Detection of statistical asymmetries in non-stationary sign time series: Analysis of foreign exchange data

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Supporting information

S2 Appendix. Normal approximation for the probability distribution of the number of pairs in a stationary Markov process.

The probability $\mathcal{P}$ of a stationary sign binary Markov process $\{\mu, \nu\}$ to generate a sequence with numbers of pairs N_{++}, N_{+-}, N_{-+}, N_{--}, the first symbol s_1 and the last symbol s_w, given length w and parametrized using N_+ and N_{++}, is given by:

$$\mathcal{P} = \binom{N_+ - 1}{N_{++}} \binom{N_- - 1}{N_{--}} \times \frac{1}{\mu+\nu} (1-\mu)^{N_{++}} \mu^{N_+ - N_{++} - \varepsilon(+,+) + \varepsilon(-,-)} \nu^{N_- - N_{--} + \varepsilon(+,+) - \varepsilon(-,-)} (1-\nu)^{N_{--}}, \quad (1)$$

for $0 < N_+ < w$, with $N_- = w - N_+$ and $N_{--} = w - 2N_+ + N_{++} + \varepsilon(+,+) - \varepsilon(-,-)$.

The above expression can be rearranged in order to explicit two terms resembling binomial distributions:

$$\mathcal{P} = \frac{\nu^{1+\varepsilon(+,+)-\varepsilon(-,-)} \mu^{1-\varepsilon(+,+)+\varepsilon(-,-)}}{\mu+\nu} \times \binom{N_+ - 1}{N_{++}} (1-\mu)^{N_{++}} \mu^{N_+ - N_{++}} \binom{N_- - 1}{N_{--}} (1-\nu)^{N_{--}} \nu^{N_- - N_{--}}. \quad (2)$$

Now we use the normal approximation for the binomial distribution:

$$\binom{n}{k} p^k (1-p)^{n-k} \approx \frac{1}{\sqrt{2\pi np(1-p)}} exp\left(-\frac{(k-np)^2}{2np(1-p)} \right). \quad (3)$$

And finally we obtain the approximate expression for the $\mathcal{P}$:

$$\mathcal{P} \approx \frac{\nu^{1+\varepsilon(+,+)-\varepsilon(-,-)} \mu^{1-\varepsilon(+,+)+\varepsilon(-,-)}}{\mu+\nu} \times \frac{1}{\sqrt{2\pi(N_+ - 1)\mu(1-\mu)}} exp\left(-\frac{[N_{++} - (N_+ - 1)(1-\mu)]^2}{2(N_+ - 1)\mu(1-\mu)} \right) \times \frac{1}{\sqrt{2\pi(N_- - 1)\nu(1-\nu)}} exp\left(-\frac{[(N_{--} - (N_- - 1)(1-\nu)]^2}{2(N_- - 1)\nu(1-\nu)} \right). \quad (4)$$