S4 Text: Optimal estimator for 0,1 cost function is MAP estimator

The cost function associated with the L0 norm is given by

$$\gamma\left(\hat{X},X\right)=E\left[\left(\hat{X}-X\right)^{0}\right]$$

$$\gamma\left(\hat{X},X\right)=\sum_{X}\left(\hat{X}-X\right)^{0}p\left(X\right)$$

Splitting the summation for all $\hat{X}$ that do and do not equal X

$$\gamma\left(\hat{X},X\right)=\sum_{X_k=\hat{X}}0\,p\left(X_k\right)+\sum_{\forall X\neq\hat{X}}p\left(X\right)$$

The first term disappears and the second term can be rewritten

$$\gamma\left(\hat{X},X\right)=1-p\left(X_k\right)$$

The value of $\hat{X}$ that corresponds to the maximally probable X minimizes the cost.

$$\hat{X}^{opt}=\arg\max_{X}p\left(X\right) \tag{S23}$$

Thus, for the 0,1 cost function, the maximum a posteriori (MAP) estimator is the optimal estimator.