

RESEARCH ARTICLE

Describing Medellín as a complex tropical high Andean urban system

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Abstract

We assess the structural complexity underlying the spatial configuration of urban state variables in Medellín. Leveraging the concept of the urban fractal, we describe the general organizational mode of the urban plane in terms of movement and mobility, while employing fractal dimension to quantify the dynamic affinities among different urban state variables. Additionally, we introduce a spatial entropy metric to evaluate the deviation of state variables from entropic equilibrium. To illustrate the contrasting distributions of urban variables across the territory, we utilize diffusive cartograms at the district scale. Our findings reveal the tension between Medellín's organic and planned structures, shaped by physiographic, environmental, social, functional, and economic factors.

Author summary

Medellín is a Colombian city undergoing a transition from its violent past to becoming a modern metropolitan hub. However, this transition has not been adequately planned, leading to a patchwork of urban development approaches that reflect the priorities of successive local governments and consolidate the city's territorial memory. In this study, we examine Medellín's urban state by analyzing its geometric organization and the spatial distribution of urban functions across its Administrative Units. Our findings highlight significant imbalances: while the housing system is gradually overcoming the city's orographic challenges, networks supporting habitability are lagging, and sustainable mobility networks are even further behind. This uneven development results in segregation and localized deficiencies in essential urban functions. Using cartograms, we visualize these disparities, offering a powerful tool to map the city's inherent inequalities.

Data availability statement: Data source provided from public dataset at: <https://datacatalog.worldbank.org/search/dataset/0038272/World-Bank-Official-Boundaries>; <https://www.usgs.gov/3d-elevation-program>. Sentinel 2 satellite images from Copernicus: <https://dataspace.copernicus.eu/>; <https://www.colombiaenmapas.gov.co/>; <https://www.medellin.gov.co/geomedellin/datosAbiertos/>.

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These insights provide a valuable basis for central planning decisions, helping determine where urban functions need to be reinforced.

Introduction

The city is the modern setting where social transformations are recorded, serving as a scenario of continuous change and adaptation built on the interaction and integration of individuals and institutions constrained by material, environmental, and cultural conditions. The city is a network of networks [1]. Its formation is an emergent result of both implicit and explicit agreements arising from the tension between individual and collective well-being [2].

By inhabiting the city, individuals become sampling instruments, able to recognize specific places where a particular good or service can be acquired or to identify zones that are unsafe to wander. Locals can guide other residents based on a mental map each person stores, resembling moves in a chess match between experienced players. These recordings form the empirical knowledge of citizens, shaping how individuals spontaneously interact with each other and the environment, summarizing the microscopic relationships that configure the physical space and the city's collective knowledge. We shall henceforth refer to this as the *organic city* or the *in-fact city* [3]. The organic city grows in a fractal-like manner, adapting to its environment as a living organism would within an ecosystem. In nature, scaling provides advantages that help optimize energy consumption [4]. However, there is a significant difference between organisms and urban systems: organisms must operate within the limits of available environmental resources, whereas urban systems respond to scarcity by importing resources, thereby establishing a continuous inflow of external supplies.

In agreement with [2], the organic city embeds three levels of information: physical information, related to encoding and decoding environmental data; semantic information, concerning the meaningfulness of environmental data; and enacted information, which pertains to day-life actions that shape interactions based on this environmental data. These levels of information materialize in the geometric organization of streets, public spaces, green spaces, urban artifacts, natural and artificial networks, and, more broadly, in the state variables that define the city as a physical system.

In contrast, the planned city exists within land-use plans and follows central (often governmental) control that organizes the territory into smaller administrative units. The organic modern city rarely aligns with the planned city [5]. This dilemma is especially evident in developing cities, where governments respond to citizens' needs in a contingent manner. Furthermore, rapid urban expansion and increasing social inequality characterize these cities. Poverty, violence, informality, and unequal access to education and culture are key elements of urban dynamics and focal points in the Sustainable Development Goals [6,7].

Medellín is a city evolving under the constant tension between the planned and organic city. While the municipal administration, with its apparatus for social organization, creates an image of regularity, the microscopic interactions between individuals, institutions, and artifacts lead to a spontaneous amplification of self-organizing forms. These forms emerge in a space shaped by complex and varied landforms. The competition between the organic city and the planned city is the main mechanism that stimulates the permanent processes of adaptation and change, resulting in the continuous reconfiguration of the territory [8–10].

This work characterizes Medellín as a complex system from an interdisciplinary perspective, integrating urban systems methodologies and tools from physics within a comprehensive theoretical framework (as suggested, for instance, in [11,12]). We analyze

the qualitative and quantitative fractal structure of the urban state to classify and differentiate the geometric properties that represent the city's forms and functions, determining how far the system's state is from entropic equilibrium. Additionally, we examine the state's evolution based on the transversal concept of urban state variables, visualizing these variables through fluid cartograms to highlight the inherent differences in their spatial distribution across administrative scales from neighborhoods to the city.

Medellín: The urban entanglement

Medellín is the capital of the Department of Antioquia and the second most important city in Colombia. The city covers an urban area of 110 km² and has a population of 2,653,729, accounting for 44.5% of the Department of Antioquia [13]. Located in the Aburrá Valley, a tropical depression between the central branches of the Colombian Andes (Fig 1), Medellín is traversed by the Aburrá River, which flows from south to north. This river serves as the natural axis that shapes the macro-scale of the city and connects it to the surrounding territories [14,15]. The landscape is steep and irregular, with pronounced slopes in whose hillsides stand a heterogeneous housing system, while the flatter areas concentrate the urban functions. The city is divided into administrative units (AUs) ranked by social stratum, from 1 (lower,

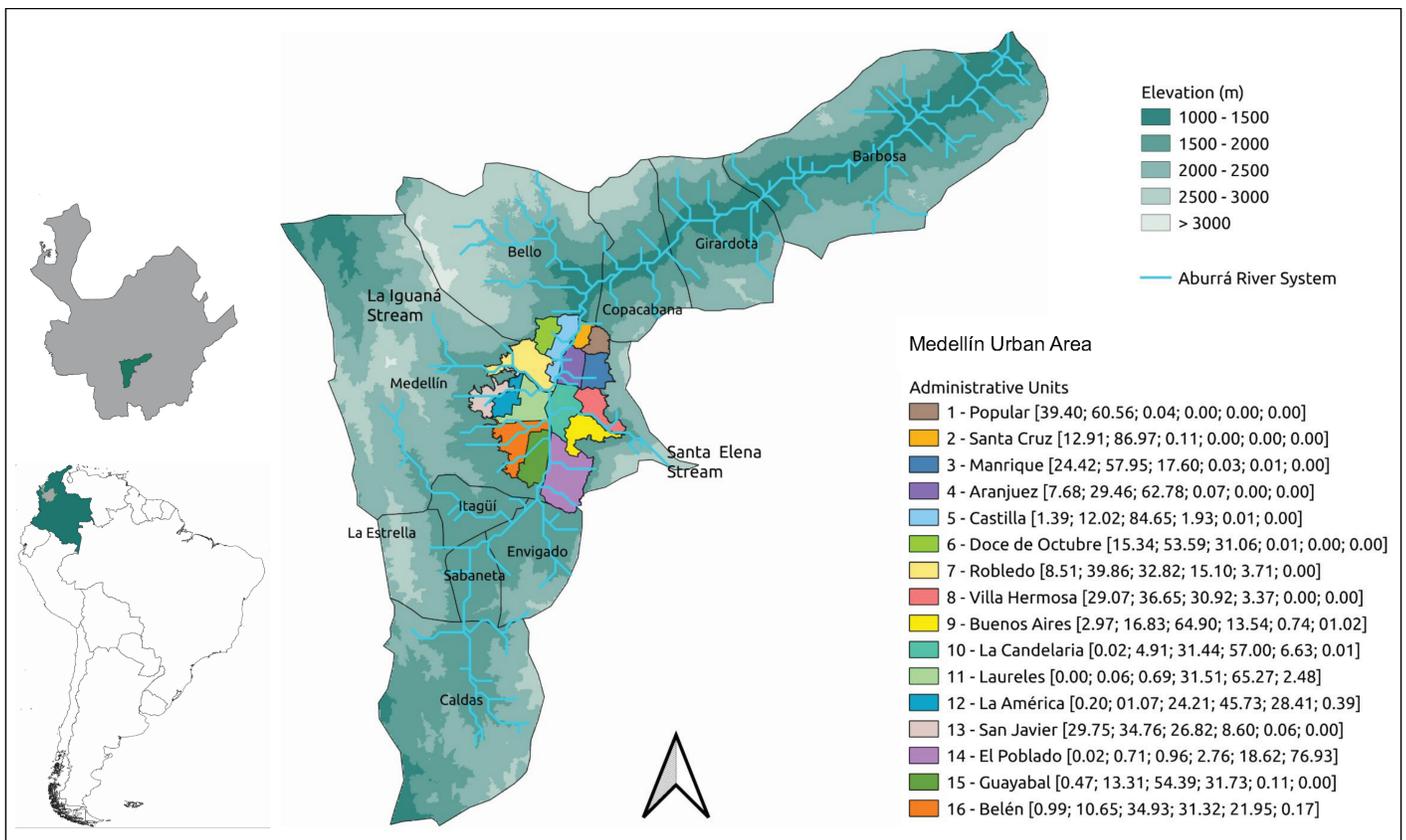


Fig 1. Study area. Aburrá Valley and administrative units of the urban area of Medellín. Percentage of housing social stratum composition in each administrative unit in square brackets, from lower (1) to higher (6), estimated by the MEDData report [18] (<https://medata.gov.co/dataset/3ba48b35-29ff-4798-bd20-d4b41dc10f85>). Elevation from USGS 1 arc-second resolution [19] (<https://www.usgs.gov/3d-elevation-program>). Official boundaries from World Bank (<https://datacatalog.worldbank.org/>) and Instituto Geográfico Agustín Codazzi (<https://www.colombiamapas.gov.co/>).

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for poorer people) to 6 (higher, for wealthier people), which are used to assign functions and uses to the land. This ranking applies to the normative, geometric, and entropic levels [16]. The south-north configuration scheme is also reflected in the social composition or strata. The northernmost AUs are characterized by a high percentage of the lower strata while the balance is reversed southward. In the north, the hills show a large expansion of informal settlements inhabited by people facing transversal basic needs. In contrast, the hills in the south are the luxury and well-being dream, manifesting spatial structural segregation as it is observed in other cities [17].

From an urban planning perspective, different generations of urban planners have sought to translate the milestones of their discipline into the urban space of Medellín [20]. However, the biophysical aspects (such as the water network and environmental corridors) were not adequately represented or valued in the rationalized urban space, despite multiple attempts at formal inclusion in urban planning [21,22]. As a result, the material realization of the city remained far from the technical ideal. The built-up city has a systemic relationship with the valley's topographic constraints and the water network, which decisively influence the evolution and consolidation of the settlements along the valley. The Santa Elena stream is considered the historical foundational watercourse, while the La Iguaná stream works like the new foundational channel of the expanding contemporary city [23]. Currently, the Aburrá River serves as a focal point for urban development, with multiple specialized functions and services situated along both banks. This structural role of the river was envisioned in 1950 as part of the Pilot Plan for Medellín [24,25].

The city has long benefited from the ecological and cultural diversity that the Aburrá River represents. With the growth, development, and consolidation of the city, the integrating function of the river has mutated towards the provision of services, supporting the internationalization of the economy, and contributing to urban marketing. Various nuances of urban development coexist in the city as remnants of what is adopted in each administrative program: the sustainable city, the tactical urbanism, the city branding, the hostile urbanism, and others. The shape of the city is transformed accordingly. The urban landscape is interrupted by high-rise buildings, discontinued green spaces, and expressways, among other features, fostering heterogeneous, poorly integrated, and segregated social dynamics that contribute to the city's urban entanglement.

As it stands, Medellín strives to position itself as a metropolitan city. In the 21st century, the city has gained international relevance through cooperation efforts and the promotion of infrastructures for information services, communication flows, and tourism, aiming to overcome the stigma of its violent past (in the last decade, Medellín has received multiple awards, including recognition as an innovative and smart city). These infrastructures have become increasingly significant, operating on a scale where speed, especially as an attribute of mobility, clashes with the rhythms of urban daily life. The city's shape appears as an assembly of textures that combine technological remains from other times [26]. However, in everyday life, the inhabitants organically transform the urban hierarchy, and as a result, the city is also consolidated from the multiple ways of living in it [2,6,27]. The rhythms, the wanderings, the trajectories, and the diversity of practices and actors that shape the city are not fully represented by urban planning perspectives [28]. From a morphogenetic standpoint, the coexistence of connectivity and the movement of various elements (people, information, energy) gives rise to different mechanisms of territorial memory, that is, a process of construction-production-representation of the urban territory, or according to [29], experimental, spatial, experiential, and projected memory of the habitat.

The land use plan of Medellín has embraced a modernist vision for the city, conceptualizing it as a city of functions (work, study, leisure, and rest). Over time, land use has evolved

to accommodate the goal of transforming the city into a regional and national hub for international tourism. This shift provides an opportunity to explore the complexity of the city's dynamics through fractality and urban fragmentation concepts, particularly as they manifest along the Aburrá River [29,30]. How is the city produced? How is the city (self)-organized? What is the shape of the built city? The discussion about the form is an approach to the urban habitat system of Medellín as a material substrate where life unfolds. Movement and mobility express a clear connection between geometry and space. The processes and functions establish urban patterns of networks, flows, infrastructures, routes, and itineraries that empirically evidence the change and growth of the city.

Measuring the city's underlying complexity

Complex systems exhibit three basic properties: i) emergent behavior from simple relationships between their components, ii) production and transfer of information at different spatio-temporal scales, and iii) adaptation and co-evolution with the environment [31–39]. Cities are open and complex systems where several interactions and processes converge, and their dynamics cannot be described in reductionist terms.

Urban systems are highly heterogeneous, their components cooperate, compete, and interact due to the exchange of matter, energy, and information. As live organisms, the cities remain out of thermodynamic equilibrium. The input energy flow for a biological system comes from the sun, a reduction potential, or a metabolic sugar. Similarly, cities are great energy sinks, they absorb energy in the form of goods, food, fossil fuels, natural resources, etc [2,40]. In practice, we can record the city's morphogenesis evolution through a time series of dynamic variables or their spatial mapping [41]. For example, the change in land use and vocation could determine the structure of urban settlements, on the other hand, the development of infrastructure affects the mobility within the city, and the mobility network conditions the social network since it promotes or impedes the spatio-temporal interactions between individuals [28].

Systems far from thermodynamic equilibrium exhibit self-organization emergent properties resulting from energy inflows [42,43]. Self-organization is a spontaneous phenomenon that does not require an external agent and manifests itself in scale-free space-time structures [44]. In this context, fractality is a valuable metric to quantify complexity [36].

Fractals are geometric objects characterized by self-similarity, wherein at smaller scales of observation, iterative copies of the initial object are obtained. As a consequence, the topological dimension of these objects is a non-integer number. The self-organization that manifests in urban morphology obeys a fractal-like structure with self-similarity at different scales of observation. Fractal geometry offers a more suitable description of urban systems (shape, growth, size, densities, etc.) since it avoids the simplifications of Euclidean methods [3,39,45,46]. A natural fractal approximates the mathematical fractal since fractal properties in natural systems are limited to certain scales and restricted by the multiplicity of interactions in open systems. The richness of fractal objects lies in their continuum of dimensions from points to lines, planes, and volumes (see the figure 2.10 in [3]). Fractal geometry is a way of understanding the city and its relationship with the habitat and with the inhabiting.

To determine the complexity of Medellín, we studied, among several methodologies, its fractal structure qualitatively and quantitatively. We measure the fractal dimension of a set of variables that synthesize processes that are fundamental in urban morphology and therefore determine what can be called the urban state.

Qualitative fractality

According to [47], understanding the city involves a problem of organized complexity, which is expressed through visual order. In this context, visual order refers to the mosaic-like composition of urban landscape functions, where predominant forms and functions stand out, and central artifacts play a key role in shaping this composition.

The present analysis begins by visualizing urban order in Medellín through its urban functions, using a methodological approach that includes extensive fieldwork and tracking the city's shape transformation over time. The emerging geometric configuration results from the tension between governmental planning guidelines and the everyday spatial practices of its inhabitants. The amorphous and fluid forms observed in daily life contrast with the Euclidean geometry of smooth, orderly designs that have long dominated architectural and land-use planning paradigms. The rough, blurred, diffuse, and organic aspects of urban life are now being examined through the geometries of irregularity, revealing a deeper underlying order [3].

We introduce the notion of the urban fractal, which describes functional similarities in territorial organization across different urban scales. This qualitative fractality emerges from the natural configuration shaped by the geo-forms and bio-forms of the ecological environment, as well as from human actions influenced by urban artifacts and the imagined territory. To illustrate this concept, we examine two key spatial scales. The first considers the entire city as a planimetric system with the Aburrá River axis serving as the organizing element of the urban fabric (Fig 2). The origin of this Cartesian plane is defined at the intersection of the Aburrá River and San Juan Avenue, recognized as the modern structural center of the

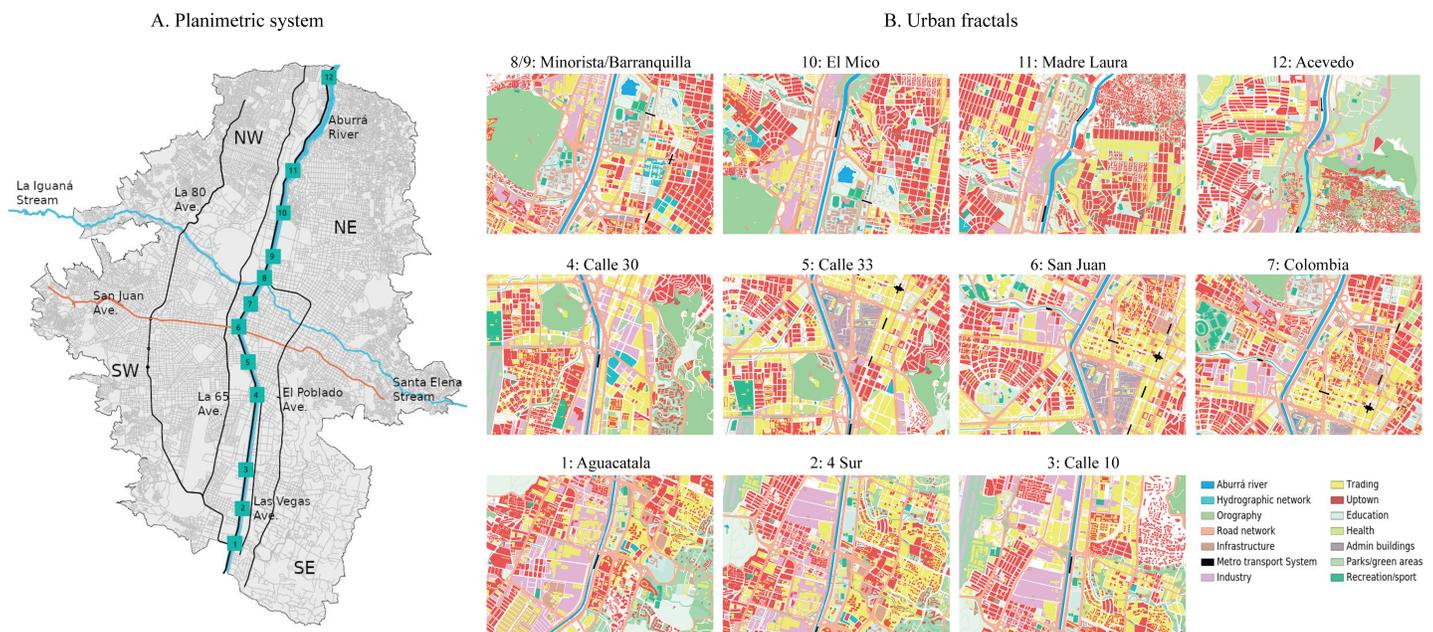


Fig 2. Qualitative fractality in Medellín. A) Planimetric system of Medellín. The origin of the Cartesian plane is defined as the intersection of the Aburrá River with the San Juan Avenue. From this origin, the four quadrants (and their visual order) are defined: SE (FF), SW (RFI), NE (HD), and NW (BOP). Target bridge intersections are highlighted in numbered squares. B) Emerging urban fractals at the intersections of the Aburrá River. The different visual orders in the urban landscape are determined by fieldwork and are portrayed in a color code to facilitate their identification by the composition of urban functions. Basemap data from the OpenStreetMap project [48] (<https://planet.openstreetmap.org/>) and GeoMedellín Open Data [49] (<https://www.medellin.gov.co/geomedellin/datosAbiertos/>).

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city [24]. The second scale focuses on the surroundings of key bridges, which work as central artifacts in the fractal-like local organization of urban functions.

Describing the visual order of the general urban plane in terms of its quadrants provides a useful analytical framework (Fig 2A). In the southeastern quadrant (SE), the city is transitioning from an industrial hub to a service-oriented economy, marked by large-scale housing projects, the expansion of financial capital, and speculative land rents. This area primarily functions as a bedroom community, enclosed by roads with varying speed levels and densities that compete with the steep slopes of a dispersed and fragmented urban landscape. The city is expanding vertically and spatially, increasing in height and complexity, constrained by the orography and the corporate business model. This city section defines its visual order through *financial flows* (FF).

In the southwestern (SW) quadrant, the urban landscape transitions from an industrial-based to a service-oriented city. The landscape reflects a mix of traditional residential areas, middle-class neighborhoods, and vast vacant lots once occupied by industrial facilities, now awaiting revaluation and real estate speculation. Here, the *remnant former industry* (RFI) defines the visual order.

In the northeastern (NE) quadrant, remnants of the historical city are evident, reflected in a hybrid street layout that merges planned structures with organic settlement patterns, adapting to the steep terrain. The urban landscape features traditional residential areas, working-class districts, informal settlements, and slums. Despite ongoing transformations, the imprint of land use planning endures through social investment projects. This quadrant is home to residents who live in the north but work in the south, experiencing the city without reliance on car ownership, luxury amenities, or air conditioning. Here, the visual order is defined by a *hybrid densification* (HD).

Finally, in the northwestern (NW) quadrant, planned urbanism and self-built settlements coexist, creating a hybrid urban landscape. On the outskirts, communities have collaboratively constructed their homes, giving rise to popular neighborhoods. Moving toward the central west, the city takes on a more contemporary character, expanding along the steep slopes that define the western valley's geomorphology. This area hosts small industries—such as pottery and metalworking, some now in decline—alongside new land uses, emerging services, retail businesses, urban renewal projects, and the implementation of integrated transport systems, contributing to rising property prices. Here, the visual order is defined by a *balance between organic and planned* (BOP) urbanism.

Following the planimetric methodology of quadrants, we focused on the intersection of bridges with the Aburrá River (Fig 2B). The bridges, originally conceived as connectors for urban mobility, also serve as metaphors for connection itself, where symbolic and imagined associations reshape the experience of inhabiting the city. This duality reveals two sides of the same coin: what appears static and immobile integrates dynamic forms associated with movement. What exactly do these bridges connect? What moves across them? These questions prompt us to think beyond the bridge as a mere artifact and its obvious engineering function of linking one place to another.

At this local scale, the scenes in Fig 2B exhibit a visual order similar to that of the general urban scale, meaning that these city fragments iteratively replicate the global urban pattern. This process forms a qualitative urban fractal, encapsulating multiple repeated actions contributing to territorial memory. This fractality implies the city resists homogenization in these sampling scales. Each type of visual order is associated with a distinctive combination of key urban functions, represented using a color code to facilitate the identification of their specific configurations around target bridges.

Regarding urban functions, the FF visual order exhibits a dense trading function with a reticulate uptown and organic high-rise construction that adapts to the ascending orography. The road network varies in density and speed, adjusting to the residential system. Here, we find scattered parks and green areas, along with an overlap of health and education functions with trading zones, reflecting the influence of business interests on the urban landscape. The RFI visual order comprises the industry and trading functions and a pronounced reticular uptown exerting pressure on them. In the HD category, the residential visual order combines planned development and organic growth, shaped by the hydrographic network and orography. Small-scale trade adapts to the inhabitants' needs, while social investment—achieved through community action—is materialized in small educational centers, sports courts, informal parks, green spaces, and medical centers. In the BOP visual order, the uptown is characterized by reticular urban renewal projects accompanied by a high-density road network and organic remnants shaped by the hydrographic network and orography. This area also includes small industrial zones, an increasing presence of small businesses, and territorial planning reflected in social infrastructure such as educational, health, recreational facilities, and parks. As in natural fractals, the self-similarity is not perfectly fulfilled, however, the different visual orders are determined by the composition of urban functions. Table 1 provides the geographic reference where the urban functions associated with the distinct visual orders of the general urban plan are identified at each bridge scale. For further explanations of how the visual order is defined in bridge-scale, see S1 Fig.

Measuring the self-organization level via the fractal dimension

Describing the city as a physical system implies adopting a mechanistic language for urban dynamics [8–10,50]. The city evolves by changing its state variables [41]. These variables link the functionalities of the city to its environment, following fractal-like spatial patterns [46, 51–56], evidencing organically self-organization built from the bottom up [3]. This fractality is inherent in the architectural spaces, streets, public spaces, infrastructures, and green areas configurations [57]. The fractal dimension talks about how the variable is geometrically dimensioned and how the urban function fills the urban fabric. A value of fractal dimensions near 2 indicates that the variable almost covers the plain, while values near 1 indicate that the variable barely exceeds the dimension of the line.

We define a set of 15 variables representing the urban state in Medellín: Terrain curvature, Leaf Area Index (LAI), ecological connectivity network, hydrographic network, trees, residential system, urban facilities, public services networks, public space, pedestrian network,

Table 1. The planimetric visual order iterates at bridge-scale. The geographic reference where the planimetric visual order is observed as a distinctive combination of key urban functions within each bridge scene.

Bridge	FF	RFI	HD	BOP
1: Aguacatala	NE	W	NW	SE
2: 4 Sur	E	NW	SE	SW
3: Calle 10	SE	W	NE	NW
4: Calle 30	SE	SW	NW	SW
5: Calle 33	NE	SE	SW	NW
6: San Juan	NE	NW	SE	SW
7: Colombia	SE	SW	NE	NW
8/9: Minorista / Barranquilla	SE	NW	SW	NE
10: El Mico	SW	NW	SE	NE
11: Madre Laura	SE	SW	NW	NE
12: Acevedo	NE	SW	NW	SE

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urban structural axes, road network, public transport routes, Metro Transport System, and bike paths. Each variable brings dynamic information in the space scale of the city and encompasses a diversity of categories such as habitat, geography, mobility, infrastructure, land use, and nature. [S1 Table](#) presents a summary of data description and sources. [Fig 3](#) displays the spatial distribution of each variable. The fractal dimension was determined using

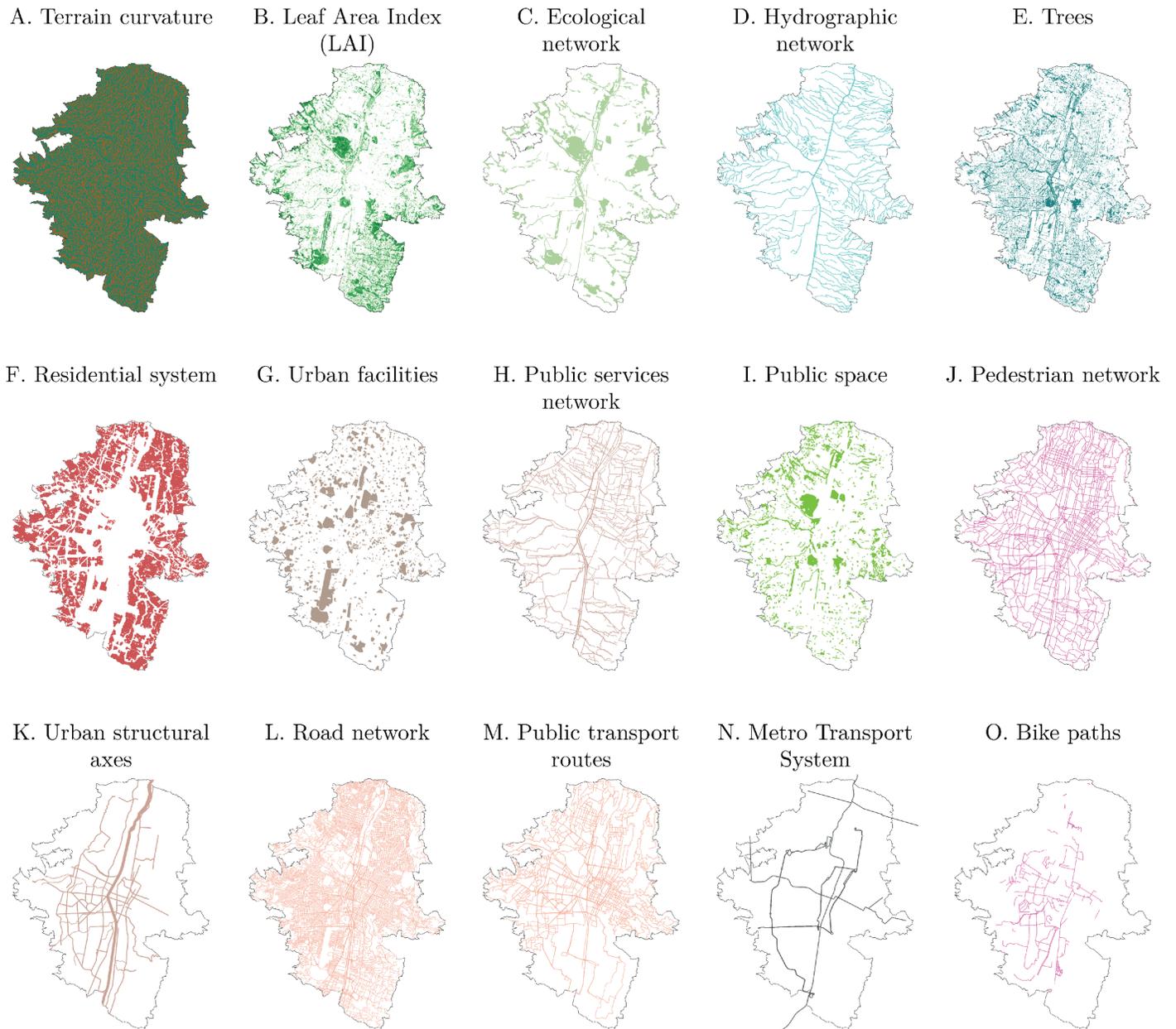


Fig 3. State variables of the Medellín urban system. A) Terrain curvature (Calculated from elevation map provided by USGS, <https://www.usgs.gov/3d-elevation-program>). B) Leaf Area Index (LAI, calculated from Sentinel 2 satellite images, <https://dataspace.copernicus.eu/>, following [61]). C) Ecological connectivity network. D) Hydrographic network. E) Trees. F) Residential system. G) Urban facilities. H) Public services networks. I) Public space. J) Pedestrian network. K) Urban structural axes. L) Road network. M) Public transport routes. N) Metro Transport System. O) Bike paths. Basemap C to D, from GeoMedellin Open Data [49] (<https://www.medellin.gov.co/geomedellin/datosAbiertos/>).

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the classical box-counting method over the binarized (info/no-info) images [58–60]. All data were converted to a squared image size of 2048 pixels covering the entire city's geographic area. A bootstrap sampling was generated by swapping the stroke width in the binarization image preprocessing from 0 to 100% in steps of 10%. S2 Fig presents a statistical summary of bootstrapping for each variable.

The fractal dimension of the urban state of Medellín ranges from 1.32 to 1.74, revealing structural differences in the form and function that each variable adopts on the urban fabric. To better visualize these differences, results are clustered by an Euclidean distance dendrogram (Fig 4) that quantifies affinities among data, based on the mean fractal dimension. This dynamical affinity can be understood as a universal property that footprints complex systems [62]. In the dendrogram, three main branches define qualitative categories in the city, which we define as follows:

- Urban shape. This group of variables organizes the structural geometric configuration of the city. This group accounts for the terrain curvature, LAI, road network, and residential areas, with high average fractal dimensions from 1.68 to 1.74. In the organic city, the habitational system fits the landscape inhomogeneities (terrain curvature) and the road network, which is the main representative of mobility within the city, adjusts and grows following the evolution of the organic city.
- Habitability networks support. This group of variables is composed of the networks supporting the residential system. It includes pedestrian networks, public transport routes, urban facilities, public service network, ecological network, hydrographic network, urban

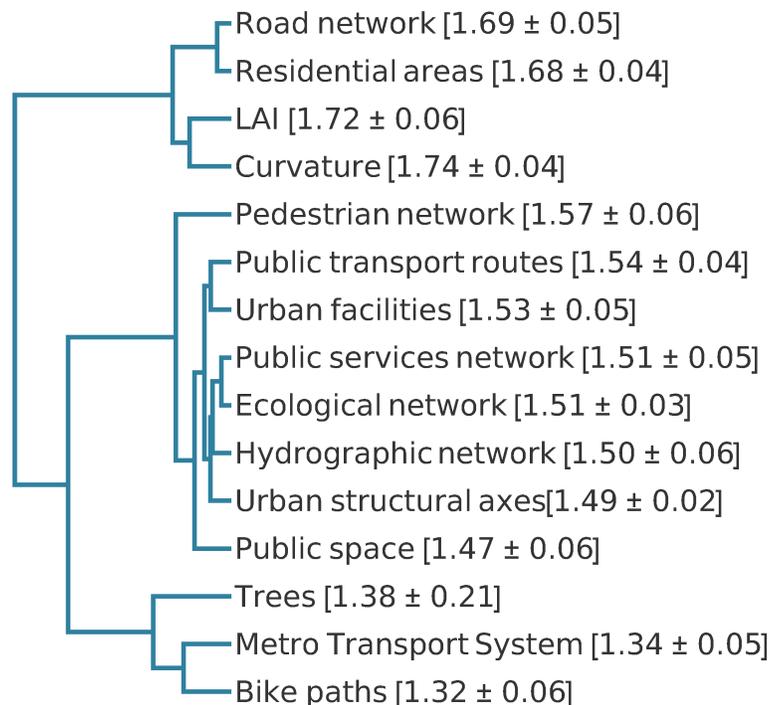


Fig 4. Fractal classification dendrogram of the variables that define the urban state in Medellín. Error is estimated as the standard deviation of the sample from bootstrapping.

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structural axes, and public space, with intermediate average fractal dimensions ranging from 1.47 to 1.57.

- Sustainable mobility networks. This class groups variables related to the sustainable mobility system of the city: bike paths, Metro transport system, and trees, with the lowest average fractal dimensions of 1.32, 1.34, and 1.38, respectively.

The road network in Medellín has an average fractal dimension of 1.69 in agreement with the reported in the literature for cities of similar size [63]. This value is quite similar to the fractal structure of the residential areas, indicating the transport network has evolved following the manifest demand in housing construction. In a sustainable urban system, efforts are made to ensure that mass public transport is efficient, and homogeneously distributed, minimizing distances and travel time [6]. However, the variables that provide services and support for living have a lower fractal dimension, which implies a lack of functionalities supporting the housing system. Sustainable mobility networks are even further from reaching the housing system. This analysis does not allow us to distinguish whether the lack of these functions is homogeneous in the city or is concentrated in some particular areas. These issues will be addressed in the following sections.

Measuring how far from entropic equilibrium is Medellín urban system

According to the second law of thermodynamics, systems evolve towards the state of highest entropy which corresponds to the macrostate compatible with the highest number of microscopic configurations or equivalently, the macrostate of highest probability. C. E. Shannon generalizes the concept of entropy in the context of physical information [64]. The information content of Shannon measures the number of bits that encode the expected value of the uncertainty of the information source that emits messages with probability p_i :

$$H = - \sum_i p_i \log_2 p_i. \tag{1}$$

[65] introduces a spatial observation window that allows differentiating microscopic configurations by weighting the probability as a function of the area fraction Δx_i associated with a certain region, which better estimates the information content in geospatially distributed systems.

$$H_{Batty} = - \sum_i p_i \log_2 \frac{p_i}{\Delta x_i}. \tag{2}$$

However, we are dealing with state variables inhomogeneously distributed over different areas a_i , as depicted in S3 Fig. In statistical terms, the problem is to find the entropy functional for a state variable X , distributed with probability $p_x = \frac{x_i}{\sum_i x_i}$, over a distribution of areas with probability $p_a = \frac{a_i}{\sum_i a_i}$. Following the idea underlining in Batty’s spatial entropy, we propose a formal structure of probability as follows:

$$p_i = k \frac{p_x}{p_a} = k \frac{A}{W} \frac{x_i}{a_i}, \tag{3}$$

where k is the normalization factor, $A = \sum_i a_i$ is the total area, and $W = \sum_i x_i$. By applying the normalization constraint $\sum_i p_i = 1$, we have $k = \frac{W}{AZ}$, with $Z = \sum_i \frac{x_i}{a_i}$. Finally, the spatial information content

$$H(p_i) = - \sum_i p_i \log_2 p_i = - \log_2 \frac{1}{Z} - \frac{1}{Z} \sum_i \frac{x_i}{a_i} \log_2 \frac{x_i}{a_i} = H(x_i, a_i). \tag{4}$$

Entropic properties like non-negativity, maximal at equal probabilities, and extensivity, can be easily proven from Eq (4). Expanding the concept of entropy as a measure of urban ordering presented in [66], we develop an entropic analysis in Medellín, based on the spatial information content $H(x_i, a_i)$. We consider urban state variables distributed in the administrative units, such as a proper spatial scale to capture the marked differentiation of social structure in Medellín (shown in Fig 1).

According to Fig 5, the urban state variables are mostly out of tune with the state of maximum entropy, which is calculated assuming that state variables are homogeneously distributed across the area of each administrative unit. The variables furthest from the maximum entropy value have a more inhomogeneous geometric distribution on the spatial units (Fig 3O) and consequently, an inhomogeneous distribution of the related urban function. The bike paths variable exhibits the lowest value of spatial entropy explained by the restricted

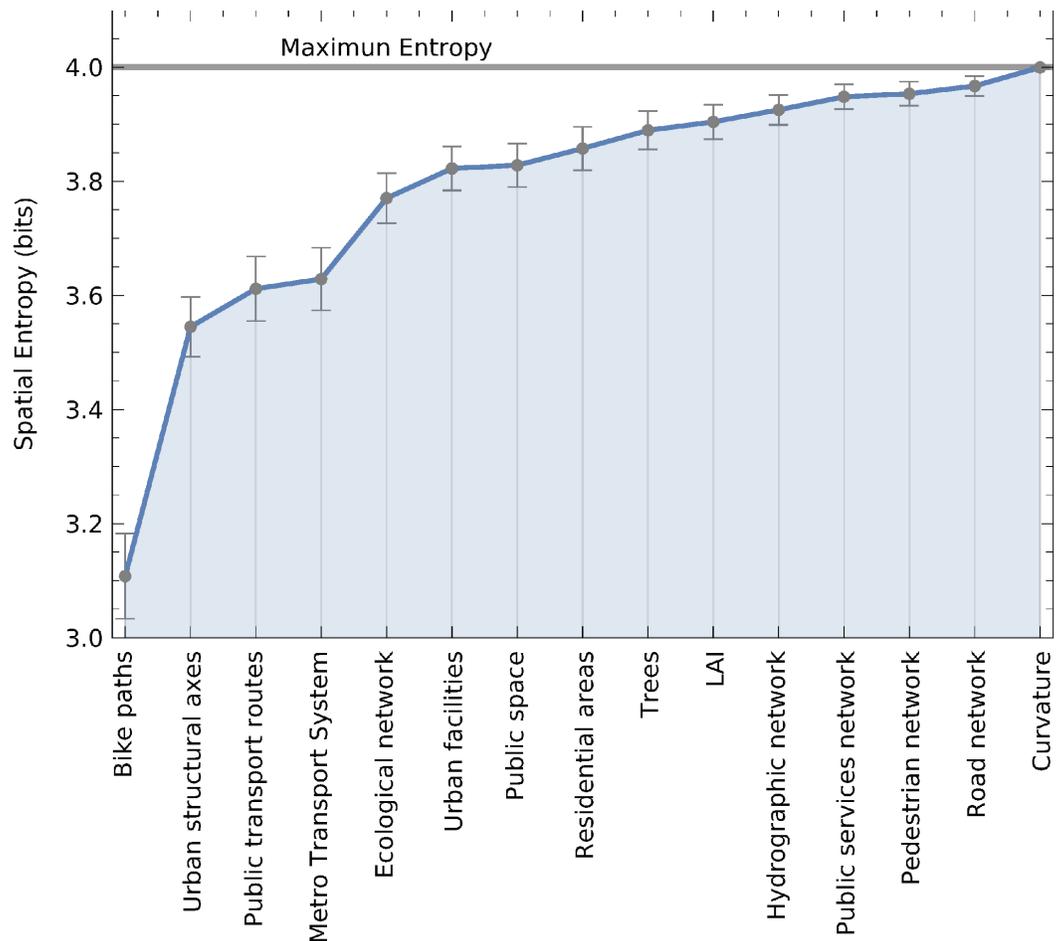


Fig 5. Spatial entropy $H(x_i, a_i)$ for urban state variables in Medellín. Maximum entropy corresponds to the homogeneous distribution of urban state variables across the i -th administrative units with area a_i . The error is estimated using the standard deviation.

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spatial distribution in the city. Urban structural axes, public transport routes, and the Metro transport system have similar values of entropy showing a similar statistical inhomogeneity in the city which means the urban function is more concentrated in certain areas. The entropy of ecological network, urban facilities, public spaces, residential areas, trees, LAI, hydrographic network, public services network, pedestrian network, road network, and curvature, have a variety of values gradually increasing towards the maximum spatial entropy, as the urban-related function is more homogeneously distributed in the administrative units (Fig 3).

In the city, fractal dimension is to urban shape as spatial entropy is to urban ordering. The lack of functionalities supporting the habitability network detected by the fractal dimension is not evenly distributed over the city. For instance, public transport routes evidence a high degree of spatial inhomogeneity, which means the urban function is concentrated in some administrative units at the expense of a lack in others. Whereas, road network, pedestrian networks, and public services have high values of spatial entropy, evidencing a high degree of homogeneity in the city.

Urban variables like LAI and trees, largely differentiated by fractal dimension, have similar statistical spatial inhomogeneities, in contrast, variables with low fractal dimensions and low values of entropy like bike paths, urban structural axes, and Metro Transport System are characterized by a high degree of the lack of urban function in the city but also a large inhomogeneity of the urban function. In the following section, we apply the novel framework of diffuse cartograms to evidence the differences in urban function concentration in terms of the administrative units.

Mapping divergent realities

Urban evolution can also be represented by adopting the analogy of a fluid city. In this metaphor, the territory lies on a liquid layer of uniform density, and the urban variables behave as a solute with concentration ρ_i for the i -th AU. In this context, the city expands in the geographical space constrained by the border conditions imposed by the territory and the dynamics of the habitat. Administrative boundaries are simulated as porous walls allowing the mixing up of information, and the system evolves toward the equilibrium state according to Fick's law of diffusion [67,68].

Considering a differential element of rectangular area $dx dy$, enclosing each spatial point (x,y) , where the density of the urban variable is $\rho(\mathbf{r})$, the current density in the standard diffusion problem, is defined as:

$$\mathbf{J} = \mathbf{v}(\mathbf{r}, t)\rho(\mathbf{r}, t), \tag{5}$$

where $\mathbf{v}(\mathbf{r}, t)$ is the time-dependent velocity field of the fluid. Consistent with thermodynamic irreversibility, the direction of the diffusion process is from regions of highest to lowest concentration and does so more rapidly when the gradient is steeper, satisfying:

$$\mathbf{J} = -\nabla\rho(\mathbf{r}). \tag{6}$$

The continuity equation must also be fulfilled since there are no sources or sinks for this system, leading to the standard form of the diffusion equation with a diffusion coefficient equal to one:

$$\nabla^2\rho - \frac{\partial\rho}{\partial t} = 0. \tag{7}$$

From Eqs 5 and 6, we obtain an expression for the velocity in terms of the density:

$$\mathbf{v}(\mathbf{r}, t) = -\frac{\nabla \rho}{\rho}. \quad (8)$$

The cartogram is obtained by numerically solving Eq 7, with initial values of urban state variables $\rho(\mathbf{r}, t = 0)$. This solution determines the cumulative displacement $\mathbf{r}(t)$ of any point on the map at time t , solving numerically the Volterra integral of the second type:

$$\mathbf{r}(t) = \mathbf{r}(0) + \int_0^t \mathbf{v}(\mathbf{r}, t') dt'. \quad (9)$$

In the asymptotic limit, each of the points of the original map, defines the equilibrium state of the cartogram. Neutral buoyancy conditions are imposed on external regions, with uniform density $\bar{\rho}$ to have the map floating on a *sea* of uniform density that contains its borders. The mapped area must be enclosed within a much larger rectangle imposing Neumann boundary conditions that prevent any outward flow. These choices ensure that any perturbation outside the map does not affect its final shape. In practice, a rectangle two or three times larger than the area to be mapped is enough.

This methodology has been developed in [67,68], and previously applied to Medellín in [69]. This mechanism leads to an asymptotic evolution of the city according to the proportion in which the variables are distributed throughout the territory. The representation of the state of equilibrium of the diffuse city (or cartogram), allows us to combine geographic information with statistical information to directly observe the (in)homogeneity in the distribution of the variables [70,71].

The actual urban state for Medellín, that is, the initial condition for fluid cartograms (Fig 6B–6P) is provided in S2 Table. Comparing the administrative map of Medellín (Fig 6A) with cartograms (Fig 6B–6P) reveals some features hidden in the canonical geographic representation. Lower distortions of administrative units (AUs) concerning the administrative map indicate the variable is more homogeneous distributed before the diffusive process and the function is proportional to the administrative area. The global cartogram distortion is quantified by the average percentage of deformation regarding the administrative map. A high degree of average deformation is quite consistent with a large distance of entropic equilibrium, except for the Metro transport system and pedestrian network variables that are more inhomogeneously distributed in administrative units (Fig 6C and 6L) than could be inferred from spatial entropy rank (Fig 5). The fluid city expands far from the equilibrium state due to the permanent input of matter and energy, which leads to forced self-organization that competes with the evolution toward the state of maximum entropy [42].

Fig 6 evidences differences in the urban function dynamics across administrative units. It is clear how some urban functions are concentrated in certain administrative units (with a noticeable expansion in the diffusive equilibrium state) and a lack of functionality in others (with a noticeable shrinkage even until almost disappearing). The deformation of each variable summarizes the coevolution of the planned city and the organic city under the forcing of the microscopic realization of day-to-day life, geometric constraints, and territorial planning. This manner of representing the city returns to the morphogenetic point of view of multiple information movements represented in territorial memory.

The bike paths variable is the most distorted, with an average deformation of 78.20%. The urban function is lost in the peripheral administrative units in the north of the city (AUs 1, 2, 3, 6, 8, 9, and 13) while the urban function is concentrated in the central and southwest AUs

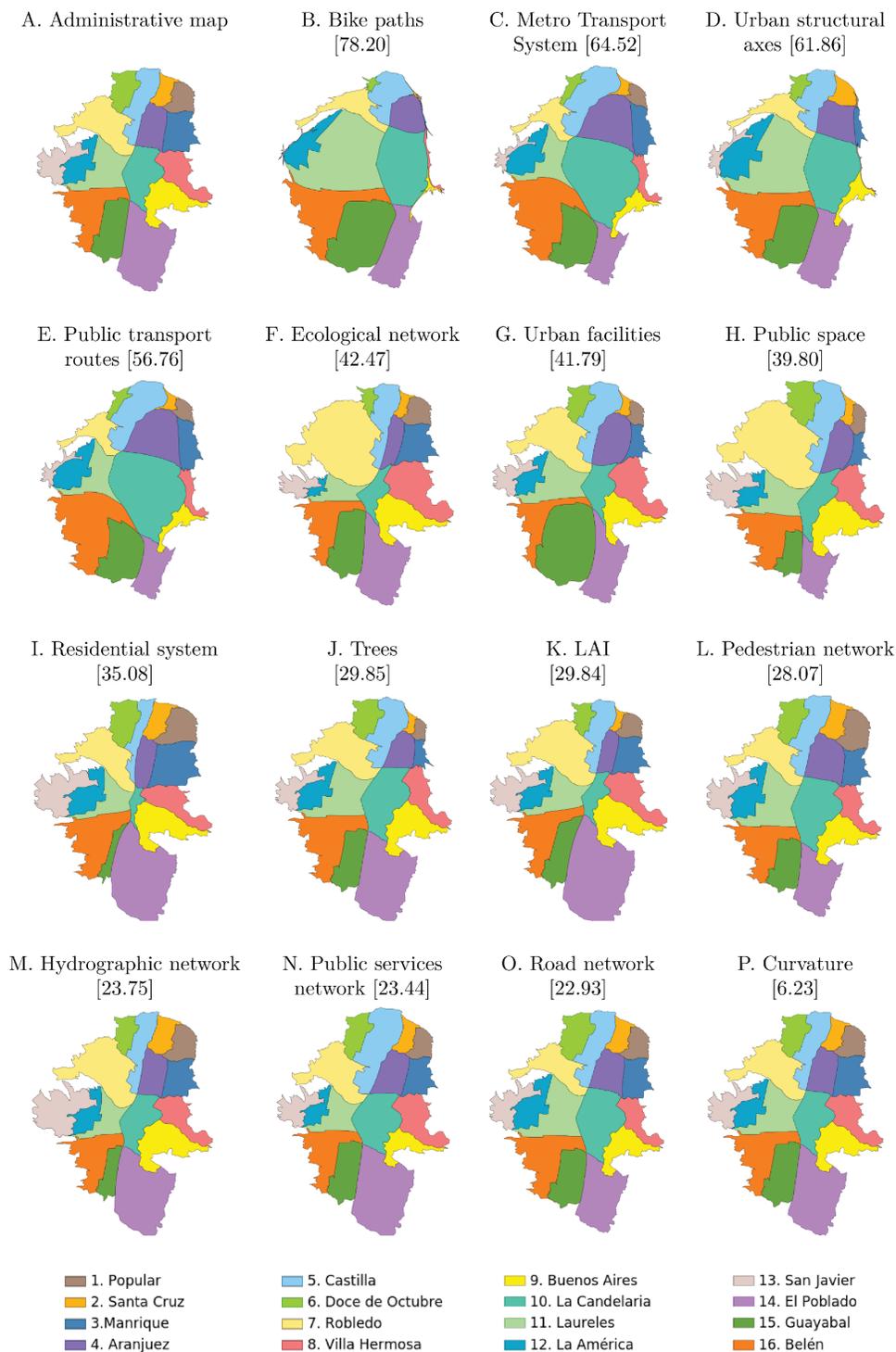


Fig 6. Fluid representation of urban variables in Medellín. A) Administrative map of Medellín. B) to P) Urban state variables ordered from higher to lower percentage deformation, in square brackets [%]. Basemap source for state variables as in Fig 3.

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(10, 11, 12, 15, and 16). This picture could be explained by, geographical restrictions (bike routes are preferably available in plain terrains) and socioeconomic conditions (low income neighborhoods). The city dynamics related to sustainable mobility and city marketing have promoted the development of cycling path systems in the touristic city. The Metro transport system is the second most inhomogeneous urban state variable with an average deformation of 64.52%. The function is limited by the elevation gradient and the peripheral administrative units (AUs 1, 2, 6, 7, 8, 9, 13, and 14) show a functionality reduction. By contrast, the central and plain areas show a functionality increase (AUs 4, 10, and 16). The urban structural axes variable, with an average deformation of 61.86%, shows a lack of function in the northwestern city (AUs 1, 3, and 8) and central west (AU 13), meaning these areas are outside the planned city. Public transport routes variable, with an average deformation of 56.76%, exhibits a loss of functionality in the southernmost city (AUs 14, 15, 16). These areas are characterized by the well-being and the higher income neighborhoods where private transport is more common than public transport. By contrast, AUs 10 and 11 accumulate this function in agreement with the centrality of daily activities in the services city. The ecological network variable, with an average deformation of 42.47%, presents a predominance of the ecological connectivity function in AUs 7 and 8, due to El Volador Tutelar Hill and La Ladera Natural Park, respectively. The deformation of urban facilities variable is dominated by the inner city airport in the AU 15 and its average deformation is 41.79%.

Similarly, the deformation of the public space variable is dominated by the presence of El Volador in AU 7 and a decrease in functionality in AUs 3 and 14. The residential areas variable, with an average deformation of 35.08%, shows a differentiated behavior in the habitational system. The northeast (low-income population) and southeast (high-income population) corners (AU 1, 2, 3, and 14) exhibit a concentration of function, whereas, the center and southwest exhibit a significant lack of this function (AUs 10 and 15). This cartographic picture contrasts the bedroom city against the services city.

Trees, LAI, pedestrian, hydrographic, public services, and road networks show similar structural behavior with average deformation ranging from 29.85 to 22.93%. These variables have rapidly adapted to residential growth since the end of the last century. The urban expansion towards mountain slopes pairs with a functional expansion in the road network and the associated habitational services. Finally, the curvature variable is the most homogeneous distributed variable in the city, with an average deformation of 6.23%, according to the maximum entropy measure.

Visualizing the city in terms of cartograms allows an organic approach to urban reality. The stronger deformation reveals the emergence of macroscopic order due to local microscopic organization (i.e. neighborhood decisions or uses) against global forcing such as urban planning.

Concluding remarks

Fractality serves as a fundamental framework for understanding how the urban system of Medellín organizes and distributes its functions in space. On one hand, the city reflects the morphological imprint of traditional urban paradigms, shaped by modern Euclidean planning principles. On the other hand, it can be interpreted as a socio-spatial construct where fractal geometric forms manifest across the global urban plane and various observational scales.

The complex organization of Medellín emerges from daily experiential dynamics, such as access to services and urban functions, which we synthesize in static representations of the spatial distribution of urban state variables. Analyzing these variables facilitates mono-functional spatial connections, providing a critical tool for evaluating information exchange

across scales, from the individual to the city. Medellín's urban state variables cluster according to universal physical properties characterized by their fractal dimension [62].

We identified three primary modes of self-organization in Medellín: urban shape, habitability support, and sustainable mobility. The city's road network, shaped by the geomorphological constraints of the Valley, co-evolves with the housing system. However, networks supporting the housing system lag behind its geometric demands for habitability, while sustainable mobility networks remain misaligned with residential needs. Here, fractal dimension is to urban shape as spatial entropy is to urban ordering.

The interaction of streets, sidewalks, proximity between places, urban furniture, and spatial distribution of connectivity mechanisms—alongside the preservation of the city's technical memory—plays a pivotal role in facilitating information exchange across scales. Urban attributes must adapt not only to the city's physical conditions but also to generate and integrate processes of information production and transfer. Urban systems with high geometric structural organization inherently require significant energy resources for maintenance [72], driving them away from entropic equilibrium and toward spatial order. In Medellín, this dynamic highlights a stark lack of urban functions in certain administrative units, limiting inhabitants' opportunities for social integration and interaction, consistent with the (in)homogeneous distribution of urban functions.

Cartographic representations reveal pronounced disparities in resource allocation across the city. While certain areas concentrate urban functions, others remain underserved, reflecting spatial heterogeneity in the urban state. These disparities provide valuable insights into the dynamics and development trajectories of different city sectors, highlighting areas where urban functions may need reinforcement in central planning.

Our findings support a tangible agenda for urban transformation that prioritizes efficiency, economic viability, ecological sustainability, and public equity. In alignment with [73], our proposals aim to mitigate social segregation and inequality. Ultimately, this study provides a robust analytical framework for understanding the organic interplay between urban landscapes and functions, offering a pathway to conceptualize and shape future urban developments.

Supporting information

S1 Fig. Identification of the visual order in terms of a distinctive combination of key urban functions. Color code facilitates the identification of specific configurations of urban functions around the target bridges (exemplified in the Aguacatala bridge). Photos show the related visual order in the general urban plain.

(TIF)

S2 Fig. Boxplot of the box-counting method to obtain the fractal dimension over each urban state variable's binarized (info/no-info) image. All data were converted to a squared image size of 2048 pixels covering the entire city's geographic area. A bootstrap sampling was generated by swapping the stroke width in the binarization image preprocessing from 0 to

100% in steps of 10%.

(TIF)

S3 Fig. Schematic representation of how a state variable x_i (trees, in this case) is distributed over different areas (a_i , in arbitrary units).

(TIF)

S1 Table. Data description. Elevation map from USGS (<https://www.usgs.gov/3d-elevation-program>). Sentinel 2 satellite images from Copernicus: <https://dataspace.copernicus.eu/>.

Data from the GeoMedellín Open Data Project (GeoMed, 2023) are updated to 2023. Data can be accessed using the corresponding reference code RC at <https://www.medellin.gov.co/geomedellin/datosAbiertos/RC>.

(PDF)

S2 Table. Data input for cartograms in Administrative Units.

(PDF)

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Map data copyrighted OpenStreetMap contributors and available from <https://www.openstreetmap.org>. The authors thank Daniela Velásquez Ciro for producing LAI basemap.

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