# NUFEB: A Massively Parallel Simulator for Individual-based Modelling of Microbial Communities

#### Supporting Information (SI)

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## Model validation against biofilm benchmark problems BM2 and BM3

Validation is a critical step to ensure that both the software and the model have been built correctly with respect to the real biological system. To validate NUFEB, we refer to the two biofilm benchmark problems BM2 and BM3 proposed by the International Water Association (IWA) task group on biofilm modelling [1, 2]. For each benchmark problem, we compare the NUFEB simulation results with previous models.

Problem BM2 evaluates how fluid dynamics affects mass transfer in a mono-microbial functional group (heterotroph) biofilm system with spatially heterogeneous architectures. To match the reference model [3], we assume the biomass density is uniform throughout the biofilm and the microbes are stationary, i.e, particles motion is not considered. The three-dimensional biofilm geometries are constructed on pre-defined lattice points in order to keep similar spatial properties (e.g, biofilm average height, area enlargement factor) with the other models. Two geometries are created in this way: a strictly flat biofilm and a heterogeneous (wavy) biofilm morphology. For simplicity, in BM2 we ignore all biological processes except the consumption of organic substrate in the biofilm compartment. The chemical process considered is substrate mass transport due to diffusion and advection, generated from the fluid dynamics. Note that the mass transport process in the biofilm region can be slightly affected by advection due to biofilm porosity. This is different from the model described in [3] where the transport process in the solid region is only governed by diffusion.

Table A shows the BM2 results for the NUFEB simulations and other reference models [4, 1]. Note that all the models (including NUFEB) are deterministic. The results are further analysed using Student's t-test to show if the NUFEB and reference results are statistically different from each other. For each biofilm geometry, we investigated the average surface concentration of substrate under different flow velocities  $\mathbf{U}_f$ . It can be seen that there is a good agreement in the flat biofilm cases (p-values > 13.7%). The results clearly show that the substrate concentration increases as the fluid velocity becomes higher, since fluid transports more substrate to the downstream area as a result of increased advection. The simulation results for heterogeneous biofilm structure show the same trend. However, the average concentrations in the high  $\mathbf{U}_f$  are about 10% lower than the mean value of other simulation results (with p-value = 8.7%). This may due to the variance of biofilm morphologies.

The benchmark problem BM3 models a multi-microbial functional groups and multi-substrate biofilm system. Briefly, two microbial functional groups are considered in the biofilm compartment: aerobic autotrophic

nitrifiers that oxidize ammonium  $NH_4^+$  to nitrate  $NO_3^-$ , and aerobic heterotrophs that use organic substrate as the electron donor and oxygen  $O_2$  as the electron acceptor. Initially, microbes are randomly distributed on the bottom surface and then grow until the biomass reaches a prescribed value ( $250\mu m$  biofilm height). The bulk liquid compartment is assumed to be completely mixed and fluid flow is not taken into account. Therefore, nutrient concentration is only governed by diffusion in the boundary layer compartment, and by diffusion and reaction in the biofilm. Problem BM3 investigates three different cases including a standard case and two special cases with varying initial ammonia concentration. For each case, we evaluated two key variables at biofilm steady states: the concentration of substrate  $C_{\rm S,bulk}$  and ammonia  $C_{\rm N,bulk}$  in bulk liquid. Table B summarises the comparative results between NUFEB and previous models [2]. Each case is run for five replicates due to stochastic initial microbe distribution and cell division, and the average result is then calculated. The stochastic effects of the results from the biofilm morphology are negligible due to the flat biofilm structure. However, one should notice that the difference can be significant when irregular biofilm structures are formed. The Hotelling's  $T^2$  test for two dependent samples is performed for each case to show the differences between the multivariate means of different results [5]. The results show a good agreement except for  $C_{\text{N,bulk}}$  in standard N:COD case which is 10% higher than the mean value (with p-value = 6.8%). This may be expected as the value is sensitive to idiosyncrasies of different models [6].

Table A: BM2 results from NUFEB simulations and other reference models: values are average biofilm surface concentration [kg m<sup>-3</sup>] of a flat and a wavy biofilm with variation of fluid velocity  $\mathbf{U}_f$ . All the models (including NUFEB) are deterministic. The results of the reference models are taken from [1] and [4].

Model	Flat:High $\mathbf{U}_f$	Flat:Low $\mathbf{U}_f$	Wave:High $\mathbf{U}_f$	Wave:Low $\mathbf{U}_f$
N3c(3D)	3.82	2.86	2.50	1.48
N2b(2D)	3.83	2.87	2.35	1.41
N2d(2D)	3.99	2.94	3.02	1.38
N1s(1D)	3.83	2.89	2.61	1.60
Mean	3.86	2.89	2.62	1.47
NUFEB(3D)	3.95	2.92	2.26	1.38
p-values from T-test	0.137	0.19	0.087	0.171

Table B: BM3 results from NUFEB simulations and previous models: values are bulk concentrations of substrate and ammonia in a baseline case, a case with high initial ammonia, and a case with low initial ammonia. Each case in NUFEB is run for five replicates due to the stochastic processes. We take the mean values with the deviation. The results of the reference models are taken from [2] and [6].

	Standar	rd Case	High I	N:COD	Low N:COD		
Model	$\begin{array}{cc} C_{ m S,bulk} & C_{ m N,bulk} \\ g_{ m COD}/{ m m}^3 & g_{ m N}/{ m m}^3 \end{array}$		$rac{C_{ m S,bulk}}{ m g_{ m COD}/m^3}$	$C_{ m N,bulk} \  m g_N/m^3$	$rac{C_{ m S,bulk}}{ m g_{ m COD}/m^3}$	$C_{ m N, bulk} \  m g_{ m N}/m^3$	
W(1D)	5.39	1.59	5.86	18.93	4.39	0.48	
M1(1D)	4.84	1.45	5.35	20.26	4.98	0.45	
CP(2D)	5.14	1.50	5.45	18.15	5.19	0.44	
DN(2D)	5.14	1.74	5.56	20.26	4.66	0.48	
iDynoMiCS(3D)	5.23	1.46	5.74	17.3	5.05	0.53	
Mean	5.15	1.55	5.59	18.98	4.86	0.48	
NUFEB(3D)	$5.21_{\pm 0.10}$	$1.72_{\pm 0.14}$	$5.74_{\pm 0.19}$	$18.42_{\pm 0.13}$	$\boldsymbol{5.18}_{\pm 0.17}$	$\boldsymbol{0.53}_{\pm 0.08}$	
p-values from $T^2$ test	0.148		0.	471	0.068		

### Nutrient mass balance

The nutrient mass balance equation is discretised on a Marker-And-Cell (MAC) uniform grid. The concentration scalar S is defined at the centre of the voxel (cubic grid element), and velocity components  $U_x, U_y$ , and  $U_z$  are defined at the centres of six surfaces of the voxel (Fig A). The temporal and spatial derivatives of the transport equation are discretised by Forward Euler and Central Finite Differences, respectively. For a given nutrient concentration field at time t, the concentration field at next time step can be calculated using following discretised equation:

$$\frac{\overline{S_{i,j,k}^{n+1} - S_{i,j,k}^{n}}}{\Delta t} + \overline{U}x_{i,j,k}\frac{\overline{S_{i+1/2,j,k}^{n} - S_{i-1/2,j,k}^{n}}}{\Delta x} + \overline{U}y_{i,j,k}\frac{\overline{S_{i,j+1/2,k}^{n} - S_{i,j-1/2,k}^{n}}}{\Delta y} + \overline{U}z_{i,j,k}\frac{\overline{S_{i,j,k+1/2}^{n} - S_{i,j,k-1/2}^{n}}}{\Delta z}}{\Delta z} \\
= \frac{Jx_{i+1/2,j,k} - Jx_{i-1/2}, j, k}{\Delta x} + \frac{Jy_{i,j+1/2,k} - Jy_{i,j-1/2}, k}{\Delta y} + \frac{Jz_{i,j,k+1/2} - Jz_{i,j,k-1/2}}{\Delta z} + R_{i,j,k},$$

where

$$\overline{U}x_{i,j,k} = \frac{Ux_{i+1/2,j,k} - Ux_{i-1/2,j,k}}{2}; \overline{U}y_{i,j,k} = \frac{Uy_{i,j+1/2,k} - Uy_{i,j-1/2,k}}{2}; \overline{U}y_{i,j+1/2,k} = \frac{Uy_{i,j+1/2,k} - Uy_{i,j+1/2,k}}{2}; \overline{U}y_{i,j+1/2,k} =$$

Here D is the diffusion coefficient of the nutrient and  $R_{i,j,k}$  is the nutrient consumption rate at grid (i, j, k).

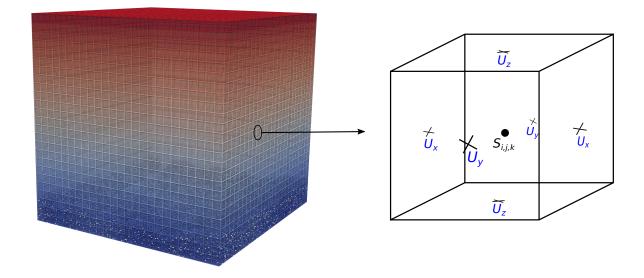


Figure A: A MAC grid (right). Velocity components,  $U_x, U_y$ , and  $U_z$ , are stored at the centres of six surfaces of the voxel. Nutrient concentration S is stored at the voxel centre.

#### pH calculations and thermodynamics

An explicit pH calculation module was implemented in NUFEB to handle both hydration reactions (e.g.,  $CO_2 + H_2O \rightarrow H_2CO_3$ ) and up to three deprotonations (e.g.,  $H_2CO_3 \rightarrow HCO_3^- \rightarrow CO_3^{2^-}$ ). The dissociations are assumed to occur instantaneously with respect to the rate of other phenomena considered and are modeled as equilibrium processes. For example, the dissociation reaction for  $NH_3$  is  $NH_3 + H_2O \leftrightarrow NH_4^+ + OH^-$ . The equilibrium constants ( $K_{eq}$ ) are computed as:

$$K_{eq} = e^{\frac{-\Delta G_{\text{dissociation}}}{RT}},\tag{1}$$

where  $\Delta G_{\text{dissociation}}$  is the Gibbs free energy corresponding to the dissociation reaction, which is derived from the standard free energies of formation (for example, for the chemical species NH<sub>3</sub>,  $\Delta G_{f,\text{NH}_3} = -26.57$  and  $\Delta G_{f,\text{NH}_4^+} = -79.37$  [7]), R is the ideal gas constant (kJ/mol/K), and T is the temperature (K). Moreover, the computation of the dissociation constants and the Gibbs energy of the anabolic and catabolic reactions are corrected for temperature:

$$\Delta G = \Delta G^0 + RT \ln Q \,, \tag{2}$$

where  $\Delta G_r^0$  is the standard Gibbs energy, Q is the reaction quotient, R is the ideal gas constant and T is the temperature.

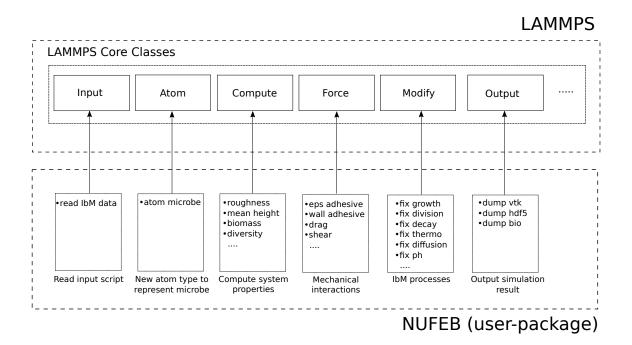


Figure B: **NUFEB tool architecture.** Each small box in the upper level refers to a class in LAMMPS, and each small box in the lower level refers to a collection of classes implemented in NUFEB that inherits from the corresponding parent class in LAMMPS.

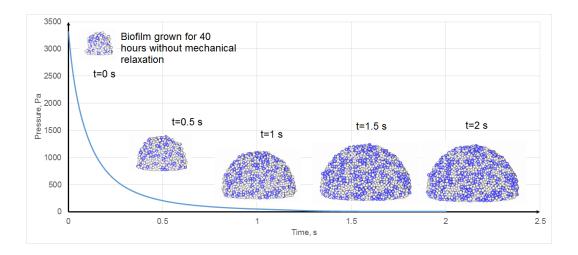


Figure C: Mechanical relaxation of a biofilm. The biofilm grows for 40 hours without mechanical relaxation, and then apply the relaxation for 2 seconds. Within 0-1 seconds, the biofilm pressure decreases rapidly and the biofilm shape expands due to the relaxation. The system reaches mechanical equilibrium after 1 second. Correspondingly, the biofilm shape would not change significantly at this stage.

Parameters	$\mathbf{Symbol}$	Value	Unit
Domain dimensions	$L_X \times L_Y \times L_Z$	$100\times40\times100$	$\mu m$
Fluid grid cells	$N_X \times N_Y \times N_Z$	$15\times6\times15$	nodes
Liquid density	$ ho_{water}$	1000	${ m kg}~{ m m}^{-3}$
Kinematic viscosity for water	nu	$1 \times 10^{-6}$	$m^2 s^{-1}$
Hamaker coefficient for cohesion	$H_a$	$1 \times 10^{-16}$	J
Damping constant for normal contact	$\gamma_n$	$1 \times 10^6$	$s^{-1}$
Elastic constant for normal contact	$K_n$	1	N/m
Particle diameter	Dia	$1 \times 10^{-6}$	m
Particle density	$\rho_X$	1000	${\rm kg}~{\rm m}^{-3}$

Table C: List of physical parameters used in Case Study 1.

Parameters	Symbol	Value	Unit
Heterotroph (HET)			
Maximal specific growth-rate	$\mu_{ m H}$	$6.9  imes 10^{-5}$	s
Yield	$Y_{H}$	0.61	m kg/kg
Decay rate	$b_{ m H}$	$9.17 \times 10^{-7}$	S
Maintenance rate	$bm_{ m H}$	$3.69  imes 10^{-6}$	S
Affinity constant for Substrate	$\mathrm{K}_{\mathrm{H,sub}}$	$4 \times 10^{-3}$	${ m kg}~{ m m}^{-3}$
Affinity constant for $O_2$	$K_{H,O_2}$	$2 \times 10^{-4}$	${ m kg}~{ m m}^{-3}$
Affinity constant for $NO_2$	$K_{H,NO_2}$	$3  imes 10^{-4}$	${ m kg}~{ m m}^{-3}$
Affinity constant for $NO_3$	K <sub>H,NO3</sub>	$3  imes 10^{-4}$	${\rm kg}~{\rm m}^{-3}$
reduction factor in anoxic condition	$\eta_{ m H}$	0.6	
Ammonia Oxidizer (AOB)			
Maximal specific growth rate	$\mu_{ m A}$	$2.37  imes 10^{-5}$	s
Yield	Y <sub>A</sub>	0.15	kg/kg
Decay rate	$b_{\mathrm{A}}$	$1.27 \times 10^{-6}$	S
Maintenance rate	$bm_{\rm A}$	$1.5  imes 10^{-6}$	s
Affinity constant for $NH_4$	$\rm K_{\rm H, NH_4}$	$2.4  imes 10^{-3}$	${\rm kg}~{\rm m}^{-3}$
Affinity constant for $O_2$	$K_{H,O_2}$	$6  imes 10^{-4}$	${\rm kg}~{\rm m}^{-3}$
Nitrite Oxidizer (NOB)			
Maximal specific growth-rate	$\mu_{ m N}$	$1.68 \times 10^{-5}$	s
Yield	$Y_N$	0.041	kg/kg
Decay rate	$b_{ m N}$	$1.27 \times 10^{-6}$	S
Maintenance rate	$bm_{ m N}$	$6.94 \times 10^{-7}$	s
Affinity constant for $NO_2$	$K_{H,NO_2}$	$5.5  imes 10^{-3}$	${\rm kg}~{\rm m}^{-3}$
Affinity constant for $O_2$	$K_{H,O_2}$	$2.2  imes 10^{-3}$	${\rm kg}~{\rm m}^{-3}$
EPS			
Yield	$Y_{\rm E}$	0.18	kg/kg
Decay rate	$b_{ m E}$	$1.97  imes 10^{-6}$	S
Diffusion coefficient			
Substrate	$\mathrm{D}_{\mathrm{Sub}}$	$1.16 \times 10^{-9}$	$\mathrm{m}^2~\mathrm{s}^{-1}$
Oxygen	$D_{O_2}$	$2.3 \times 10^{-9}$	$\mathrm{m}^2~\mathrm{s}^{-1}$
Ammonium	$\mathrm{D}_{\mathrm{NH}_4}$	$1.97 \times 10^{-9}$	$\mathrm{m}^2~\mathrm{s}^{-1}$
Nitrite	$D_{NO_2}$	$1.85  imes 10^{-9}$	$\mathrm{m}^2~\mathrm{s}^{-1}$
Nitrate	$D_{NO_3}$	$1.85  imes 10^{-9}$	$\mathrm{m}^2~\mathrm{s}^{-1}$

Table D: List of kinetics parameters used in Case Study 2. The parameters for HET and EPS are chosen from [8], and the parameters for AOB and NOB are chosen from [9].

Parameters	Symbol	Value	Unit
Computational Domain			
Dimensions	$L_X \times L_Y \times L_Z$	$600\times 600\times 200$	$\mu m$
Cartesian grid cells	$N_X \times N_Y \times N_Z$	$150\times150\times50$	nodes
MPI domain decomposition	$P_X \times P_Y \times P_Z$	$10\times 10\times 1$	processors
Reactor parameters			
Reactor volume	V	$1.25  imes 10^{-3}$	$\mathrm{m}^3$
Biofilm surface area	$A_F$	0.1	$\mathrm{m}^2$
Flow rate	Q	$2.31  imes 10^{-7}$	$\mathrm{m}^3~\mathrm{s}^{-1}$
Boundary layer thickness	$L_L$	20	$\mu m$
Substrate influent concentration	$S_{in}^{Sub}$	$3 \times 10^{-3}$	$\rm kg~COD~m^{-3}$
Oxygen influent concentration	$S_{in}^{O_2}$	$1 \times 10^{-2}$	${\rm kg}~{\rm m}^{-3}$
Ammonium influent concentration	$S_{in}^{NH_4}$	$2 \times 10^{-2}$	$ m kg~N~m^{-3}$
Nitrite influent concentration	$S_{in}^{Sub} \\ S_{in}^{O_2} \\ S_{in}^{NN4} \\ S_{in}^{NO_2} \\ S_{in}^{NO_3} \\ S_{in}^{NO_3}$	0	$\rm kg~N~m^{-3}$
Nitrate influent concentration	$S_{in}^{NO_3}$	0	${ m kg}~{ m N}~{ m m}^{-3}$
Physical and system parameters			
Biomass density	$\rho_X$	32	$\rm kg~COD~m^{-3}$
EPS density	$ ho_{EPS}$	30	${\rm kg}~{\rm m}^{-3}$
Division diameter	$Max_{dia}$	1.3	$\mu m$
Maximum mechanical iterations		3000	
Biological timestep	$dt_{bio}$	1200	s
IbM processes			
Biological: Monod-based growth, division, EPS production, death			
Chemical: nutrient mass balance			
Physical: contact force, EPS adhesion			

Table E: List of reactor and physical parameters, and IbM processes used in Case Study 2.

Table F: Stoichiometric matrix for particulate and soluble components implemented in Monod-based growth model. Here  $Y_i$  is the yield for microbial functional group i (i = HET, AOB, NOB, EPS and DEAD),  $\mu_i$  and  $b_i$  are the maximum specific growth and decay rates, respectively,  $\eta_i$  is the reduction factor in anoxic conditions,  $S_j$  is the concentration of nutrient j (j = Substrate,  $O_2$ , NH<sub>4</sub>, NO<sub>2</sub>, NO<sub>3</sub>),  $K_{i,j}$  is the affinity constant between nutrient j and functional group i,  $bm_i$  is the maintenance RATE for i, and X is the biomass density

	Process		cessSoluble				Particulate					Kinetic Expression
			$S_{O_2}$	$S_{\rm NH_4}$	$S_{\rm NO_2}$	$S_{\rm NO_3}$	$X_{\rm H}$	$X_{\rm A}$	$X_{\rm N}$	$X_{\rm E}$	$X_{\rm D}$	
	Aerobic growth	$-rac{1}{Y_H}$	$-rac{1-Y_H-Y_E}{Y_H}$				1					$\mu_{\rm H} \frac{S_{\rm sub}}{K_{\rm H, \ O2} + S_{\rm sub}} \frac{S_{\rm O2}}{K_{\rm H, O2} + S_{\rm O2}} X_{\rm H}$
	Anoxic growth on $NO_2$	$-rac{1}{Y_H}$				$-\frac{1-Y_{\rm H}-}{2.86Y_{\rm H}}$	$\frac{Y_E}{H}$ 1					$\eta_{\rm H} \mu_{\rm H} \frac{S_{\rm sub}}{{\rm K}_{\rm H,  sub} + S_{\rm sub}} \frac{S_{\rm NO_2}}{{\rm K}_{\rm H, NO_2} + S_{\rm O_2}} \frac{{\rm K}_{\rm H, O_2}}{{\rm K}_{\rm sH, O_2} + S_{\rm O_2}} X_{\rm H}$
	Anoxic growth on $NO_3$	$-\tfrac{1}{Y_{H}}$		-	$-\frac{1-Y_{\rm H}-Y_{\rm E}}{1.17Y_{\rm H}}$	-1	1					$\eta_{\rm H} \mu_{\rm H} \frac{S_{\rm sub}}{K_{\rm H, sub} + S_{\rm sub}} \frac{S_{\rm NO_3}}{K_{\rm H, NO_3} + S_{\rm O_2}} \frac{K_{\rm H, O_2}}{K_{\rm H, O_2} + S_{\rm O_2}} X_{\rm H}$
<b>H</b> ET	Aerobic Maintenance		-1				-1					$bm_{\mathrm{H}} rac{S_{\mathrm{O}2}}{\mathrm{K}_{\mathrm{H},\mathrm{O}2} + S_{\mathrm{O}2}} X_{\mathrm{H}}$
	Anoxic Maintenance on NO <sub>2</sub>				$-\frac{1}{1.17}$		-1					$\eta_{\rm H} b m_{\rm H} \frac{S_{\rm NO_2}}{K_{\rm H,NO_2} + S_{\rm NO_2}} \frac{K_{\rm H,O_2}}{K_{\rm H,O_2} + S_{\rm O_2}} X_{\rm H}$
	Anoxic maintenance on NO <sub>3</sub>					$-\frac{1}{2.86}$	-1					$\eta_{\rm H} b m_{\rm H} \frac{S_{\rm NO_3}}{K_{\rm H, NO_3} + S_{\rm NO_3}} \frac{K_{\rm H, O_2}}{K_{\rm H, O_2} + S_{\rm O_2}} X_{\rm H}$
	Decay						-1					$b_{ m H}X_{ m H}$
	Aerobic growth		$-\tfrac{3.42-Y_A}{Y_A}$	$-rac{1}{Y_A}$	$\frac{1}{Y_A}$			1				$\mu_{\rm A} \frac{S_{\rm NH4}}{K_{\rm A, NH4} + S_{\rm NH4}} \frac{S_{\rm O2}}{K_{\rm A, O2} + S_{\rm O2}} X_{\rm A}$
AOB	Maintenance		-1					-1				$bm_{\mathrm{A}} \frac{S_{\mathrm{O}_2}}{\mathrm{K}_{\mathrm{A},\mathrm{O}_2} + S_{\mathrm{O}_2}} X_{\mathrm{A}}$
	Decay							-1				$b_{ m A}X_{ m A}$
	Aerobic growth		$-\tfrac{1.15-Y_N}{Y_N}$		$-\frac{1}{Y_N}$	$\frac{1}{Y_{N}}$			1			$\mu_{\rm N} \frac{S_{\rm NO_2}}{K_{\rm N,NO_2} + S_{\rm NO_2}} \frac{S_{\rm O_2}}{K_{\rm N,O_2} + S_{\rm O_2}} X_{\rm N}$
NOB	Maintenance		-1						-1			$bm_{\mathrm{N}} \frac{S_{\mathrm{O2}}}{\mathrm{K}_{\mathrm{N},\mathrm{O2}} + S_{\mathrm{O2}}} X_{\mathrm{N}}$
	Decay								-1			$b_{ m N}X_{ m N}$
$\mathbf{E}PS$	Decay	1								-1		$b_{ m E}X_{ m E}$
DEAD	) Decay	1									-1	$b_{\mathrm{D}}X_{\mathrm{D}}$

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