To study the dynamical changes in $x_H$ (Hertzian deformation) and $x_b$ (beam-bending deformation) and their contribution to the total deformation $X = x_H + x_b$ (see Fig. 4 in the main text), we employed the method of Lagrange multipliers. This method allows us to find the values of $x_H$ and $x_b$ that minimize the total deformation force,

$$F(x_H, x_b) = k_H x_H^{3/2} + K_b x_b s(x_b)$$

(see the inset Fig. 4 in main text), subject to the constraint: $X = x_H + x_b$. To that end, we constructed the Lagrange function

$$\Lambda(x_H, x_b, \lambda) = F(x_H, x_b) + \lambda g(x_H, x_b),$$

where

$$g(x_H, x_b) = x_H + x_b - X$$

Here $\lambda$ is the Lagrange multiplier. By calculating the partial derivatives of $\Lambda(x_H, x_b, \lambda)$ with respect to each of the two variables $x_H$ and $x_b$, we obtained equations of the form

$$\nabla_{x_H, x_b} F(x_H, x_b) = -\lambda \nabla_{x_H, x_b} g(x_H, x_b)$$

Next, by eliminating $\lambda$ we arrived at the system of two equations:

$$3/2 k_H x_H^{1/2} = K_b s(x_b) + K_b x_b s'(x_b)$$

$$X = x_H + x_b$$

For small deformations $x_b$, we expand the Weibull survival probability $s(x_b) = \exp[-(K_b x_b / F_b^*)^m]$ in powers of the exponent $z = K_b x_b / F_b^*$, and retain the terms up to the first order in $z$. Then, Eq. (S4) becomes:

$$3/2 k_H x_H^{1/2} - K_b (1 - \left( \frac{K_b x_b}{F_b^*} \right)^m) (1 - m \left( \frac{K_b x_b}{F_b^*} \right)^m) = 0$$

(S6)

Eq. (S5) allows us to eliminate $x_H$ by substituting $x_H = X - x_b$ into Eq. (S6) above:

$$3/2 k_H (X - x_b)^{1/2} - K_b (1 - \left( \frac{K_b x_b}{F_b^*} \right)^m) (1 - m \left( \frac{K_b x_b}{F_b^*} \right)^m) = 0$$

(S7)

Simplifying Eq. (S7) and grouping terms of the same power in $x_b$, we arrive at the following polynomial equation:

$$a_1 x_b^4 + a_2 x_b^3 + a_3 x_b^2 + a_4 x_b + a_5 x_b + a_6 = 0$$

(S8)
with the following constant coefficients:

\[
\begin{align*}
\alpha_1 &= m^2 K_b^2 \left( \frac{K_b}{F_b^*} \right)^{4m} \\
\alpha_2 &= -2m(1 + m)K_b^2 \left( \frac{K_b}{F_b^*} \right)^{3m} \\
\alpha_3 &= (1 + 4m + m^2)K_b^2 \left( \frac{K_b}{F_b^*} \right)^{2m} \\
\alpha_4 &= -2(1 + m)K_b^2 \left( \frac{K_b}{F_b^*} \right)^m \\
\alpha_5 &= \frac{9}{4} k_H^2 \\
\alpha_6 &= K_b^2 - \frac{9}{4} k_H^2 X
\end{align*}
\]

Eq. (S9) can be solved numerically (for example, using Mathematica software) for a given set of parameters \(k_H, K_b, F_b^*\), and \(m\), and for each specified value of the total deformation \(X\). The obtained numerical solution for \(x_H\) and \(x_b\) can then be substituted into the expression for \(F(x_H, x_b)\) (Eq. (14) in the main text).

**References**