**Text S1. Mathematical analysis of the stability of the normal orixate phyllotactic pattern in DC1**

In the present section, we considered the stability of normal orixate phyllotaxis, which has ideal periodic repetition of a sequence of divergence angles consisting of exactly $180°$, $90°$, $-180°$, and $-90°$.

Mathematical analysis was performed for the DC1 system, in which the radius of the shoot apical meristem $R\_{0}$is 1 and $L\_{i}$ is the $i$th leaf primordium located at $\left(r\_{i}\cos(θ\_{i}),r\_{i}\sin(θ\_{i})\right)$ with $r\_{i}>1$. The $L\_{i}$’s inhibitory effect $E\left(x\right)$ at $\left(\cos(θ),\sin(θ)\right)$ on the SAM periphery is dependent solely on $d\_{i}(θ)$, the distance from $L\_{i}$. When the $n$th primordium $L\_{n}$ is arising, the inhibitory field strength $I\left(θ\right)$ at the position $\left(\cos(θ),\sin(θ)\right)$ is calculated by summing the inhibitory effects from all existing primordia, as follows.

$$\begin{array}{c}I\left(θ\right)=\sum\_{k=1}^{n-1}E\left(d\_{k}\left(θ\right)\right)=\sum\_{j=0}^{3}\sum\_{i=1}^{\left⌊\frac{n-1+j}{4}\right⌋}E\left(d\_{n-4i+j}\left(θ\right)\right)\#\left(S1\right)\end{array}$$

When the normal pattern of orixate phyllotaxis is stably maintained, the inhibitory field strength should give a minimum at $θ=θ\_{n-4i}$. Hence, when setting $θ\_{n-4i}=0$, the following equation should be satisfied:

$$\begin{array}{c}\left.\frac{dI\left(θ\right)}{dθ}\right|\_{θ=0}=0\#\left(S2\right)\end{array}$$

Because $d\_{k}\left(θ\right)=\sqrt{r\_{k}^{2}+1-2r\_{k}\cos(\left(θ-θ\_{k}\right))}$, we obtain:

$$\begin{array}{c}\left.\frac{dd\_{k}\left(θ\right)}{dθ}\right|\_{θ=0}=\left\{\begin{array}{c}0 \left(θ\_{k}=0, π\right)\\\mp \frac{r\_{k}}{ϱ\left(r\_{k}\right)} \left(θ\_{k}=\pm \frac{π}{2}\right)\end{array}\right.\#\left(S3\right)\end{array}$$

$$\begin{array}{c}d\_{k}\left(0\right)=\left\{\begin{array}{c}r\_{k}-1 \left(θ\_{k}=0\right)\\r\_{k}+1 \left(θ\_{k}= π\right)\\ϱ\left(r\_{k}\right) \left(θ\_{k}=\pm \frac{π}{2}\right)\end{array}\right.,\#\left(S4\right)\end{array}$$

where $ϱ\left(r\right)≡\sqrt{r^{2}+1}$.

Thus,

$$\begin{array}{c}\left.\frac{dE\left(d\_{k}\left(θ\right)\right)}{dθ}\right|\_{θ=0}=\left.\frac{dE\left(x\right)}{dx}\right|\_{x=d\_{k}\left(0\right)}\left.\frac{dd\_{k}\left(θ\right)}{dθ}\right|\_{θ=0}=\left\{\begin{array}{c}0 \left(θ\_{k}=0, π\right)\\\mp f\left(r\_{k}\right) \left(θ\_{k}=\pm \frac{π}{2}\right)\end{array}\right.,\#\left(S5\right)\end{array}$$

where $f\left(r\right)≡\frac{r}{ϱ\left(r\right)}\left.\frac{dE\left(x\right)}{dx}\right|\_{x=ϱ\left(r\right)}$.

Regarding the arrangement of primordia, there are two geometrical situations; in situation 1, the divergence angle between the newly arising primordium, $L\_{n}$, and the last primordium, $L\_{n-1}$, is $\pm 90°$ ($\pm π/2$), while it is $180°$ ($π$) in situation 2 (Fig S1A).

(Situation 1)

Situation 1 is represented by setting $θ\_{n-4i+j}$ as:

$$\begin{array}{c}θ\_{n-4i+j}=\left\{\begin{array}{c}0 \left(j=0\right)\\π \left(j=1\right)\\-\frac{π}{2} \left(j=2\right)\\\frac{π}{2} \left(j=3\right)\end{array}\right..\#\left(S6\right)\end{array}$$

The application of this condition to Eq S5 yields:

$$\begin{array}{c}\left.\frac{dE\left(d\_{n-4i+j}\left(θ\right)\right)}{dθ}\right|\_{θ=0}=\left\{\begin{array}{c}0 \left(j=0, 1\right)\\f\left(r\_{n-4i+j}\right) \left(j=2\right)\\-f\left(r\_{n-4i+j}\right) \left(j=3\right)\end{array}\right..\#\left(S7\right)\end{array}$$

Hence,

$$\begin{array}{c}\left.\frac{dI\left(θ\right)}{dθ}\right|\_{θ=0}=\sum\_{j=0}^{3}\sum\_{i=1}^{\left⌊\frac{n-1+j}{4}\right⌋}\left.\frac{dE\left(d\_{n-4i+j}\left(θ\right)\right)}{dθ}\right|\_{θ=0}\\ =\sum\_{i=1}^{\left⌊\frac{n+1}{4}\right⌋}f\left(r\_{n-4i+2}\right)-\sum\_{i=1}^{\left⌊\frac{n+2}{4}\right⌋}f\left(r\_{n-4i+3}\right).\#\left(S8\right)\end{array}$$

Because $E\left(x\right)$ is a monotonically decreasing function,$f\left(r\right)$ is always negative:

$$\begin{array}{c}\left.\frac{dI\left(θ\right)}{dθ}\right|\_{θ=0}\geq \sum\_{i=1}^{\left⌊\frac{n+1}{4}\right⌋}\left\{f\left(r\_{n-4i+2}\right)-f\left(r\_{n-4i+3}\right)\right\}.\#\left(S9\right)\end{array}$$

(Situation 2)

Situation 2 is represented by setting $θ\_{n-4i+j}$ as:

$$\begin{array}{c}θ\_{n-4i+j}=\left\{\begin{array}{c}0 \left(j=0\right)\\-\frac{π}{2} \left(j=1\right)\\\frac{π}{2} \left(j=2\right)\\π \left(j=3\right)\end{array}\right..\#\left(S10\right)\end{array}$$

The $θ$-derivative of $I\left(θ\right)$ can be calculated as in the case described for situation 1:

$$\begin{array}{c}\left.\frac{dI\left(θ\right)}{dθ}\right|\_{θ=0}=\sum\_{i=1}^{\left⌊\frac{n}{4}\right⌋}f\left(r\_{n-4i+1}\right)-\sum\_{i=1}^{\left⌊\frac{n+1}{4}\right⌋}f\left(r\_{n-4i+2}\right)\\ \geq \sum\_{i=1}^{\left⌊\frac{n}{4}\right⌋}\left\{f\left(r\_{n-4i+1}\right)-f\left(r\_{n-4i+2}\right)\right\}.\#\left(S11\right)\end{array}$$

According to the distance dependency of the inhibitory effect assumed in DC1, $E\left(ϱ\right)=kϱ^{-η}$. Using this assumption and noting that $η>0$ and $r>1$, we obtain:

$$\begin{array}{c}\frac{df\left(r\right)}{dr}=\frac{d}{dr}\left(\frac{r}{ϱ}\frac{d}{dϱ}kϱ^{-η}\right)=kηϱ^{-η-4}\left\{\left(η+1\right)r^{2}-1\right\}>0.\#\left(S12\right)\end{array}$$

As $f\left(r\right)$ increases monotonically with $r$, $f\left(r\_{n-4i+1}\right)>f\left(r\_{n-4i+2}\right)>f\left(r\_{n-4i+3}\right)$, and then $\left.\frac{dI\left(θ\right)}{dθ}\right|\_{θ=0}>0$ in both situations. This indicates that the total inhibitory field strength cannot satisfy Eq S2, which demonstrates that normal orixate phyllotaxis cannot be established in DC1.