PLOS COMPLEX SYSTEMS

RESEARCH ARTICLE

Persistence of wealth inequality from network effects

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Citation: Naggi EF, Rossini S, Andrade Jr. JS, La Porta CAM, Zapperi S (2025) Persistence of wealth inequality from network effects. PLOS Complex Syst 2(6): e0000050. ttps://doi.org/10.1371/journal.pcsy.0000050

Editor: Dariusz Siudak, Lodz University of Technology: Politechnika Lodzka, POLAND

Received: February 14, 2025

Accepted: May 6, 2025

Published: June 4, 2025

Peer Review History: PLOS recognizes the benefits of transparency in the peer review process; therefore, we enable the publication of all of the content of peer review and author responses alongside final, published articles. The editorial history of this article is available here: https://doi.org/10.1371/journal.pcsy.0000050

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Data availability statement: Longitudinal wealth and income data for Italy was obtained from the Bank of Italy (BoI) for the years 1991-2020 ("Indagine sui bilanci delle famiglie italiane") last accessed on March 22, 2023,

Abstract

Addressing wealth and income inequality requires a thorough understanding of the mechanisms driving these disparities. Agent-Based Models (ABMs) offer a powerful tool for simulating these complex systems, capturing the intricate interplay of individual behaviors and emergent macroeconomic trends. Here we consider two existing ABM classes: one, exemplified by the Nirei-Souma (NS) model, which simulates how individuals accumulate wealth through income from work, returns on investments, and consumption, and the other, represented by the Bouchaud-Mezard (BM) model, which emphasizes the role of wealth exchanges and random returns in shaping the wealth distribution. Drawing on empirical evidence of wealth and income distribution in Italy, we benchmark both models revealing that they effectively captures Pareto-like wealth distribution, but fail to fully account for the persistent lack of social mobility observed in empirical data. To overcome this limitation, we propose an interacting version of the NS model, integrating it with wealth exchange mechanisms. Through this interacting model, we can show the influence of network topology on wealth distribution and dynamics. Simulations on hierarchical networks yield results that align more closely with empirical observations compared to regular random graphs, highlighting the importance of hierarchical interactions in shaping wealth inequality and social mobility. The model is further analyzed to reveal the interplay between income sources and wealth accumulation.

Author summary

In most countries, a small fraction of the population controls a disproportionate share of wealth leading to large social disparities. Large wealth inequality tend to be also persistent, with little mobility between different classes. In this work, we investigate basic mechanisms leading to a persistent wealth inequality by comparing the predictions of

from https://www.bancaditalia.it/statistiche/basi-dati/rdc/index.html

Funding: JSA gratefully acknowledges the FUNCAP Award 06849573/2023, the CNPq Award 303765/2017-8 (Bolsa PQ), and the CAPES Award 88887.311932/2018-00 (CAPES/PRINT) for financial support. CAMLP acknowoledge funding from FAIR - Future Artificial Intelligence Research: Adaptive AI methods for Digital Health" grant number PNRR_BAC24GVALE_01 PE_0000013, CUP D53C2200238000 and from CAPES (PrInt, process n. 88887.937852/2024-00). SZ acknowledges support from CAPES (PrInt, process n. 88887.937759/2024-00) and from the PRIN 2022 project METACTOR funded from the European Union - Next Generation EU, Mission 4 Component 1 CUP G53D2300164000. CAMLP and SZ thank for their hospitality the Physics department of the Universidade Federal do Ceara' and the School of Public Health of the state of Ceara', where this work has been completed. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing interests: The authors have declared that no competing interests exist.

simple agent based models with survery data on income and wealth in Italy. Our results highlight the importance of network effects in shaping the wealth distribution and the dynamics of wealth accumulation. We show that persistent wealth inequality naturally stem from the hierarchical organization of the interactions among agents. Weakly connected agents can only exchange wealth with few other agents and are trapped at the lower end of the wealth distribution, relying only on income from labor. Well connected agents tend instead to accumulate wealth through increasing returns on their investments. Models like ours could be used to simulate policy interventions aimed at redressing excessive wealth inequality.

Introduction

Inequalities are increasingly prominent in the contemporary world and reflect profound disparities in opportunities, resources, and rights among individuals within the society. Recent reports highlight a sharp increase in wealth and income inequality with wealth becoming ever more concentrated [1], reaching levels comparable to those seen in the early 20th century [2]. As shown by Piketty and Saez, income and wealth inequality gradually decreased in Europe and USA until the 1970s but then started to grow, especially in the USA [3]. This general trend affects both developing and emerging countries, driven by policies that have amplified its effects such as the reduction of progressive taxation which disproportionately benefit high-income and high-wealth individuals, and cuts to welfare programs that have disproportionately impacted lower-income groups and undermined social mobility [4]. Additionally, structural disparities in income and wealth influence the saving capacity of low income households. This trend has been exacerbated in recent years also by economic crises, pandemics, and geopolitical conflicts [4].

For households without significant assets, income is essential for survival [5]. Conversely, higher income levels facilitate wealth accumulation, net of fixed expenses. Income, however, is not limited to wages: wealth itself, which might otherwise remain static, can become a source of income through investments [6]. Lower income levels not only increase vulnerability to economic shocks and uncertainty but also hinder access to diverse revenue streams. Inequality is also deeply intertwined with social mobility. According to the so called "Great Gatsby effect" [7], societies with high levels of inequality also tend to have lower levels of mobility. As mentioned earlier, wealth concentration in the hands of a few leaves the majority with insufficient resources to improve their social standing. Conversely, a disadvantaged social position limits access to crucial elements such as education, healthcare [8,9], and investment opportunities, all of which are essential for upward mobility. This creates a vicious cycle where poorer families remain trapped in their conditions, while wealthier ones continue to accumulate resources, further widening the gap. Wealth and income inequality are often also associated to geographical segregation, so that different income groups become also effectively physically separated [10,11].

Addressing the persistent dynamics of inequality requires policies grounded in a thorough understanding of the underlying mechanisms. The study of such complex phenomena demands robust modeling approaches. In this work, we employ Agent-Based Models (ABMs) [12], a widely used tool for analyzing socio-economic systems and building econophysics theories [13–18]. ABMs simulate the behavior of individuals, households, or institutions within an economic context, capturing interactions and dynamics that often result in emergent phenomena—patterns that are difficult to predict using traditional models. In this framework, agents interact at a microscopic level, leading to macro-level behaviors and trends. To put ABMs in a realistic context, we consider empirical evidence of inequality and mobility in Italy in the past twenty years. Longitudinal survey data from the Bank of Italy provide a comprehensive view of wealth and income distribution, as well as their evolution over time. A recent analysis [19] reveals that the wealth distribution follows a Pareto-like pattern [20], a power-law distribution indicating the existence of a small fraction of individuals holding a disproportionate share of resources. Similarly, income data highlight growing disparities, showing that wealthier individuals tend to have higher earnings, primarily derived from investment returns rather than wages [19]. Finally, low social mobility patterns are evident, as individuals at the extremes of the wealth distribution rarely shift their economic status. This aligns with the notion that affluent families or individuals continue to consolidate wealth, while those at the lower end of the spectrum struggle to improve their financial standing.

Using this empirical evidence as a benchmark, we critically analyze two classes of existing ABMs, each offering unique insights into the dynamics of inequality. In the first class of models, popular in the economics literature, agents follow independent stochastic differential equation that accurately depict individual economic variables such as wages, consumptions, investments and taxation. As an example of this class of models, we consider explicitly the model introduced by Nirei and Souma (NS) [21]. The main limitation of this model is that it does not describe how income and wealth of an agent may depend on the behavior of the other agents. At the other hand of the spectrum are ABMs, mostly studied in the econophysics literature, focusing on the interplay between wealth exchanges among agents and random returns from investments [13–18,22]. Among this class of ABMs, we study the model introduced by Bouchaud and Mezard [22] that captures the behavior of the wealthiest strata, reproducing Pareto tails in wealth distribution. The model, when simulated in fully connected or random graphs, underestimates the large persistence of agents in the top wealth class [19]. Building on this insight, we propose a new model that combines the strengths of the two approaches. The goal is to create a more comprehensive framework able to represent the dynamics of inequality in a way that aligns more closely with empirical data.

Methods

Data

Longitudinal wealth and income data for Italy was obtained from the Bank of Italy (BoI) for the years 1991-2020 ("Indagine sui bilanci delle famiglie italiane") last accessed on 22/3/2023 from https://www.bancaditalia.it/statistiche/basi-dati/rdc/index.html. We analyzed the microdata from a panel of $N = 8000$ households for years ranging from 1991 to 2020.

Statistical analysis

The statistical analysis follows closely the method discussed in [19]. To estimate wealth distributions, we employ logarithmic bins. Given the wealth $W_i(t)$ of household i at time t, we first normalize the variable by its weighted mean as $w_i = W_i/\langle W_i \rangle$, where $\langle W_i \rangle = \sum_i p_i W_i$. The normalized weights $p_i = M_i/\sum_i M$ take into account the statistical weights M_i associated to each household in the survey, which are designed to compensate for unequal selection probabilities, missing responses and attrition (see methodological section at https://www.bancaditalia.it/statistiche/basi-dati/rdc/index.html). We select the positive values $W_i > 0$ and log-transform the normalized variable $X_i = \log_{10}(W_i/\langle W_i \rangle)$. We then construct the histogram of X_i by defining a set of linearly-spaced intervals summing the statistical weights of the data points falling in each bin. The resulting counts in each interval are then divided by the length of the interval. Zero and negative values are analyzed separately. We then classify each household according to the decile interval to which it belongs within the wealth distribution. Given a variable x whose cumulative distribution is $F(x)$, we define decile x_q as $F(x_q) = q/10$ for $q = 0, ..., 9$. In practice, we sort households according to their wealth and divide them in $Q = 10$ intervals of equal weight. Thus, to each data point x_i we associate a number $0 \leq q_i \leq 9$, where $q = 0$ represent the bottom 10% of the wealth distribution and $q = 9$ the top 10%. We refer to this number as the wealth decile q_i of household i. We study the dynamics $q_i(t)$ of the wealth deciles associated to each household by following its time evolution for the time span available in the longitudinal surveys. From the decile trajectories $q(t)$, we estimate transition probability matrices $P_Q(q', t|q, t_0) = P(q_i(t) = q'|q_i(t_0) = q)$ describing the probability that $q_i = q'$ at time t when $q_i = q$ at time t_0. The time-dependent persistence of quantile q is then defined as the probability to remain in the initial quantile

$$\Phi_Q(t|q, t_0) = P_Q(q, t|q, t_0). \tag{1}$$

For a discussion of persistence in rank see also [23].

Models

Bouchaud-Mezard model. The model proposed by Bouchaud and Mezard (BM) [22] describes the evolution of a population of N agents, where the wealth W_i of the agent i follows a stochastic differential equation

$$\frac{dW_i}{dt} = \eta_i(t) W_i + \sum_j (J_{ij} W_j - J_{ji} W_i), \tag{2}$$

where J_{ij} controls wealth redistribution between agent i and agent j and $\eta_i(t)$ is a Gaussian random noise, interpreted in the Stratonovic sense, with mean m and variance σ. The definition of the model relies on an interaction matrix $J_{ij} = J_0 A_{ij}/c(j)$, where A_{ij} is the adjacency matrix of the network formed by the agents ($A_{ij} = 1$ if the node i is connected to j and zero otherwise), and $c(j)$ is the degree of node j. In this paper, we will explore different possibilities for the interaction network, as discussed below.

To integrate numerically the equations, we express them in terms of an N-dimensional system of equations for the vector $X = (W_0, ..., W_N)$ which obeys

$$dX_i = \sum_j D_{ij} X_j dt + B_i X_i d\psi_i, \tag{3}$$

where $D_{ij} = A_{ij} J_0/c(j) + \delta_{ij}(m - J_0/c \sum_k A_{ki})$, $B_i = \sigma$ and $d\psi_i$ are independent Wiener processes (with variance 1 and zero mean). We exploit the scaling invariance of the equations [22] and study the model as a function of the reduced parameter J_0/σ^2, measuring time in units of $1/\sigma^2$. Since we are only interested in relative inequality, we rescale the wealth W_i by the total wealth $W_T = \sum_i^N W_i$ defining the wealth fraction $\tilde{w}_i = W_i/W_T = N w_i$ belonging to agent i [17], so that the results are independent on the parameter m.

Nirei-Souma model. The idea behind the model developed by Nirei and Souma (NS) [21] is that an individual embedded in the socio-economic fabric performs three basic actions: accumulating wealth, consuming part of the accumulated wealth, and receiving income from work. Based on this, the authors write discrete-time equations for the evolution of three quantities: wealth $W(t)$, income $I(t)$, and consumption $C(t)$. The wealth of the i-th agent evolves according to a multiplicative process defined as follows:

$$W_i(t+1) = \eta_i(t)W_i(t) + I_i(t) - C_i(t).$$

The random returns $\eta_i(t)W_i$ involve a noise term $\eta_i(t)$ assumed to follow a log-normal distribution, so that the distribution of $\log \eta(t)$ is normal with mean r and variance σ. The second contribution to wealth accumulation corresponds to income from labor, $I_i(t)$, which follows an additive random process with a lower limit set by a minimum wage $\bar{I}(t)$:

$$I_i(t+1) = \begin{cases} uI_i(t) + s\epsilon_i(t)\bar{I}(t) & \text{if } uI_i(t) + s\epsilon_i(t)\bar{I}(t) > \bar{I}(t) \\ \bar{I}(t) & \text{otherwise,} \end{cases} \tag{4}$$

where the minimum wage grows as $\bar{I}(t) = \nu^t$. According to this rule, income from labor has a growth rate given by the variable u, while the minimum wage grows more or less quickly depending on the value of ν. Setting these two values to be larger than one, one assumes that wages do not decrease over time. In Eq 4, there is also a stochastic component representing individual productivity differences, with $\epsilon(t)$ following a standard normal distribution. This is scaled by a coefficient s that regulates the magnitude of fluctuations. Finally, the function representing consumption is written as:

$$C_i(t) = \bar{I}(t) + b(W_i(t) + I_i(t) - \bar{I}(t)). \tag{5}$$

Eq 5 indicates that the consumption of the i-th agent consists of subsistence consumption plus a fraction of savings, corresponding to the wealth owned and the income surplus above the subsistence level. The savings parameter b control the saving propensity: when $b = 1$, all savings are consumed, while when $b = 0$, consumption equals the subsistence level and saving is maximal. Here we assume that the saving propensity is independent on wealth, so that saving is a linear function of wealth. One could consider also situations in which the relation is not linear, to model larger saving propensity for high wealth agents.

The interacting Nirei-Souma model. We propose here an interacting version of the NS that combines the interesting features of the NS model, namely the combination of returns from investments, income and consumption, with a wealth exchange mechanism as the one described by the BM model. Our main assumption is that the income of an agent is proportional to the aggregate consumption of other agents. This simple assumption could be justified from the Keynesian multiplier concept [24], whereby an increase in aggregate spending has multiplicative effects on demand and thus income, or even from the idea of a circular economy [25], where consumption and services generate value flows that become the income of other actors in the system. Under this assumption, we consider N agents whose wealth and consumption are described as in the NS model

$$\begin{aligned} W_i(t+1) &= \eta_i(t)W_i(t) + I_i(t) - C_i(t) \\ C_i(t) &= \bar{I}(t) + b(I_i(t) - \bar{I}(t) + W_i(t)), \end{aligned}$$

while we modify the evolution of the income taking into account the interactions with other agents

$$I_i(t+1) = \begin{cases} \sum_{j\neq i} \frac{A_{ij}}{c_j} C_j(t) + s\epsilon_i(t)\bar{I}(t) & \text{if } I_i(t+1) > \bar{I}(t+1) \\ \bar{I}(t+1) & \text{otherwise} \end{cases}$$

In other words, the income of agent i at time $t+1$ is resulting from the consumptions of the other agents j interacting with i, according to the adjacency matrix A_{ij}.

Network topologies

The model described above depends on the particular structure of the adjacency matrix A_{ij} which corresponds to the interaction network connecting the agents. In this work we consider three different networks, namely the regular random graph [19,22], the sun graph [26] and the onion graph [27,28]. In all simulations, we considered graphs composed by $N = 1000$ nodes.

Regular random graph. In the regular random graph each node has the same degree c, and connections are generated randomly. The degree of the regular graph was set to $c = 4$ so that it is much smaller than the number of nodes. This choice might be justified by research on social networks [29], showing that the degree of node connections is much lower than the total number of nodes in the network. The regular random network allows for an analytical solution of the BM model [30,31]. The main limitation is the extreme regularity that does not match known relations between individuals.

Sun graph. The second network assumes that a restricted number of agents forms a strongly connected core, while most individuals have only few connections to the rest of the network [32]. To model this type of socio-economic network, we considered a sun graph [26] where out of N nodes, N_C nodes belong to a fully connected core, while the remaining nodes have a single connection to a randomly selected nodes in the core. The BM model on the sun graph was studied numerically in [26].

Onion graph. We finally adopt the onion network which could be considered as intermediate between the two previously defined graphs. The onion network model was originally introduced by Schneider et al. [27] as an optimized structure for enhancing the resilience of scale-free networks against malicious attacks. The model is characterized by a hierarchical organization in which high-degree nodes form a densely connected core, with peripheral nodes arranged in layers of decreasing degree, resulting in improved robustness against targeted node removal. This structural resilience was later examined by Wu and Holme [33], who demonstrated that networks with large spectral gaps, indicative of strong expander properties, naturally exhibit an onion-like topology. They also proposed a simplified generative algorithm capable of directly constructing onion networks without the need for optimization, while preserving their key resilience features. This network has a layered system where the core is highly connected and, as one moves away from the core, connections become sparser, reaching the outermost layers. The graph is designed so that each layer is characterized by a different level of connectivity, and a mechanism is introduced to favor more frequent connections between nodes within similar layers while gradually decreasing the probability of connections between nodes in distant layers. Following the algorithm in [28], the process of obtaining a graph with these characteristics is divided into three steps:

1. **Degree generation**: Each node is assigned a degree c that follows a power-law distribution with exponent γ.

$$P(c) \propto c^{-\gamma}$$

   It is ensured that each node has an integer degree greater than one.
2. **Layer connections**: For each node i, a degree c_i is assigned. Based on the degree, each node has a certain number of *stubs* (equal to its degree), which are pending connections

awaiting an endpoint. Once the list of *stubs* is created, nodes i and j will be connected with a probability:

$$P(i,j) \propto \frac{1}{1 + a|c_i - c_j|},$$

where a controls how sharply the probability decreases. Nodes with similar degrees have a higher probability of being connected.

3. **Regularization**: To avoid excessive stratification, random connections are added between nodes with a degree difference greater than one, with a fixed probability of 0.05. Additionally, checks are performed to remove self-loops, if present, and to ensure that the graph is connected, meaning no isolated components are allowed.

It is clear that the structure of the graph can be manipulated by varying the values of γ and a. Specifically, γ is the exponent of the power-law governing the degree distribution. Lower values of γ result in a less steep distribution, meaning more nodes with high degrees, creating a more homogeneous network. Conversely, higher values of γ lead to a steeper distribution, with very few nodes having many connections and most nodes having few. This creates a more heterogeneous network. In summary, γ controls the variation in node connectivity, influencing the hierarchical structure of the network. The parameter a affects the probability of connections between nodes based on the degree difference. When a is low, the probability decreases only slightly as the degree difference increases, allowing a more connected network where nodes with different degrees are more easily linked. This results in greater "mixing" between layers. As a increases, the probability of connections between nodes with significant degree differences decreases rapidly. This creates a network where nodes tend to connect primarily with nodes of similar degrees, generating greater segregation between layers. High-degree nodes preferentially connect to other high-degree nodes, and the same occurs for low-degree nodes. The network becomes more "layered." For both parameters, there are well-defined limits: γ, being a power-law exponent, should satisfy $\gamma > 2$, while a needs only to be positive. Here, we used parameters $\gamma = 2.5$ and $a = 0.3$ to achieve a smoother and less hierarchical stratification.

Results

Dynamics of wealth inequality in Italy

We summarize here a set of empirical observation on wealth distribution, its time evolution and the relation with income in Italy, as previously studied in [19]. These results will serve as a benchmark for evaluating the models. Examining the probability distribution of the wealth share in a log-log plot (Fig 1a), it is apparent that the tails follow a linear trend, indicating that the distribution for higher wealth levels exhibits Pareto-like behavior. Inequalities are evident in income disparities, not only in the sense that wealthier households have disproportionately higher incomes (Fig 1b), but also analyzing the income sources for households of different wealth (Fig 1c). Labor income is the primary source for the poorest households, whereas for wealthier households, the main source of income originates from investments.

Inequalities also manifest in the limited mobility (Fig 1d) and the persistence within a specific economic condition (Fig 1e). Assigning households to deciles based on their wealth, we observe that the most extreme deciles exhibit reduced mobility over the course of 20 years, with households in these groups tending to move, at most, to adjacent deciles. In other words, the wealthiest and poorest households are the most persistent, remaining in their respective wealth class for the longest time.

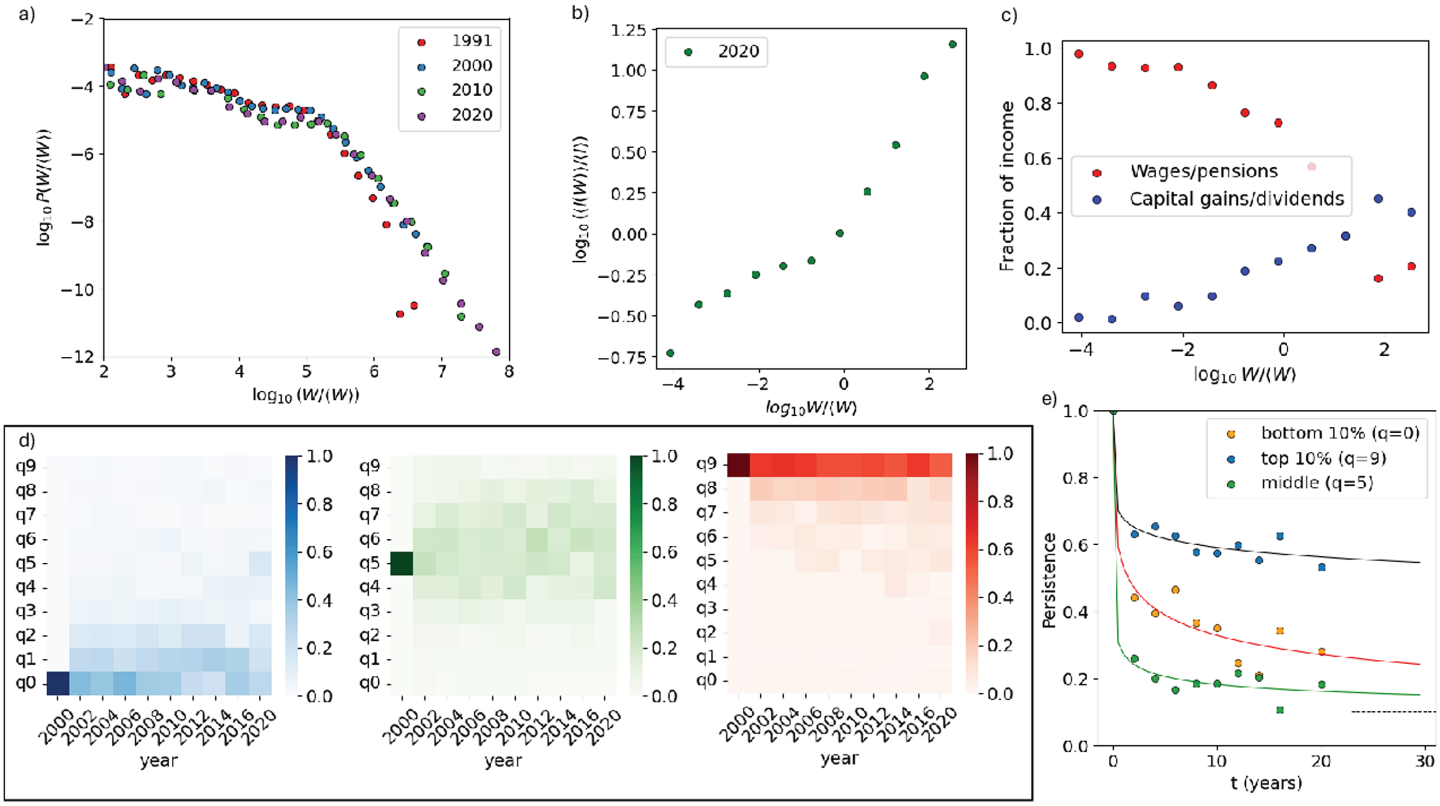

Fig 1. Key features of wealth inequality in Italy. (**a**) The distribution of household wealth share plotted in double logarithmic scales (data from the Bank of Italy). The distribution displays a Pareto tail for large wealth. (**b**) The expected value of household income given the household wealth in double logarithmic scale (only positive wealth is considered). (**c**) Fraction of household income due to wages pensions and transfers, dividends and capital gains for households of given net wealth. (**d,e**) Two graphical representations of the persistence of bottom (blue), middle (green) and top (red) deciles over time.

https://doi.org/10.1371/journal.pcsy.0000050.g001

The INS model reduces to the BM model in the limit of large wealth

Before studying in more details the dynamics of the BM and INS models on different interaction graphs, it is instructive to show that the evolution of the INS model reduces to the one of the BM model in the limit of large wealth. We consider for simplicity the INS model on a fully connected graph ($A_{ij} = 1$), but the following derivation can also be generalized in a straightforward manner to any other graph. We set the stochastic contribution to wages to zero and assume that wages are always above a *lower bound* represented by the minimum wage. The evolutionary equations for are wealth, consumption, and income can then be written as

$$W_i(t+1) = \eta_i(t) W_i(t) + I_i(t) - C_i(t)$$
$$C_i(t) = (1-b)\bar{I}(t) + b(I_i(t) + W_i(t)$$
$$I_i(t) = \frac{1}{N-1} \sum_{j \neq i} C_j(t-1).$$

We substitute the expression for consumption into the income term to simplify the system of equations by eliminating consumption:

$$I_i(t) = \frac{1}{N-1}\sum_{j\neq i}\left[(1-b)\bar{I}(t-1) + b(I_j(t-1) + W_j(t-1))\right] =$$
$$= (1-b)\bar{I}(t-1) + \frac{b}{N-1}\sum_{j\neq i}\left[I_j(t-1) + W_j(t-1)\right].$$

We then substitute this last equation into the equation describing the evolution of wealth

$$W_i(t+1) = \eta_i(t)W_i(t) + (1-b)\bar{I}(t-1) + \frac{b}{N-1}\sum_{j\neq i}\left[I_j(t-1) + W_j(t-1)\right] +$$
$$-(1-b)\bar{I}(t) - b\left[I_i(t) + W_i(t)\right].$$

It is possible to simplify the terms involving the minimum wage as

$$\bar{I}(t-1) - \bar{I}(t) = \nu^{t-1} - \nu^t = \nu^t\left(\frac{1}{\nu} - 1\right)$$

Since the minimum wage does not grow rapidly as $\nu \sim 1$, this term is small and can be neglected. Thus, we are left with

$$W_i(t+1) \approx \eta_i(t)W_i(t) + b\left\{\frac{1}{N-1}\sum_{j\neq i}\left[I_j(t-1) + W_j(t-1)\right] - (I_i(t) + W_i(t))\right\}.$$

Assuming that $W_i(t) \gg I_i(t); \forall i, t$ and defining $dW_i = W_i(t) - W_i(t-1)$, we obtain

$$W_i(t+1) \approx \eta_i W_i(t) + b\left\{\frac{1}{N-1}\sum_{j\neq i} W_j(t-1) - W_i(t)\right\} =$$
$$= \eta_i W_i(t) + b\left\{\frac{1}{N-1}\sum_{j\neq i}[W_j(t) - dW_j] - W_i(t)\sum_{j\neq i}\frac{1}{N-1}\right\} =$$
$$= \eta_i W_i(t) + b\left\{\frac{1}{N-1}\sum_{j\neq i}\left[W_j(t) - W_i(t)\right] - \frac{1}{N-1}\sum_{j\neq i} dW_j\right\}.$$

The average wealth variation is then $\langle dW\rangle = \frac{1}{N}\sum_{j=1}^{N} dW_j$. For a large number of agents, we can make the following approximation:

$$\frac{1}{N-1}\sum_{j\neq i}^{N} dW_i \approx \frac{1}{N}\sum_{j=1}^{N} dW_j = \langle dW\rangle,$$

With a very small exchange rate, the wealth variation of a single agent becomes negligible, this role is played here by the parameter b. For $b \to 0$, we have $dW_j \approx 0\ \forall j$.

In conclusion, we find that

$$W_i(t+1) \approx \eta_i(t)W_i(t) + \frac{b}{N-1}\sum_{j\neq i}\left[W_j(t) - W_i(t)\right]$$

Referring to the definition of $\eta_i(t)$ in the Bouchaud and Mézard model, which corresponds to a Gaussian noise with mean m and variance σ, and adopting a sufficiently symmetric log-normal $\tilde{\eta}_i(t)$, approximated as a normal distribution with mean m−1, we can write

$$W_i(t+1) \approx (\tilde{\eta}_i(t)+1)W_i(t) + \frac{b}{N-1}\sum_{j\neq i}\left[W_j(t) - W_i(t)\right].$$

Therefore, we obtain

$$W_i(t+1) \approx W_i(t) + \tilde{\eta}_i(t)W_i(t) + \frac{b}{N-1}\sum_{j\neq i}\left[W_j(t) - W_i(t)\right].$$

which is the result of integrating the evolutionary equation from the BM model in the case of a fully connected graph. The formal derivation requires setting the savings parameter b to a very small value and maintaining the order of magnitude of W significantly larger than I. This latter condition is achieved by eliminating the stochastic component and not allowing growth in the minimum wage.

Simulated wealth distributions

Fig 2 illustrates the wealth distributions obtained by simulating the BM model (middle row) and the INS model (bottom row) on different graphs (top row): the regular random graph (Fig 2a), characterized by a uniform distribution of connections; the Sun graph (Fig 2b), featuring a pronounced hierarchy between a dense core and a sparsely connected periphery; and the Onion graph (Fig 2c), defined by its progressive layering.

The wealth distributions exhibit straight tails on a log-log scale, consistent with Pareto-like behavior, although the overall distribution does not fully conform to a Pareto distribution. Comparing the Regular graph under the BM model (Fig 2d) and the INS model (Fig 2g), the BM model produces a longer tail, indicating a higher concentration of wealth among the richest nodes. In contrast, the steeper tail observed with the INS model suggests a more balanced wealth distribution.

For the Sun and Onion graphs, a two-lobed structure emerges in the distributions (Fig 2e 2h, 2i), which is more pronounced in the Sun graph due to its strong hierarchical organization. Each lobe retains Pareto-like straight tails, indicating that within both the central and peripheral groups of nodes, there is a partial preservation of wealth concentration patterns. However, in the case of the BM model simulated on the Onion graph (Fig 2f), the two-lobed structure is lost, demonstrating that the BM model fails to preserve the intrinsic stratification of the graph. Conversely, the INS model maintains the two-lobed structure in both the Sun (Fig 2h) and Onion (Fig 2i) graphs, indicating that the model replicates the hierarchical structure imposed by the network topology.

Simulated wealth persistence

The analysis of the heatmaps in Fig 3 illustrates the inter-decile mobility of agents over time, considering two models (BM and INS) and three network topologies (regular, Sun, and Onion). Each heatmap triplet represents the probability of transitioning between wealth deciles $P(q', t|q, t_0)$, highlighting how agents move between different deciles over time. The median deciles (green heatmaps) show a certain dynamism in wealth across all models and topologies, while the extreme deciles (blue heatmaps for bottom deciles and red heatmaps

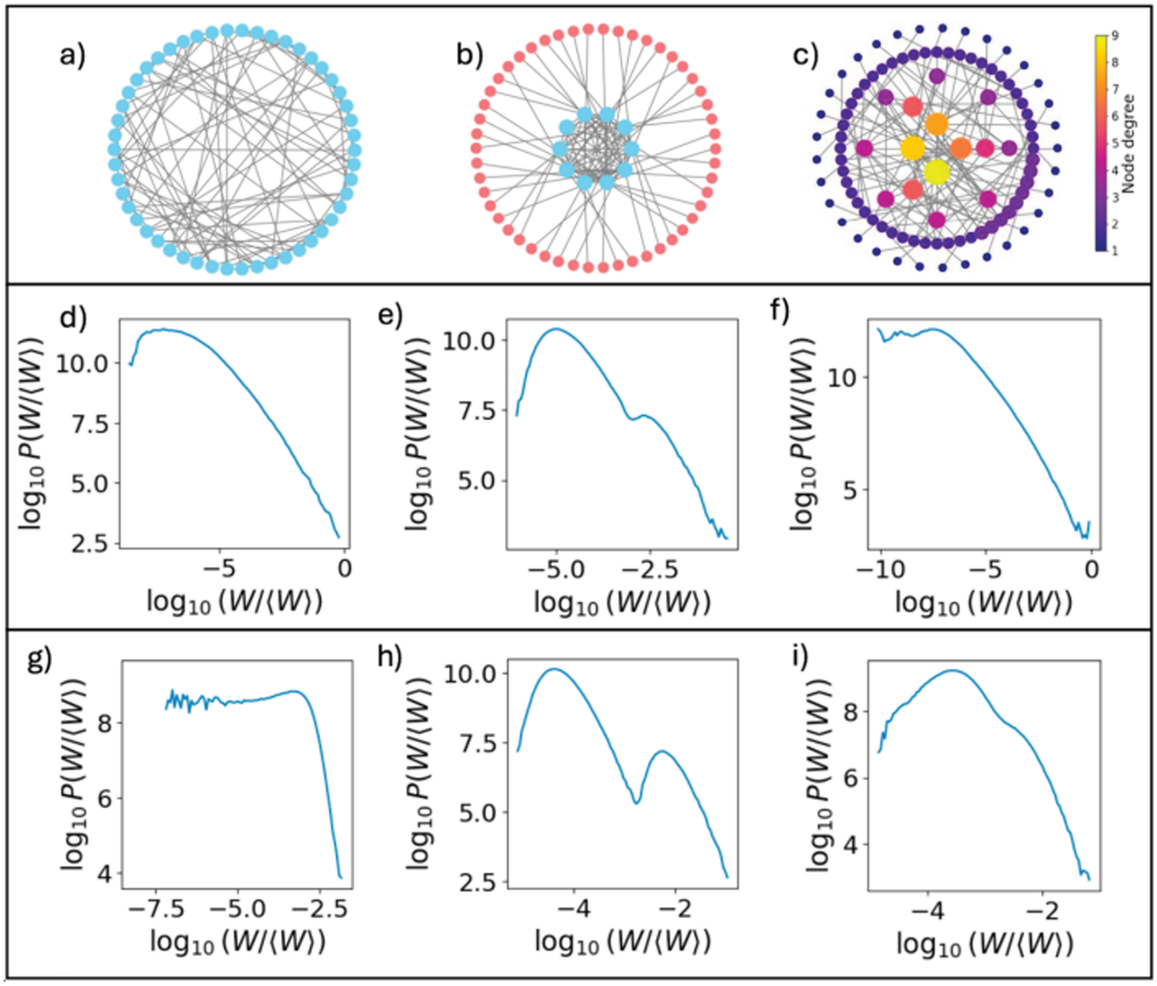

Fig 2. (a–c) Three types of networks: a regular random graph with connectivity 4 (a), a Sun graph with a fully connected core and a single link to the center (b), and an Onion graph with parameters $\gamma = 2.5$ **and** $a = 0.3$ **(c).** (**d–f**) display the wealth distribution under the BM model applied to each network, with both axes in logarithmic scale. (**g–i**) The wealth distribution under the INS model on the same networks, also in logarithmic scale.

https://doi.org/10.1371/journal.pcsy.0000050.g002

for top deciles) exhibit different behaviors, depending on the graph structure and the applied model.

For the BM model applied to the Regular graph (Fig 3a) and the Onion graph (Fig 3e), mobility in the extreme deciles is initially low. However, as time progresses, the information about the starting decile tends to dissipate. When this loss of information occurs, agents are more likely to move towards neighboring deciles, indicating a tendency towards mixing between deciles, but with a preference for movement within adjacent deciles rather than large jumps to distant ones. In contrast, the INS model applied to the random regular graph (Fig 3b) allows for significant mobility even in the extreme deciles, reflecting the non-hierarchical structure of the topology that facilitates greater interaction among agents. For the Sun graph, both the BM (Fig 3c) and INS (Fig 3d) models show no mobility for agents in the upper decile, while the lower decile demonstrates greater dynamism. This can be attributed to the privileged position of agents within the fully connected core of the graph, enabling them

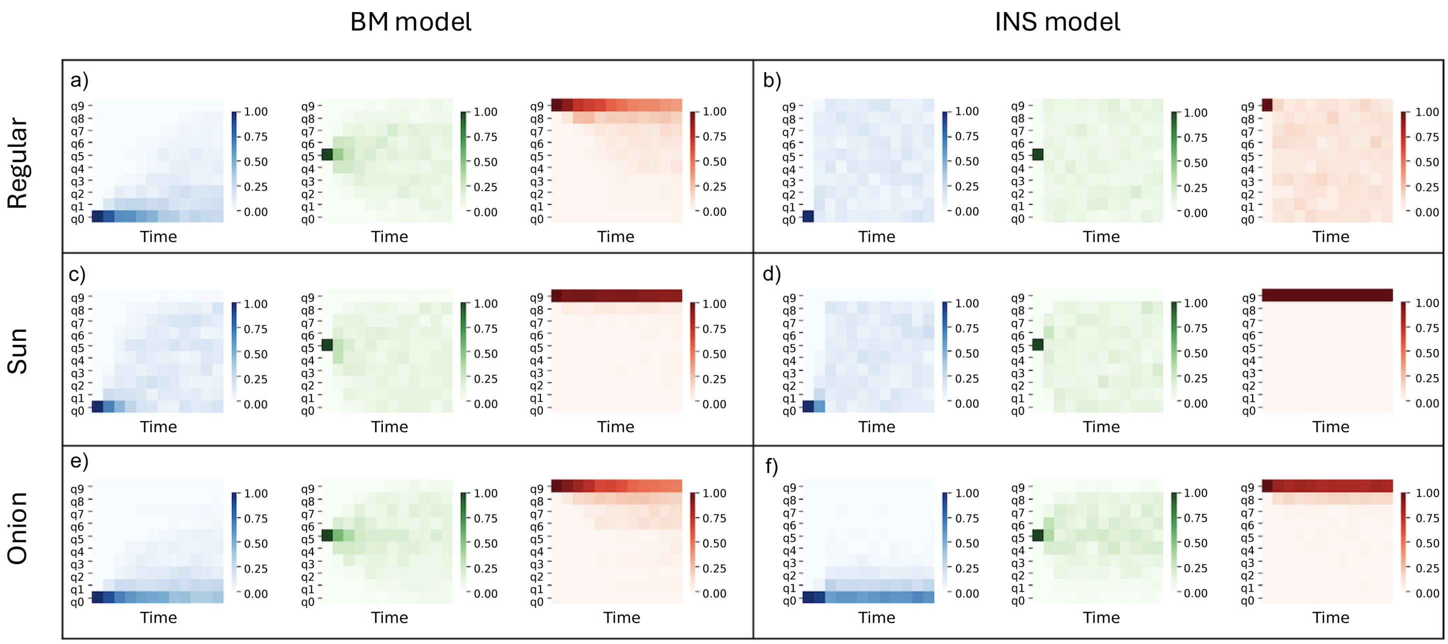

Fig 3. Transition probability matrices $P_Q(q', t|q, t_0)$ for wealth as a function of time for three values of q. Each row represents a different network structure: Regular graph (**a,b**), Sun graph (**c,d**), and an Onion graph (**e,f**). Each column corresponds to a model, with the BM model in the first column (a,c,e) and the INS model in the second (b,d,f). Each triplet of graphs shows the matrix of the Bottom (blue, $q = 0$), Middle (green, $q = 5$), and Top (red, $q = 9$) deciles over time.

https://doi.org/10.1371/journal.pcsy.0000050.g003

to maintain their wealth. In contrast, agents in the lower decile, being less stable, tend to move more frequently between deciles.

Finally, the INS model applied to the Onion graph (Fig 3f) also results in low mobility for both extreme deciles. However, an important distinction is that the information about the initial decile does not dissipate over time for either the lower or upper decile. This leads to greater persistence in the positions of agents, who largely remain in their initial deciles, with mobility being more contained compared to the other models.

The panels shown in Fig 4 focus on persistence, $P(q, t|q, t_0)$, which represents the probability that agents from a given decile q remain in the same decile over time. Each graph illustrates the persistence over time for three selected deciles: top (red), bottom (blue), and middle (green). The rows correspond to the models, with the first row representing the BM model and the second row the INS model, while the columns represent the network topologies (regular, Sun, and Onion). For the BM model applied to the regular graph (Fig 4a) and the Onion graph (Fig 4b), the persistence of the top decile consistently exceeds that of the bottom decile, while the middle decile shows the lowest persistence. However, over time, persistence for all deciles eventually approaches zero, reflecting a loss of memory regarding the initial decile.

When the INS model is simulated on a regular random graph (Fig 4d), persistence drops to zero at the very beginning for all deciles, indicating a large mobility across all deciles. Both the BM model (Fig 4b) and the INS model (Fig 4e) simulated on the Sun graph maintain a consistently high persistence for the top decile, while the persistence of the bottom and middle deciles rapidly declines to zero. This behavior highlights the stabilizing effect of the Sun graph's core for agents in the top decile. The INS model simulated on the Onion topology (Fig 4f) presents an interesting outcome: the persistence of the extreme deciles stabilizes at a constant, non-zero value. Notably, the top decile is more persistent than the bottom decile,

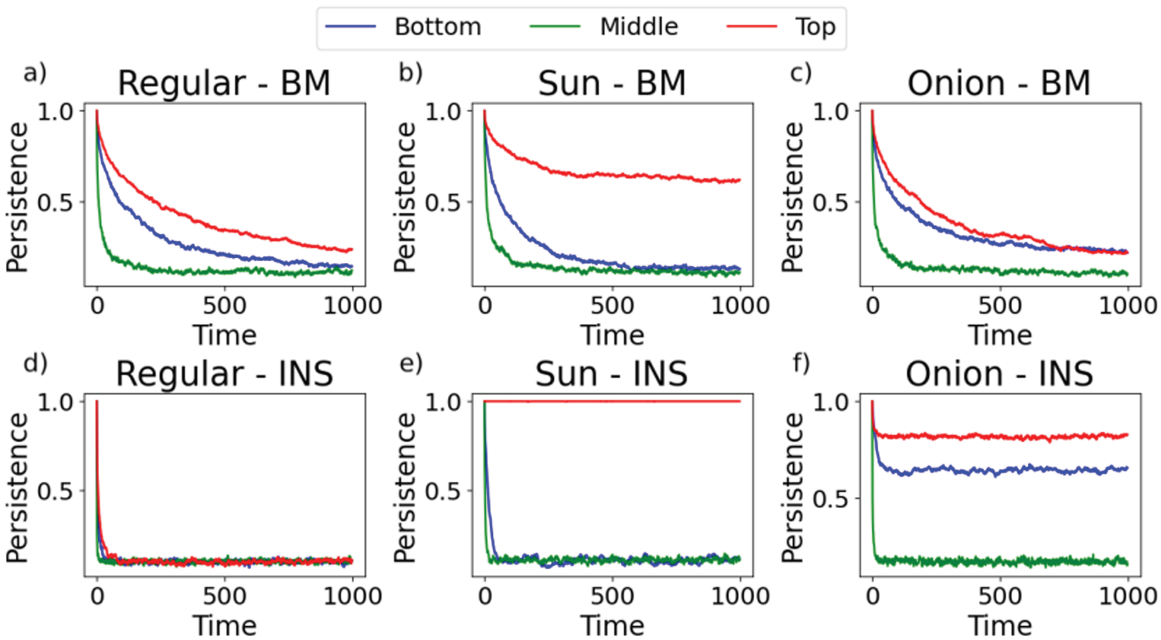

Fig 4. Persistence over time for three network topologies (Regular, Sun, and Onion) under two models (BM and INS). (**a–c**) The persistence trends for the BM model, while (**d–f**) display results for the INS model. Persistence curves are divided by deciles: bottom (blue), middle (green), and top (red).

https://doi.org/10.1371/journal.pcsy.0000050.g004

while the middle decile, as expected due to its high mobility, exhibits zero persistence. This result emphasizes the distinct dynamics induced by the Onion topology in the INS model.

Relations between income and wealth

All the graphs in Fig 5 depict the different sources of income as fractions of the total income, plotted as a function of the wealth share bin. Each graph is derived from simulations of the evolution of wealth and income variables for a chosen value of σ, which represents the standard deviation of the η term regulating investments. The first row (Fig 5a) shows the INS model simulated on the random regular graph as σ varies, while the second (Fig 5b) and the third row (Fig 5c) presents the same model simulated respectively on the Sun and the Onion graph. The INS model exhibits similar behavior across the different topologies. Specifically, when the investment strength is low, income inequality becomes evident. For lower income classes, the primary source of income are wages, whereas for higher income classes, investments dominate. As the importance of investments increases, driven by higher σ, the distinctions between wealth classes gradually become less pronounced.

In all the graphs related to the Sun topology, an irregularity is noticeable at a specific wealth share value. A closer examination of the wealth probability distribution for each simulation reveals that this value corresponds to the point where the two lobes of the distribution connect. In essence, this irregularity arises from the discontinuity in the wealth distribution, which, in turn, is caused by the highly fragmented structure of the underlying network.

In Fig 6, the relationship between income share and wealth share is shown, varying the investment power parameter in the same manner as in Fig 5. It can be observed that when income sources are unequal — specifically, when the investment power parameter is at its lowest value — the income growth exhibits a crossover, with higher wealth levels displaying

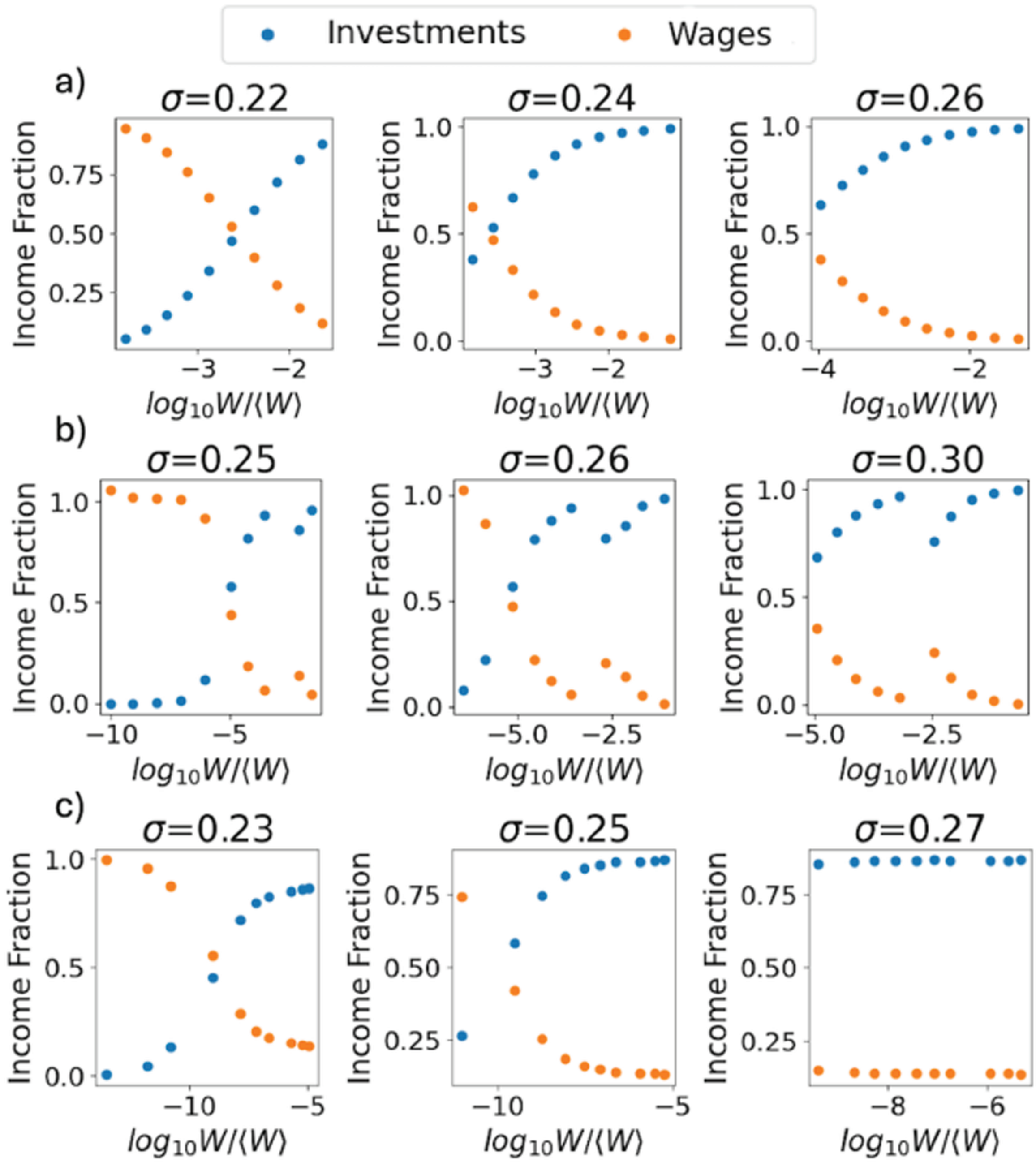

Fig 5. Income fraction plots derived from the INS model applied to the Regular (a), Sun (b) and Onion (c) network structures. In each case, the income fraction is divided into labor income (orange) and investment income (blue). As the parameter σ, representing investment strength, increases, the contribution of the investment component becomes more prominent compared to labor income.

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a linear trend. The linear trend at high wealth is expected because the INS model reduces to the BM model in this regime and the BM model displays a linear relation between income and wealth [19]. The crossover is particularly pronounced in more hierarchical networks; it is clearly visible in the Sun topology (Fig 6b), less so in the Onion topology (Fig 6c), and even smoother in the regular random topology (Fig 6a). Conversely, when the primary income

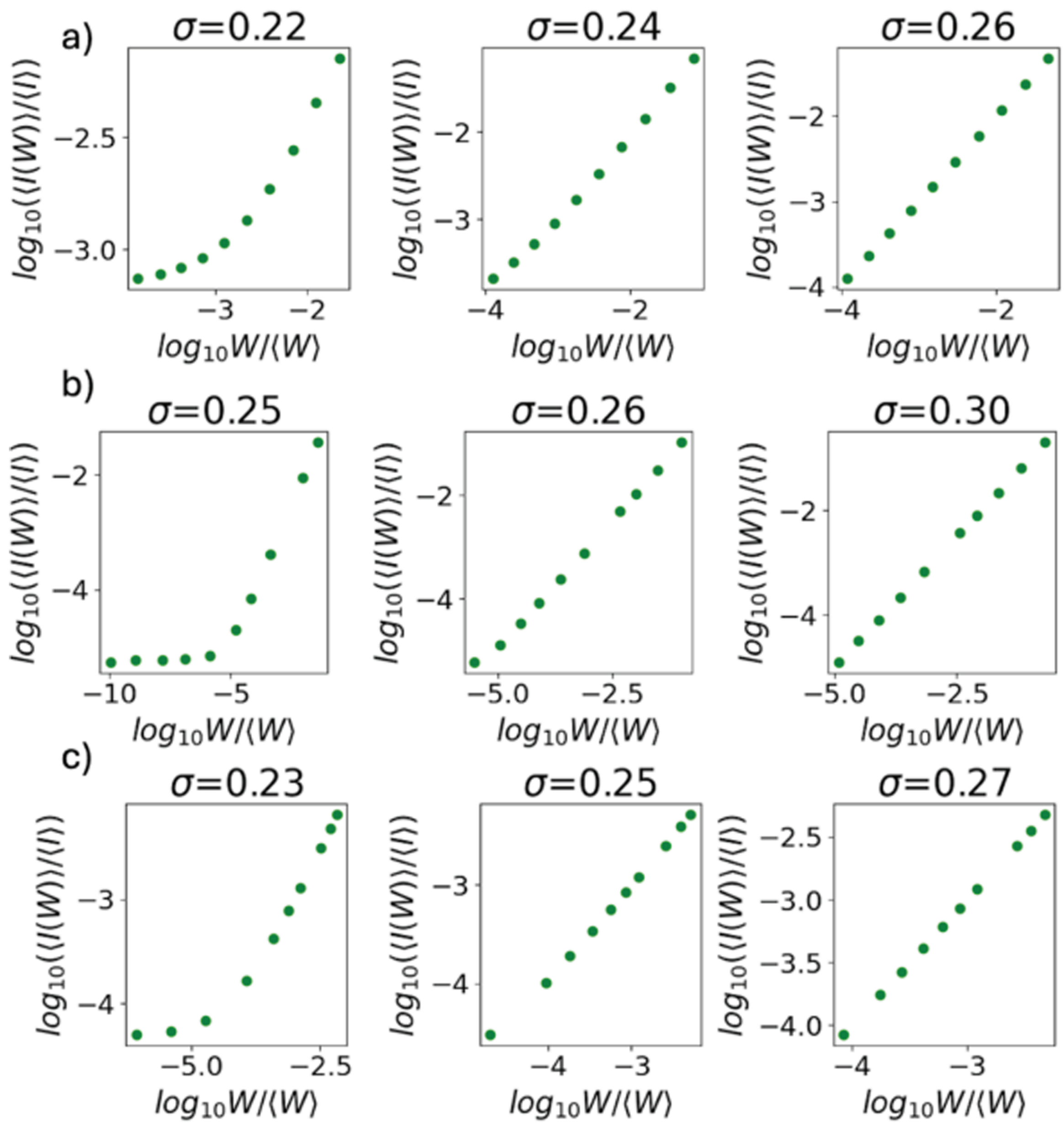

Fig 6. Relationship between income share and wealth share for the INS model applied to different network topologies as a function of the parameter σ. Each row corresponds to a specific topology: Regular (**a**), Sun (**b**), and Onion (**c**). In all cases, the total income increases with the wealth share.

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source for all agents shifts to wealth-derived investments (higher σ values), the trend becomes uniformly linear across all cases. Overall, the analysis confirms a general tendency: individuals with higher wealth are consistently those with the highest income.

Discussion

The past decades witnessed an increase in global wealth and income inequality in industrialized countries. Inequality is often associated with lack of social mobility, so that individuals remain trapped in their original wealth and income class. In this paper, we have explored the role of network interactions in describing wealth inequality, social mobility and the relation with income generation mechanism in the framework of simple ABMs. The goal was to find a minimal model that could account for a set of stylized facts related to wealth inequality which we have summarized analyzing survey data relative to Italy.

ABMs have been widely used in the economics and the physics literature to illustrate simple basic mechanisms leading to the formation of inequality and the lack of mobility. In a previous paper [19], we have investigated the relation between wealth inequality and mobility using the simple ABM of wealth condensation introduced by Bouchaud and Mezard [22] and compared the results with survey data for Italy and the USA. The model is based on two main ingredients: (i) the wealth of each agent changes randomly but in proportion to its current wealth and (ii) wealth is exchanged among agents in proportion to their respective wealth. The BM model is similar to other stochastic wealth exchange models, such as the model proposed by Angle in 1986 [34] and other models studied in the econophysics literature [35,36]. These models are simple and intuitive, but can be criticized from a classical economics perspective [37]. The BM model provides a good description of the Pareto distribution of wealth and displays a stretched-exponential relaxation in analogy with panel survey data [19]. The model differ from reality mainly because it would predict that, due to a relatively short relaxation time, the initial wealth would not matter. This is in contrast with survey data suggesting instead persistent lack of mobility at the top and, to a less extent, also at the bottom of the wealth distribution. Furthermore, survey data show that for low wealth agents income is not a linear function of wealth (Fig 1b), as assumed in the BM model. As a result of this, the model was shown to describe well the wealth dynamics of high-wealth individuals, but overestimates the mobility within the general population.

In this paper, we have proposed a new model that includes features of the BM model into an ABM introduced in the economics literature by Nirei and Souma [21]. The NS model considers that wealth variations of the agents are due to the combination of income from labor, random returns on investments and consumption. The main limitation of the model from a complex systems perspective is the lack of interactions among agents. Taking inspiration from the BM model, we include such an interaction by relating the income of each agent to the consumption of the others. We show that this interacting version of the NS model reduces to the BM model in the limit of large wealth, but departs from the BM model for low-wealth agent whose income is not dependent on wealth. We also study the structure of the wealth exchange network influences the wealth distribution and time-dependent wealth correlations. Along with a regular random graph where all the agents interact with a constant number of agents, we consider also hierarchically organized networks such as the Sun and Onion networks. Simulations of the BM and INS model on hierarchical networks display result that are closer with empirical data than those obtained in the regular random graph.

In conclusions, our results highlight the relevance of the hierarchical organization of interactions among agents in explaining the persistence of wealth inequality observed in survey data. Agents with limited exchanges are confined at the bottom of the wealth distribution, while well connected agents are more likely to remain at the top.

Acknowledgments

We thank D. Garlaschelli for useful discussions.

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