V}\right)-μ\_{V}\left(V\right)\left(1+μ\_{Sn}\right)-c\_{V}\right]V\_{nn}$, (5)

where the density-dependent birth rate function is

 $r\_{V}\left(V\right)=r\_{V0}\left(1-r\_{V1}V\right)$. (6)

The parameter $r\_{V0}$ is the birth rate in the absence of intra-species competition, while $r\_{V1}$ measures the effect of intra-species competition on birth rate.

The density-dependent mortality rate function is

$μ\_{V}\left(V\right)=μ\_{V0}\left(1-μ\_{V1}V\right)$. (7)

The parameter $μ\_{V0}$ is the death rate in the absence of intra-species competition, while $μ\_{V1}$ measures the effect of intra-species completion on death rate.

 If there are no fecundity and death rate penalties for colonization with either *Wolbachia* or *Sodalis* (or if the whole tsetse population is negative for both bacteria), the equilibrium population size is then $V^{\*}=\frac{r\_{V0}-μ\_{V0}}{r\_{V0}r\_{V1}+μ\_{V0}μ\_{V1}}$. With fecundity or death rate penalties, this provides an upper bound on the tsetse population size.

**Trypanosome Infection Model and Equations**

We assume that the presence of recombinant *Sodalis* renders a tsetse fly completely immune to trypanosome infection. Therefore, in the infection model we only consider wild type *Sodalis*-positive and *Sodalis*-negative tsetse, *V*ow and *V*on. Let *V*S be the number of susceptible tsetse, *V*E be the number of exposed tsetse, *V*I be the number of infectious tsetse, and *V*R be the number of resistant tsetse. Note that *V*S + *V*E + *V*I + *V*R = *V*ow + *V*on. Let *H*S, *H*E, *H*I, and *H*R be the number of susceptible, exposed, infectious, and recovered humans. Let *L*S, *L*E, *L*I, and *L*R be the number of susceptible, exposed, infectious, and recovered animals.

 Tsetse with wild type *Sodalis* (*V*ow) emerge from pupae according to equations 1 through 7 described above, with a fraction $ψ\_{Vw}$ of newly emerged *V*ow inherently resistant to trypanosome infection and the remaining $1-ψ\_{Vw}$ susceptible. Likewise, tsetse without *Sodalis* (*V*on) emerge with a fraction $ψ\_{Vn}$ inherently resistant and the remaining $1-ψ\_{Vn}$ susceptible. We assume that susceptible tsetse become infected with trypanosomes with probabilities $λ\_{VH}$ and $λ\_{VL} $during their blood meals taken on humans and animals, respectively, within a certain window of length ($\frac{1}{σ\_{V}}$) following emergence. Blood meals are taken at rates $α\_{H}$ and $α\_{L}$ on humans and animals, respectively, and the conservation of law of tsetse bites applies, where the number of bites taken by tsetse balances the total number of bites on hosts, applies. Tsetse trypanosome infections incubate for a period of $\frac{1}{τ\_{V}}$, after which point they become infectious. Tsetse can migrate in or out of any state at rate $c\_{V}$. Finally, tsetse die at rate .

 (8)

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 (16)

 (17)

 (18)

 (19)

where

 $λ\_{VH}=β\_{VH}\frac{H\_{I}}{H}$, (20)

 $λ\_{VL}=β\_{VL}\frac{L\_{I}}{L}$,

$λ\_{H}=β\_{H}V\_{I}$,

$$λ\_{L}=β\_{L}V\_{I}.$$

From Equations 1 through 7, the tsetse birth terms are

$B\_{V\_{ow}}^{\*}=r\_{V}\left(V^{\*}\right)\left(1-ϕ\_{Sw}\right)\left(1-h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{nw}^{\*}$, (21)

$$B\_{V\_{on}}^{\*}=r\_{V}\left(V^{\*}\right)\left(1-r\_{W}\right)\left(1-r\_{Sr}\right)ϕ\_{Sr}\left(1-ϕ\_{W}h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{pr}^{\*}$$

 $+ \left(1-r\_{W}\right)\left(1-ϕ\_{W}h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{pn}^{\*}+\left(1-r\_{Sr}\right)ϕ\_{Sr}\left(1-h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{nr}^{\*}$

 $+ ϕ\_{Sw}\left(1-h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{nw}^{\*}+\left(1-r\_{Sn}\right)\left(1-h\_{W}\frac{V\_{po}^{\*}}{V^{\*}}\right)V\_{nn}^{\*}$

 $+\frac{c\_{V}\left(V^{\*}\right)\hat{μ}\_{V}^{\*}}{a\_{H}+a\_{L}+σ\_{V}+\hat{μ}\_{V}^{\*}}$, (22)

$B\_{VS}^{\*}=\left(1-ψ\_{V\_{w}}\right)B\_{V\_{ow}}^{\*}+\left(1-ψ\_{V\_{n}}\right)B\_{V\_{on}}^{\*}$, (23)

$B\_{VR}^{\*}=ψ\_{V\_{w}}B\_{V\_{ow}}^{\*}+ψ\_{V\_{n}}B\_{V\_{on}}^{\*}$, (24)

From Equations 1 through 7, the tsetse death rate is

$\hat{μ}\_{V}^{\*}=μ\_{V}\left(V^{\*}\right)\left(1+\frac{μ\_{W}V\_{pn}^{\*}+μ\_{Sn}V\_{nn}^{\*}}{V\_{ow}^{\*}+V\_{on}^{\*}}\right)$. (25)

**Reproduction Number**

For our model equations, the basic reproduction number for trypanosome infection is calculated by the next-generation matrix [1], counting one generation as tsetse-to-tsetse transmission. This gives

$R\_{0}=\left[\frac{a\_{H}^{2}β\_{H}β\_{VH}}{γ\_{H}H}+\frac{a\_{L}^{2}β\_{L}β\_{VL}}{γ\_{L}L}\right]\frac{τ\_{V}}{\left(\hat{μ}\_{V}^{\*}+c\_{V}\right)\left(τ\_{V}+\hat{μ}\_{V}^{\*}+c\_{V}\right)}\frac{B\_{VS}^{\*}}{a\_{H}+a\_{L}+σ\_{V}+\hat{μ}\_{V}^{\*}+c\_{V}}$. (26)

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