Coding with transient trajectories in recurrent neural networks

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Supporting information

S9 Text

The exponential of the sum of two matrices \( A \) and \( B \) can be factorized as

\[
\exp(A + B) = \exp(A) \exp(B)
\]

only if \( A \) and \( B \) commute, i.e. if the commutator \([A, B] = AB - BA\) is equal to zero. In the following we compute the mean and the variance of the commutator

\[
C = [\Delta u^{(1)} v^{(1)} T, \Delta u^{(2)} v^{(2)} T]
\]

and show that

\[
\langle C_{ij} \rangle = 0, \quad \langle C_{ij}^2 \rangle \simeq \frac{2\Delta^4}{N^3}
\]

The mean of \( C_{ij} \) is given by

\[
\langle C_{ij} \rangle = \sum_{k=1}^{N} \left( u^{(1)}_i v^{(1)}_k u^{(2)}_j - u^{(2)}_i v^{(2)}_k u^{(1)}_j \right).
\]

The variance of \( C_{ij} \) is given by

\[
\langle C_{ij}^2 \rangle = \sum_{k,l=1}^{N} \left( u^{(1)}_i v^{(1)}_k u^{(2)}_j v^{(2)}_l - u^{(2)}_i v^{(2)}_k u^{(1)}_j v^{(1)}_l \right) + \frac{1}{N^4} \delta_{il} \delta_{jk} \delta_{ki} \delta_{jl} = \frac{1}{N^4} \delta_{ij}.
\]
Using Eq. (145) and Eq. (146), we obtain Eq. (142). Thus, in the limit of large $N$ we can write

$$
\exp \left( t(\Delta u^{(1)}v^{(1)}T_1 + \Delta u^{(2)}v^{(2)}T_2 - I) \right) = e^{-t} \exp \left( t(\Delta u^{(1)}v^{(1)}T_1) \right) \exp \left( t(\Delta u^{(2)}v^{(2)}T_2) \right)
$$

(147)

and recover Eq. (90).