Supplementary information for the article:  
Coupling water fluxes with cell wall mechanics in a multicellular 
model of plant development. 

Calculations for the two-cells model 

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Gathering the flux equation (Eq. 8 from main text), the wall mechanics equation (Eq. 1 from main text) with \( \frac{dP}{dt} = 0 \), and the fact that \( \dot{\gamma}_i = \dot{\varepsilon}_i \), we get

\[
\phi^a(P^M - P_0) + \frac{\phi^s}{2}(P_1 - P_0) - \phi^w(P_0 - P_0^Y)^+ = 0 
\] 
(S11)

\[
\phi^a(P^M - P_1) - \frac{\phi^s}{2}(P_1 - P_0) - \phi^w(P_1 - P_1^Y)^+ = 0. 
\] 
(S12)

First, we assume that both cells are growing \( (P_i > P_i^Y, i = 0, 1) \).

First regime: \( (P_i > P_i^Y, i = 0, 1) \). Adding (S11) and (S12) we get:

\[
P = \alpha^a P^M + (1 - \alpha^a)\overline{P}^Y, 
\] 
(S13)

where \( \alpha^a = \frac{\phi^a}{\phi^a + \phi^w} \), \( \overline{P} = \frac{P_0 + P_1}{2} \). With Eq. 1 from main text, we get

\[
\overline{\gamma} = \frac{\phi^a \phi^w}{\phi^a + \phi^w} (P^M - \overline{P}^Y), 
\] 
(S14)

where \( \overline{\gamma} = \frac{\gamma_0 + \gamma_1}{2} \). Therefore, the gathering of two cells behaves the same as one cell if one considers the mean values.

Then, we examine the heterogeneities in turgor and growth rate. Substracting (S11) to (S12), we get

\[
\Delta P = \frac{\phi^w}{\phi^a + \phi^s + \phi^w} \Delta P^Y. 
\]

Let

\[
\alpha^s = \frac{\phi^s}{\phi^s + \phi^a}. 
\]

Then the previous expression becomes

\[
\Delta P = \frac{(1 - \alpha^a)(1 - \alpha^s)}{1 - \alpha^s + \alpha^a \alpha^s} \Delta P^Y. 
\] 
(S15)

As \( (1 - \alpha^a)(1 - \alpha^s) = 1 - \alpha^a + \alpha^a \alpha^s < 1 - \alpha^s + \alpha^a \alpha^s \), we find that turgor difference \( \Delta P \) cannot exceed the value \( \Delta P^Y \). When \( \alpha^s = 0 \) (symplasmic fluxes negligible with respect to apoplasmic ones), then \( \Delta P = (1 - \alpha^a)\Delta P^Y \); when \( \alpha^s > 0 \), symplasmic fluxes tend to reduce the turgor heterogeneity between cells.

With Eq. 7 from main text we get then

\[
\Delta \dot{\gamma} = \frac{(\phi^a + \phi^s)\phi^w}{\phi^a + \phi^s + \phi^w} \Delta P^Y, 
\] 
(S16)

where \( \Delta \dot{\gamma} = \frac{\dot{\gamma}_0 - \dot{\gamma}_1}{2} \). Note that this expression is valid iff \( P_1 > P_1^Y \) or equivalently \( \dot{\gamma}_1 > 0 \). The limit \( \dot{\gamma}_1 = 0 \) corresponds to the situation where cell 0 is growing in such a way that it prevents cell 1 to grow because of
the symplasmic fluxes between them. We examine how this situation can occur depending on the values of the symplasmic conductivity \( \phi^s \) and the other parameters. We find that

\[
P_1 > P_1^Y \iff \frac{\phi^a + \phi^s}{\phi^a + \phi^w} \frac{\Delta P^Y}{P_M - P_0^Y} < \frac{\phi^a}{\phi^a + \phi^w}
\]

\[
\iff \frac{\alpha^a}{1 - (1 - \alpha^s)\alpha^a} \rho < \alpha^a
\]

\[
\iff \alpha^s < \frac{1 - \rho}{1 - \alpha^a}.
\]

For instance, \( P_0^Y = 0.25 \) MPa, \( P_1^Y = 0.5 \) MPa, and \( P_M = 0.625 \) MPa yields \( \rho = 0.5 \). The hypothesis of this study \((P_0^Y < P_1^Y < P_M)\) corresponds to the condition \( \rho \in [0, 1] \). Note that if \( \alpha^a > \rho \), then \( \frac{\alpha^a}{1 - \alpha^s} > 1 \), and the condition is verified whatever the value of \( \alpha^s \); if \( \alpha^s = 1 - \rho \), the condition is equivalent to \( \alpha^a > 0 \), which is also always verified. Fig. 2a) in main text recapitulates the regions of the parameters space \( \alpha^a \times \alpha^s \) where the condition is verified, for different values of \( \rho \). The size of the region \( \gamma_1 = 0 \) increases as \( \rho \) gets closer to 1.

**Second regime:** \( P_0 > P_0^Y \) and \( P_1 < P_1^Y \). In this case, eqs. (S11) and (S12) turn into

\[
\phi^a(P^M - P_0) + \frac{\phi^s}{2}(P_1 - P_0) - \phi^w(P_0 - P_0^Y) = 0 \quad (S17)
\]

\[
\phi^a(P^M - P_1) - \frac{\phi^s}{2}(P_1 - P_0) = 0. \quad (S18)
\]

(S18) leads to

\[
P_1 = (1 - \tilde{\alpha}^s)P^M + \tilde{\alpha}^sP_0, \quad (S19)
\]

where \( \tilde{\alpha}^s = \frac{\phi^s}{\phi^a + \phi^w} \). Adding eqs. (S17) and (S18) leads to

\[
P_0(\phi^a + \phi^w) = 2\phi^aP^M + \phi^wP_0^Y - \phi^a((1 - \tilde{\alpha}^s)P^M + \tilde{\alpha}^sP_0),
\]

then,

\[
P_0(\phi^a(1 + \tilde{\alpha}^s) + \phi^w) = \phi^a(1 + \tilde{\alpha}^s)P^M + \phi^wP_0^Y,
\]

and finally

\[
P_0 = \alpha^aP^M + (1 - \alpha^a)P_0^Y, \quad (S20)
\]

where

\[
\alpha^a = \frac{\phi^a}{\phi^a + \phi^w} \quad \text{and} \quad \phi^a = \phi^a(1 + \tilde{\alpha}^s).
\]

Hence, thanks to the symplasmic fluxes from its neighbour cell 1, cell 0 benefits from an enhanced access to the apoplasmic fluxes by a factor \( \phi^{as}/\phi^a = 1 + \tilde{\alpha}^s \). Then, from Eq. 1 in main text, the relative growth rate of cell 0 is

\[
\dot{\gamma}_0 = \frac{\phi^{as}\phi^w}{\phi^{as} + \phi^w}(P^M - P_0^Y). \quad (S21)
\]

By hypothesis, the growth rate of cell 1 is zero, and we can compute the heterogeneity in turgor: from (S19), we find that

\[
\Delta P = \frac{1 - \tilde{\alpha}^s}{2}(P_M - P_0),
\]

and hence

\[
\Delta P = \frac{1}{2}(1 - \tilde{\alpha}^s)(1 - \alpha^{as})(P_M - P_0^Y). \quad (S22)
\]