**S1 Text: Model Equations for Neurovascular Compartmental Dynamics**

The following equations are obtained from published literature models’implementation of neurovascular dynamics. Equations in bold font represent state variables of the compartments.

**1. Synaptic Space**

*(a) Potassium concentration in the synaptic space,*

**(1)**

 is potassium release from active neurons. is maximum flux, is a constant parameter that depends on extracellular sodium concentration, is the threshold value for .

**2. Astrocytic Intracellular Space**

*(a) Astrocytic Inositol trisphosphate,*

 **(2)**

 is production rate and is ratio of bound to unbound receptors and is given as

 (3)

 is input synaptic glutamate release, is Dissociation Constant, is Degradation rate and is the ratio of the activities of bound and unbound receptors.

*(b) Astrocytic intracellular calcium concentration,*

 **(4)**

 is buffering factor, is rate of concentration change due to release through IP3 binded receptors () on the endoplasmic reticulum (ER) channels, is rate of concentration change due to pump uptake into the endoplasmic reticulum (ER) and is rate of concentration change due to extracellular space through transient receptor potential vanniloid-related 4 (TRPV4) channels

 (5)

 is the maximum rate, is the dissociation constant for IP3 binding to an IP3R, is the dissociation constant for binding to an activation site on an IP3R, is the concentration in the ER.

 (6)

 is the maximum pump rate and Kp is the pump constant.

 (7)

where is determined by the steady-state flux balance.

 (8)

is electrical current through the TRPV channel, is the astrocyte cell capacitance, and is a scaling factor for relating the net movement of ion fluxes to the membrane potential.

 (9)

is maximum channel conductance, is membrane potential, is TRPV channel reversal potential, s is TRPV4 channel open probability.

*(c) Gating variable h*

 **(10)**

 is binding rate at the inhibitory site on the IP3R and is dissociation constant at the inhibitory site on the IP3R.

*(d) The TRPV4 channel open probability, s*

 **(11)**

 is time constant and is a function of

 (12)

 is perivascular concentration expressed in mM, is steady-state channel open probability and is a function of strain and

 (13)

where, is material strain gating, ε is strain on the perivascular endfoot and is given as

 (14)

x is vessel circumferential contraction and dilation variable, is strain required for half activation, is inhibitory factor and is given as

 (15)

where and are constants associated with intra and extra- cellular , respectively.

*(e) Calcium dependent EET (Epoxyeicosatrienoic Acid) production in the cell,*

 **(16)**

 is EET production rate, is minimum required for EET production and is EET decay rate.

*(f) Open BK (Big Potassium) channel probability,*

 **(17)**

is time constant associated with the opening of astrocyte BK channels and is based on statistical considerations

 (18)

 (19)

 (20)

characteristic time, is the voltage associated with the opening of half the population, is astrocyte membrane potential, is measure of the spread of the distribution, determines the range of the shift of as calcium varies.

 determines the EET-dependent shift of the channel reversal potential, and constants associated with calcium concentration/ calcium dependent constants

*(g) Astrocyte membrane potential,*

 **(21)**

 is current through BK channel

 (22)

 (23)

is channel conductance, is reversal potential, is the electrical current carried by the K+ influx at the perisynaptic process and is leak current

(24)

 (25)

 is the leak conductance and is reversal potential.

**3. Perivascular Space**

*(a) Perivascular potassium concentration,*

 **(26)**

 is the resting state equilibrium K+ concentration in the perivascular space. The potassium flow from the astrocyte and SMC are and corresponding to big potassium (BK) and inward rectifying potassium (KIR) respectively. And, and are the volume ratios of perivascular space to astrocyte and SMC, respectively. is the rate at which perivascular K+ concentration decays to its baseline state.

(27)

(28)

 is the SMC (smooth muscle cell) capacitance.

 (29)

Here is channel conductance, is reversal potential and k is open probability. These all depend on the perivascular K+ concentration

 (30)

 is the conductance when the perivascular K+ concentration is 1mM.

 (31)

 and are constants

*(b) Perivascular Ca2+ concentration,*

 **(32)**

 is calcium current from the arteriole SMC, is the decay rate of perivascular Ca2+ concentration

 (33)

 (34)

is channel conductance, is reversal potential

 (35)

 and are constants.

**4. Arteriole smooth muscle cell (SMC) intracellular space**

*(a) Open KIR (Inward Rectifying Potassium) channel probability, k*

 **(36)**

 (37)

 (38)

 (39)

 (40)

are constants.

*(b) SMC membrane potential, VSMC*

 **(41)**

 (42)

 (43)

*(c) Open potassium channel probability, n*

 **(44)**

 (45)

 (46)

 (47)

 are constants.

*(d) Ca2+ concentration in SMC,*

 **(48)**

 (49)

 is the Faraday constant times cytosol volume, is the constant ratio of Ca2+ outflux to influx, is the rate constant in the calcium buffer reaction and is the total buffer concentration.

*(e) Fraction of attached cross bridges, ω*

For muscle mechanics, a given myosin-actin overlapping segment can have a fraction of cross bridges between myosin and actin filaments represented as ω and is given as

 **(50)**

 is rate constant and is a constant, is given as

 (51)

Cam and q and are constants.

*(f) Mean circumference of the vessel, x*

 **(52)**

τ is time constant related with the wall internal friction and is force on the vessel due to transmural pressure which is expressed as

 (53)

Here, A is cross-sectional area of vessel given as,

 (54)

is the outer radius of vessel, is the inner radius of vessel, is the mean radius of vessel and .

Considering a segment of the vessel as a cylindrical element with thickness and unit length along the axis and having the longitudinal cross-sectional surface area of . The stresses on S are represented through a Maxwell model along the mean circumference (x) which consists of a contractile component of length y, a series elastic component of length u, a parallel elastic component of length , and a parallel viscous component. The hoop forces on S are:

 , and

 (55)

 is hoop force on S due to the visoelastic stress and is hoop force on S due to the myogenic stress. is viscoelastic contributed weight, is myogenic hoop forces contributed weight, is normalized hoop stresses associated with x (circumferential) component, is normalized hoop stresses associated with u (series elastic component) component and is normalized hoop stresses associated with y (contractile component) component. For initial values, ( is the initial mean radius of the vessel).

 (56) (57)

Normalized values are , ,

 (58)

 is normalization constant and are constants.

*(g) Normalized contractile component of length, y’*

(59)

 (60)

where

a’, b’, c’ and d’ are constants, v is velocity of contraction of the contractile component at zero load, and , with is at the reference muscle activation level.