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Emergence of an unpredictable evolution in a spatial prisoner's dilemma via a player's multiple perspectives

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Abstract

Spatial prisoner's dilemma (SPD) has attracted researchers' attention as a model of conflict for players. In SPD, players have two different strategies, namely, defectors and cooperators. A defector earns a high payoff from an opponent co-operator while getting nothing from an opponent defector. On the contrary, cooperators promote a win–win relationship between the two cooperators. These mechanisms influence population dynamics in SPD, and many SPD models have been developed. However, little is known about the emergence of an unstable or unpredictable evolution in population dynamics using an SPD model, which may be observed in living systems. In addressing this issue, two SPD models were proposed. In both models, players change the neighborhood definition in accordance with their strategies and sometimes select the rule for this change using probability or local information. Result showed that our models generated characteristic population patterns that may be linked to a self-organized criticality (SOC), a term referring to many systems of interconnected, nonlinear elements that evolve over time into a critical state. In fact, the second model could be spontaneously close to the critical point using local information.

Author summary

In this study, we proposed two different Spatial prisoner's dilemma (SPD) models. In the first model, the system presents a phase transition using parameter tuning and evolves dynamically around a critical parameter value. The other model, where players coordinate their actions not by an external parameter but by local information, can be spontaneously close to a critical point in the former proposed model.

Introduction

Spatial prisoner's dilemma (SPD) is a well-known spatial game theory that addresses the evolution of two types of strategies [1–5]. In SPD, defectors and cooperators interact with each other and update their strategies with the opponent. When two defectors meet, they get a punishment payoff P, but when two cooperators meet, they have a win–win relationship by

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earning a reward payoff *R*. If a defector and a co-operator meet, then a defector receives temptation payoff *T*, while a co-operator receives sucker payoff *S*. Defectors become dominant when they obtain a larger payoff *T* during interaction with cooperators [1–8]. However, evolutions of cooperative behavior have received attention from researchers because some animals show cooperative behaviors, which may flourish the species [9,10]. In addressing this problem, many SPD models have been proposed [11–20]. For example, environmental conditions support the evolution of cooperators [11,12], memory use enhances cooperation [17–19], and the use of incentives has been considered for promoting cooperation [21].

The evolution of cooperators in SPD is an important issue, but it is only one aspect of moral behavior [22,23]. Therefore, focusing the discussion on the evolution/maintaining of cooperators in SPD is not always necessary. Living systems, including animals, demonstrate unpredictable dynamics with regard to evolution [24–26]. Therefore, SPD can be used as an evolutionary system model that represents and describes the population dynamics observed in social or biological systems. In fact, several studies have focused on evolutionary game dynamics to reveal complex population dynamics [27,28]. In SPD, population dynamics can vary in accordance with time evolution. In some time, defectors can be dominant.

Self-organized criticalities (SOCs) describe a situation in which systems evolve over time into a critical state, and represent a key feature of some living systems [24]. Around a critical point, the system shows some fluctuations and behaves unpredictably because it presents a phase transition around a critical point. Several studies have reported the occurrence of a phase transition in SPD by considering parameter effects [29,30]. Criticality or SOC has also been observed in evolutionary game theory [31–33]. Unpredictable population dynamics can emerge in SOC systems. However, previous studies have not focused on SOCs achieved via bottom-up self-organization but have instead focused on SOCs that have developed using critical parameter values [31]. On the contrary, Mahmoodi et al. (2017) reported that a bottom-up self-organization achieved SOC using an evolutionary game theory, although the decision-making model was used for strategy update, and the PD model was only partly used [33].

Thus, few studies have investigated the emergence of SOC using the extension model of pure SPD. In this study, two SPD models were developed: one was created by focusing on parameter tuning and the other involved a bottom-up (i.e., 'agent perspective') process. Moreover, from a game theory perspective, it is likely that players change the neighbors they can interact with if they want improve their circumstances. Therefore, in both models, players increase or decrease neighboring areas if they become losers in competitions with these neighbors. In both models, players' strategies determine the way of change of neighboring areas. Cooperators are open-minded and have a wide perspective while defectors are selfish and have a narrow-perspective [34]. However, it is not always reasonable for them to do so. Therefore, cooperators/defectors in our models sometimes switch expansion/reduction of neighboring area with reduction/expansion of it.

In the first model, the system presents a phase transition using parameter tuning regarding the switching of neighboring areas' expansion/reduction and evolves dynamically around a critical parameter value. The other model, where players coordinate their actions regarding the switching of neighboring areas' expansion/reduction not by an external parameter but by local information, can be spontaneously close to a critical point in the former proposed model.

Materials and methods

Space and agent

We set 10,000 players on a two-dimensional lattice (100 × 100 sizes). Individual players were randomly assigned to either a co-operator or defector, that is, the initial defector density was

Table 1. A payoff matrix.

	Cooperator	Defector
Cooperator	R(1)	S(0)
Defector	T(b)	P(0)

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0.50. Periodic boundaries were used [34]. We run 10,000 time steps for each simulation trial. 100 trials were conducted. In this study, we considered the weak version of the PD game [35]. Therefore, we set $R = 1$, $P = S = 0$ and $T = b$ where $1<b<2$ according to values reported in previous studies [34,35] (Table 1).

Model description

Probabilistic switching (PS) model. In this model, a variable known as *neighborhood*, which refers to the number of neighborhoods, was assigned to every player before starting each trial [34]. This variable is expressed as follows:

$neighborhood(i,j) \in \{1,2,\ldots,8\}$,

which shows the *neighborhood* for player (i, j).

At each time step, individual players randomly select *neighborhood* neighboring players from the Moore neighborhood (eight players) and play against individually selected neighboring players using the payoff matrix (Table 1). For example, when $neighborhood(i, j) = 3$, player (i, j) randomly selects three neighbors from the following: $(i-1,j), (i+1,j), (i,j-1), (i,j+1), (i-1,j-1), (i-1,j+1), (i+1,j-1)$ and $(i+1, j+1)$. The selected neighbors are then elements of $\omega^t(i, j)$, a variable that specifies the set of neighborhoods of player (i, j) at time t, such that the following equation is satisfied:

$$neighborhood(i,j) = |\omega^t(i,j)|$$

Then, the *score* is earned as follows:

$$score^t(i,j) = \sum\nolimits_{k\in\omega^t(i,j)} payoff^t{}_{(i,j)}(k),$$

which shows the *score* of the player (i, j) at time t. Here, $payoff^t{}_{(i,j)}(k)$ indicates a payoff that the player (i, j) earns from its neighboring player k at time t.

After all players get their *score*, they compare them with their selected neighboring players and update their strategy to the neighboring player's strategy, which earns the highest score among the neighborhoods. At that time, players become losers if they are not the player earning the highest score in their neighborhood. In that case, the variable *state* changes to *state* = *loser* from *state* = *null*. This is because *state* is set to *state* = *null* for all players at the beginning of the strategy update event. For example, for player (i, j),

$strategy^{t+1}(i,j) = \max\_strategy^t(i,j)$,

$state^{t+1}(i,j) = loser, \text{ if } \max\_score^t(i,j) \neq score^t(i,j)$.

$state^{t+1}(i,j) = null, \text{ if } \max\_score^t(i,j) = score^t(i,j)$.

Here, $strategy^{t+1}(i, j)$ and $state^{t+1}(i, j)$ are the *strategy* and *state* values for player (i, j) at time $t+1$ while $\max\_strategy^t(i, j)$ and $\max\_score^t(i, j)$ indicate the *strategy* and *score* values of the player who earns the highest score at time t among player (i, j) and all other players in its neighborhoods.

Moreover, for all players for whom the variable *state* is now set to *loser* from *null*, they can then change the variable *neighborhood* at the beginning of their next iteration [34]. According to a previous study [34], cooperators are likely to expand their neighboring area while opponent players are likely to decrease the number of neighbors. In this model however, we introduce a reverse situation where cooperators decrease the number of neighbors while defectors increase the number of neighbors. Furthermore, these two situations are controlled by a parameter p.

More precisely, player (i, j) replaces the variable *neighborhood*(i, j) at the beginning of time t whenever $state^t(i, j) = loser$ as follows:

With a probability p,
$neighborhood(i, j) = neighborhood(i, j) + 1$, if $strategy^t(i, j) = cooperator$.
$neighborhood(i, j) = neighborhood(i, j) - 1$, if $strategy^t(i, j) = defector$.
With a probability $1-p$,
$neighborhood(i, j) = neighborhood(i, j) - 1$, if $strategy^t(i, j) = cooperator$.
$neighborhood(i, j) = neighborhood(i, j) + 1$, if $strategy^t(i, j) = defector$.

In that case, updated *neighborhood* is used for *score* calculation and strategy update afterward.

Switching via local information (SLI) model

This model operates in a similar way to the PS model except for the replacement of the variable *neighborhood*, which is expressed as follows:

With a probability $\frac{\max\_score^{t-1}(i,j) - \min\_score^{t-1}(i,j)}{\max\_score^{t-1}(i,j) + \min\_score^{t-1}(i,j)}$,
$neighborhood(i, j) = neighborhood(i, j) + 1$, if $strategy^t(i, j) = cooperator$.
$neighborhood(i, j) = neighborhood(i, j) - 1$, if $strategy^t(i, j) = defector$.
With a probability $1 - \frac{\max\_score^{t-1}(i,j) - \min\_score^{t-1}(i,j)}{\max\_score^{t-1}(i,j) + \min\_score^{t-1}(i,j)}$
$neighborhood(i, j) = neighborhood(i, j) - 1$, $strategy^t(i, j) = cooperator$.
$neighborhood(i, j) = neighborhood(i, j) + 1$, if $strategy^t(i, j) = defector$.

Here, $\max\_score^{t-1}(i, j)$ and $\min\_score^{t-1}(i, j)$ indicate the highest and lowest score in the neighborhood of the player (i, j) calculated at one previous iteration, respectively (considering that *neighborhood* updates occur at the beginning of one iteration, if needed). Therefore, individual players are able to modulate the variable *neighborhood* using a probability in accordance with their local information (i.e., max _*score* and min _*score*).

Increasing or decreasing neighboring areas may promote players to improve their situation since they can interact with different players from the current set after modulating neighboring areas. This is why players in our model are allowed to increase/decrease their neighboring areas when they become losers. It is likely for cooperative players to expanding their neighboring areas, which indicating that cooperators establish win–win relationships with others due to their unselfish nature. Conversely, defectors are egocentric, reflecting their narrow perspective [34]. At the same time however, it may be unclear for players whether increasing their neighboring areas or decreasing their neighboring areas brings better results.

In the SLI model, the ratio $(\max\_score - \min\_score)/(\max\_score + \min\_score)$ indicates the degree of score difference among players within the neighborhood. If that difference is high (i.e., has a relatively large ratio of $(\max\_score - \min\_score)/(\max\_score + \min\_score)$, a cooperative player who is a loser becomes more cooperative by expanding its neighboring area and tries earning a higher score. Similarly, a defective player who is a loser becomes more selfish by decreasing its neighboring area in order to improve its situation and win the next round. In contrast, if score differences among players within the neighborhood is not so large, players adopt a reverse condition and tries keeping the size balance of their neighboring area. Thus, they change their neighboring area in an opposite way from the above situation.

Please see S1 and S2 Figs for the flowcharts.

Results

Here, we set parameter *b* to high payoff values (i.e., *b* = 1.7, 1.8 and 1.9) since a defector facing a co-operator can earn a high payoff. In fact, SPD models sometimes focus on such a high parameter value [34]. As shown in Fig 1, which demonstrates the fraction of defectors after each trial obtained from 100 trials (Fig 1A) and its standard deviations (Fig 1B), the PS model presents a phase transition with $p \approx 0.8$. The SD of the fraction of defectors around that critical value showed the highest SD, suggesting that the fraction of defectors present at the end of the trial can vary and is dependent on the trials themselves. Interestingly, the SLI model exhibits similar deviations in the fraction of defectors. This is true for all cases except *b* = 1.7, in which the SD of the SLI model is lower than the highest SD of the PS model; however, it still shows a high SD. Thus, these data confirm that the PS model presents a phase transition with $p \approx 0.8$ and that the SLI model can evolve criticality in some *b* values. However, according to Fig 2, which depicts the relationship between parameter *b* and fraction of defectors at the end of trials in the SLI model, during which parameter *b* decreases, defectors vanish, and no fluctuations in population dynamics occur thereafter. Therefore, we speculate that the SLI model can evolve criticality at high *b* values.

Next, we evaluated system dynamics according to time evolution using both models. Here, we set *b* = 1.9. Fig 3 demonstrates the time evolution of defectors obtained from one trial using

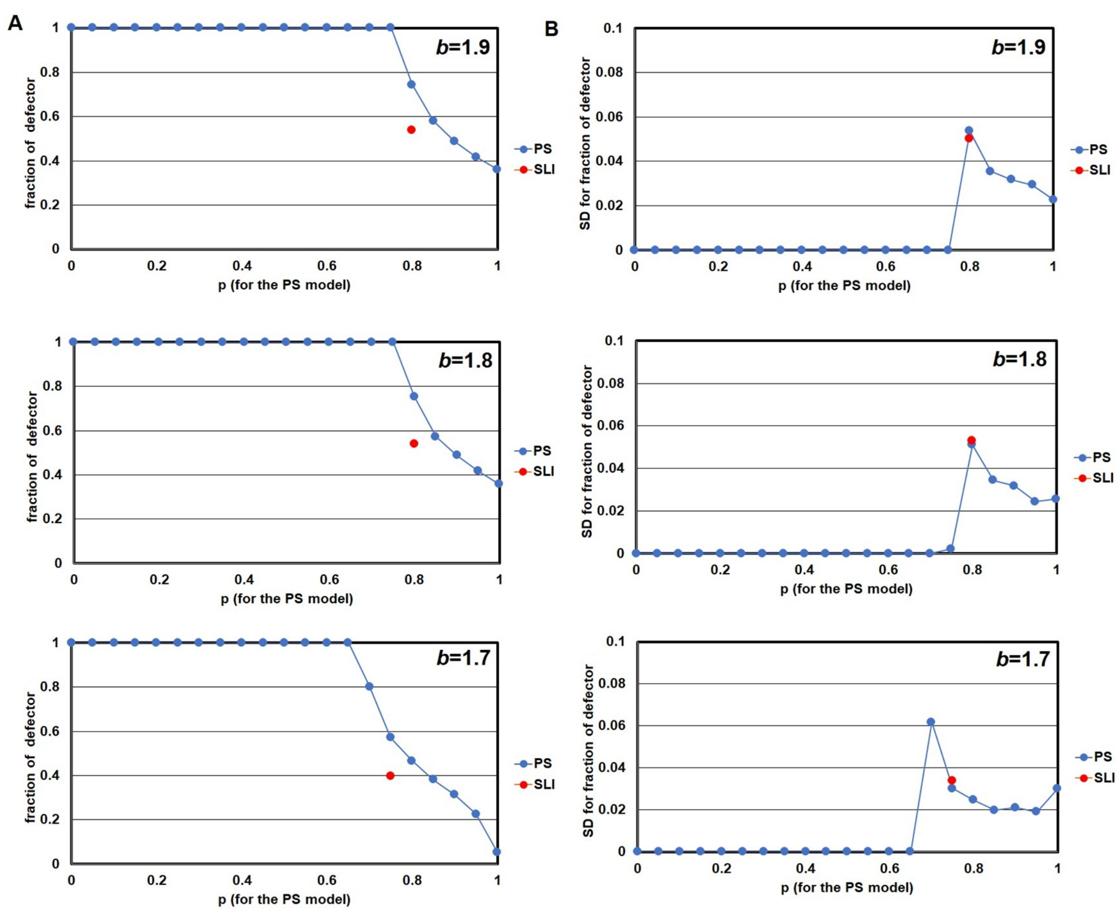

Fig 1. Fraction of defectors at the end of simulation trials (averaged over 100 trials) and its standard deviation (*b* = 1.9, 1.8 and 1.7). (A) Fraction of defectors. (B) Standard deviation of fraction of defectors.

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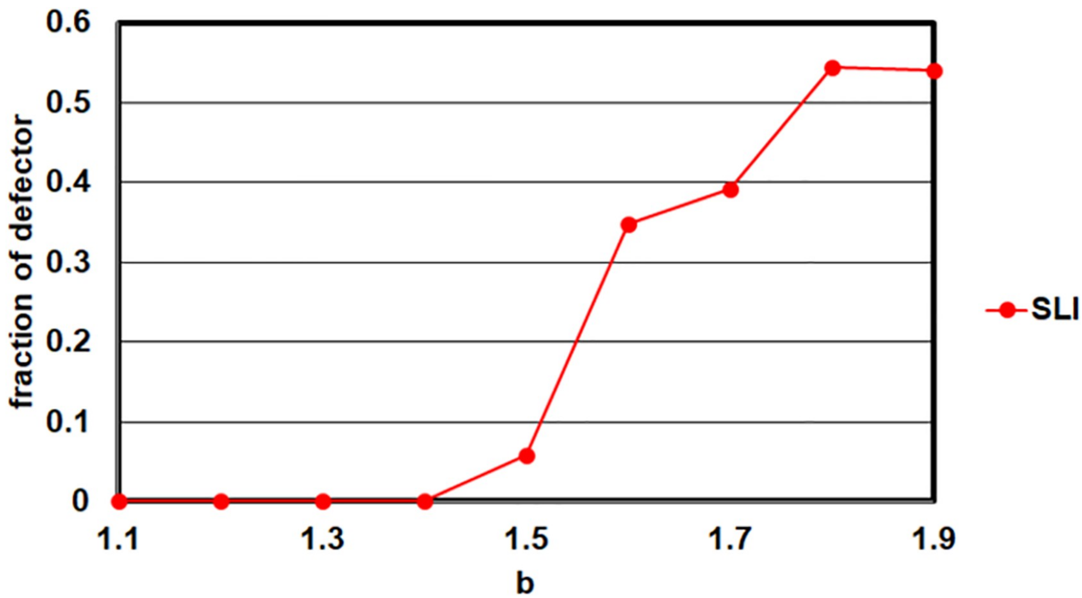

Fig 2. Fraction of defectors at the end of simulation trials (averaged over 100 trials) regarding parameter *b* in the SLI model.

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the SLI model and PS model with $p = 0.8$ and $p = 0.85$. The PS model with $p = 0.8$ and the SLI model show some fluctuations in the time evolution of defectors. Furthermore, we evaluated the time interval between becoming a loser and becoming a loser once again. Fig 4 shows the relationship between the abovementioned time interval obtained from a single player over one trial and its cumulative distribution in both models. As shown in this figure, both models exhibit a power law tailed distribution (Fig 4A: PS model with $p = 0.8$, $\mu = 2.29$, AIC weights for power law against exponential law = 1.00, Fig 4B: SLI model, $\mu = 2.37$, AIC weights for power law against exponential law = 1.00). These results indicate that the SLI model and the PS model with $p = 0.8$ evolve criticality because self-organized systems exhibit power law tailed distributions [36,37]. We also evaluated the emergence of power-law tailed distributions from a different perspective, i.e., the time elapsed during operation (e.g., time persistence for cooperation) [38]. S3 Fig confirms that the SLI model exhibits power-law tailed distributions with respect to this variable ($\mu = 1.67$, AIC weights for power law against exponential law = 1.00).

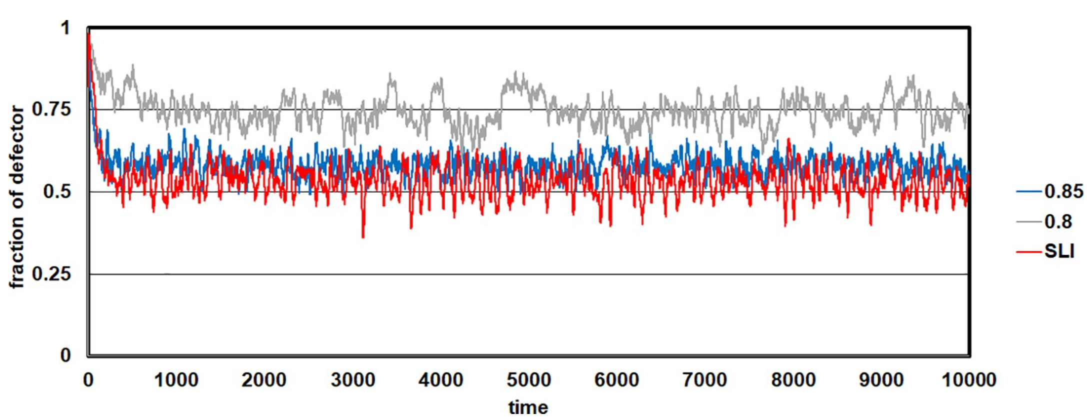

Fig 3. Time evolutions of defectors using the PS model with $p = 0.8$, $p = 0.85$, and the SLI model ($b = 1.9$).

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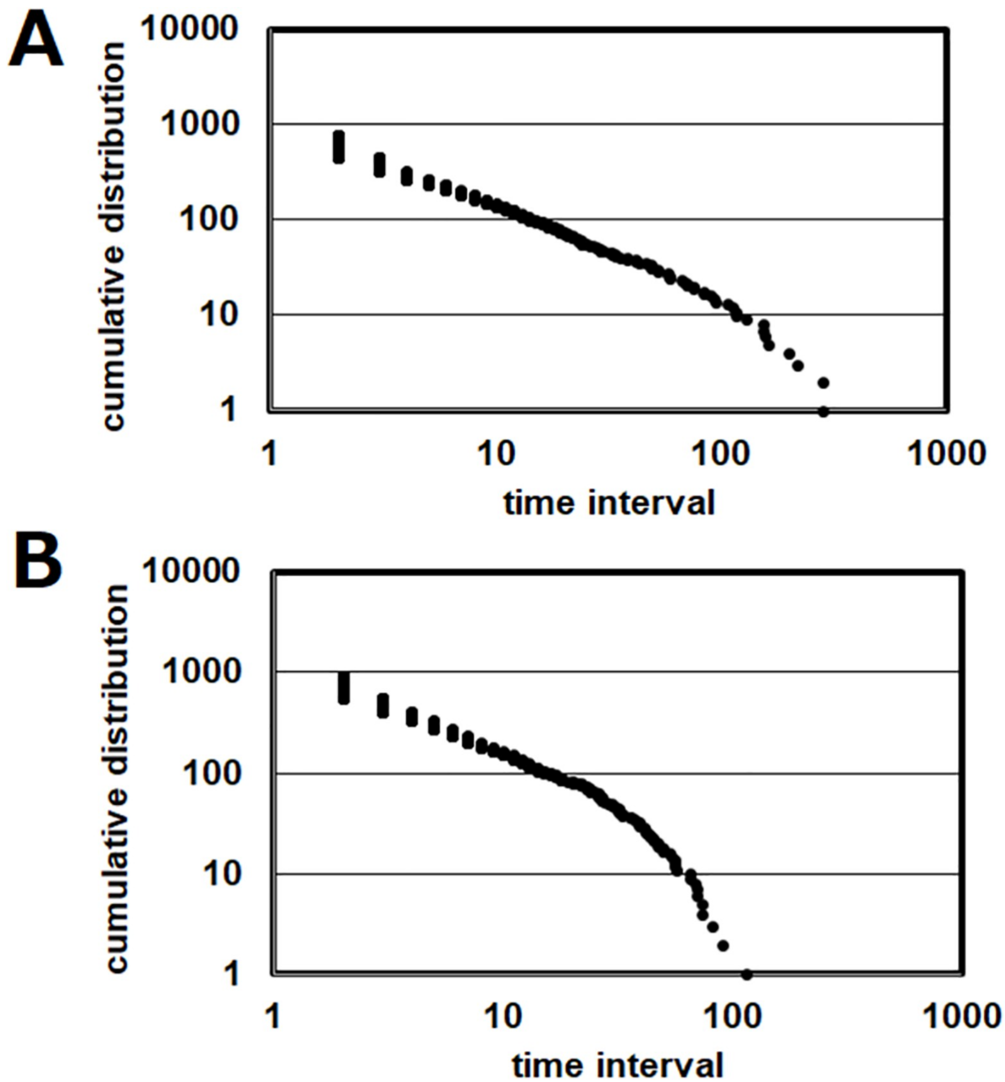

Fig 4. Time interval between becoming a loser and becoming a loser once again (b = 1.9). (A) The PS model with p = 0.8. (B) The SLI model.

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Next, we determined that the SLI model was not affected by varying initial defector density. We replaced the initial defector density from 0.50 with 0.35 or 0.65 (Fig 5A and 5B). As shown in figures, the system evolves as time progresses, suggesting that the SLI model is flexible to the initial defector densities to some extent.

Finally, we evaluated the performance of these two proposed models for the evolution of cooperators. Fig 6 presents the results regarding the fraction of defectors at the end of trials and its relationship with parameter b. Both proposed models outperformed the classical SPD model when parameter b increased. Moreover, the SLI model outperformed the PS model. Here, players in the classical SPD model always interact with the players defined as the Von Neumann neighborhood and never change the interacting opponents until the end of the trial. The other procedures are the same as those of the proposed models. Notably, the SLI model showed similar results for this analysis even after the calculation time is extended to 20,000 time steps (Fig 7). These results confirm that calculation time of 10,000 time steps is enough.

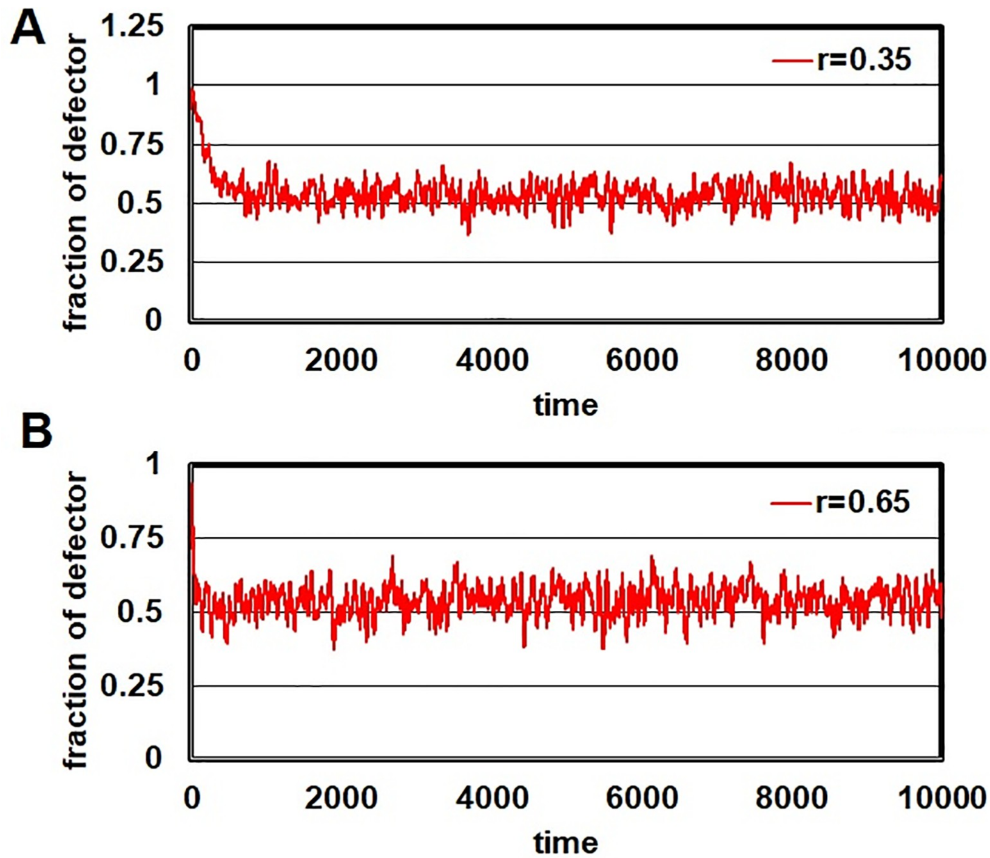

Fig 5. Time evolution of defectors in two different initial defector densities ($b = 1.9$) (A) The initial defector density: $r = 0.35$. (B) The initial defector density: $r = 0.65$.

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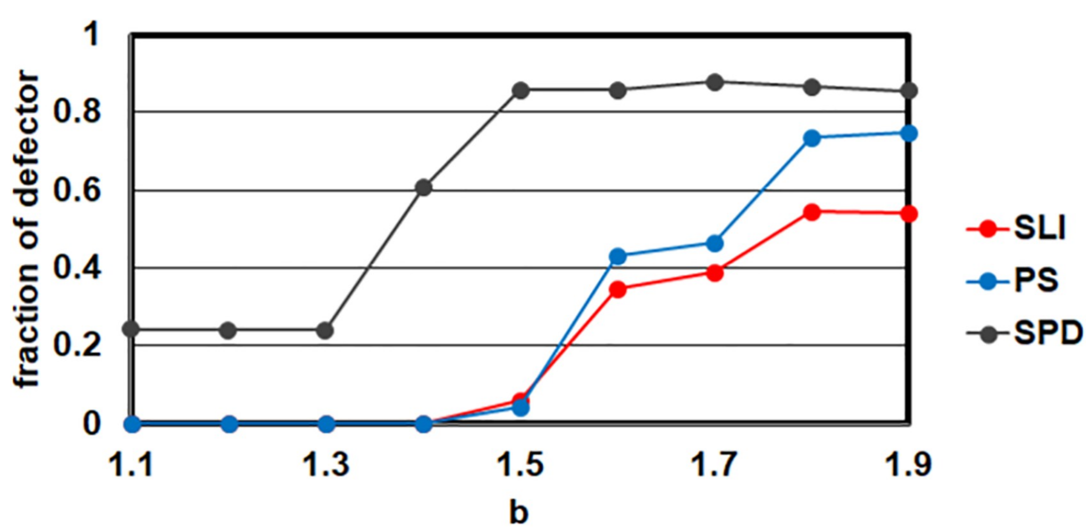

Fig 6. Fraction of defectors at the end of simulation trials (averaged over 100 trials) regarding parameter *b* (*p* = 0.80 for the PS model).

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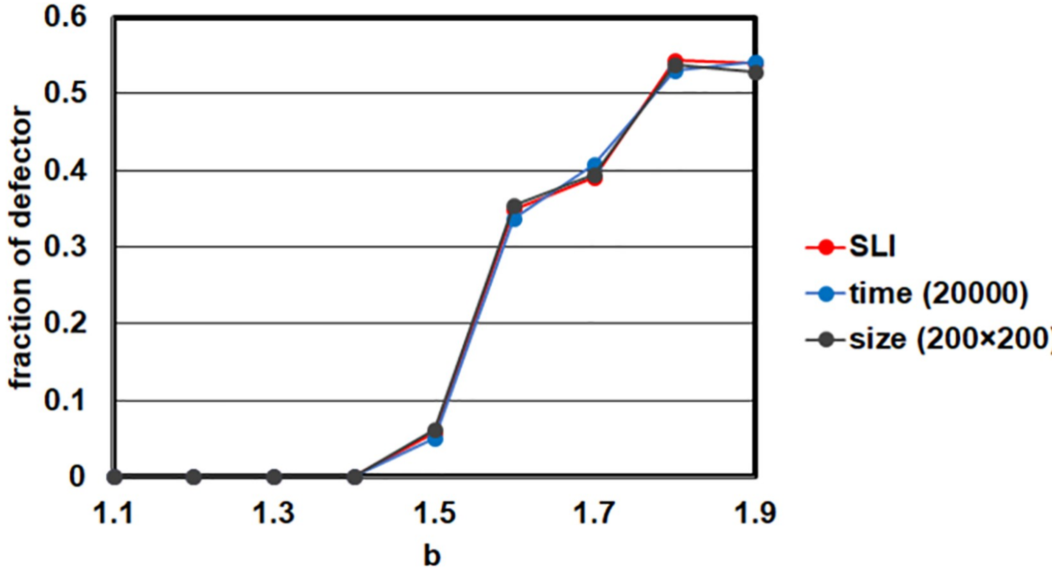

Fig 7. Fraction of defectors at the end of simulation trials (averaged over 100 trials) after varying calculation time and/or system size.

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Moreover, the SLI also exhibited similar results when the system size was expanded from 100 × 100 to 200 × 200 (Fig 7).

Discussion

In this paper, two different SPD models were proposed to induce criticality or SOC in the population. The PS model, in which players reversibly switched the rule of the modification of *neighborhood* with a fixed probability, showed a phase transition regarding population dynamics. Moreover, the SLI model where players switched reversibly the rule of variable *neighborhood* without using a fixed probability but via local information was likely to be close to the critical point autonomously. We also found that the time interval between once becoming a loser and again becoming a loser was consistent with a power law tailed distribution. This result indicates that players in these models likely become a loser at irregular time intervals. This scale-free property is linked to the SOC [36,37].

Numerous SPD models have been proposed to address the evolution of cooperators [11–20]. Nevertheless, few studies have focused on the critical behavior in population dynamics for SPD, although some studies have reported the occurrence of phase transitions in SPD [28,30]. Our proposed models have two important aspects. First, a phase transition can be induced if players use two perspectives in response to one event. In our models, players updates variable *neighborhood* in accordance with their strategy [34]. However, players sometimes switch the *neighborhood* update rule reversibly in accordance with their strategy. Thus, players have two perspectives regarding one event (the rule for the *neighborhood* update). Second, the players in the SLI model use local information to set the switching probability instead of using a fixed parameter (p). Therefore, individual players use different values regarding the switching probability, which can vary over time for every player. Such effects promote cooperative behavior in spatial game theories [39,40]. No parameter tuning is required to bring the system SOC [33]. In the SLI model, if max *_score* and min *_score* vary remarkably, then players likely increase or decrease the area of their neighborhood if they are cooperators/defectors. Therefore, cooperative players expand their neighborhood area to seek other cooperators because a single defector around them can be a winner who takes all. On the contrary, defector players likely decrease their neighborhood area to avoid competition with other defectors around them, who have higher scores than them. In this study, we introduced criticality or SOC in two SPD models. In inducing the SOC in SPD, players have to have dual perspectives regarding input information. Moreover, populations can move to the critical point using local information to trigger the dual perspectives.

As mentioned, SOCs describe a key feature of some living systems [24]. Our model may reveal what kind of decision-making processes of the players bring such properties to the system.

In future studies, we will modify and apply these models such that they can be used to study network topologies. This will permit further consideration of the link connection/disconnection and its relation to emerging criticalities.

Supporting information

S1 Fig. A flowchart of the PS model.
(TIF)

S2 Fig. A flowchart of the SLI model.
(TIF)

S3 Fig. Cumulative duration of cooperation for a single player (b = 1.9).
(TIF)

S1 Code. A program code.
(DOCX)

Author Contributions

Conceptualization: Tomoko Sakiyama.

Data curation: Tomoko Sakiyama.

Formal analysis: Tomoko Sakiyama.

Investigation: Tomoko Sakiyama.

Methodology: Ken'ichi Kojo.

Project administration: Tomoko Sakiyama.

Resources: Tomoko Sakiyama.

Software: Tomoko Sakiyama, Ken'ichi Kojo.

Validation: Tomoko Sakiyama.

Visualization: Tomoko Sakiyama.

Writing – original draft: Tomoko Sakiyama.

Writing – review & editing: Tomoko Sakiyama, Ken'ichi Kojo.

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