**S1 Appendix: Reaction-Diffusion Model Equations**

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| $\frac{∂P\left(\vec{x},t\right)}{∂t}=\left[\begin{array}{c}plant growth \\at \vec{x} at t\end{array}\right]-\left[\begin{matrix}plant loss \\at \vec{x} at t\end{matrix}\right]\pm \left[plant dispersal\right]=cg\_{max}\frac{W}{W+k\_{1}}P-dP+D\_{P}∆P$  | $$(1a)$$ |
| $$\frac{∂W\left(\vec{x},t\right)}{∂t}=\left[\begin{matrix}infiltration rate\\at \vec{x} at t\end{matrix} \right]-\left[\begin{matrix}plant water uptake\\at \vec{x} at t\end{matrix} \right]-\left[\begin{matrix}evaporation and drainage\\at \vec{x} at t\end{matrix} \right]$$$$\pm \left[water movement\right] =αO\frac{P+k\_{2}W\_{0}}{P+k\_{2}}-g\_{max}\frac{W}{W+k\_{1}}P-r\_{w}W+D\_{W}∆W $$ | $$(1b)$$ |
| $$\frac{∂O\left(\vec{x},t\right)}{∂t}=\left[\begin{matrix}rainfall rate\\at \vec{x} at t\end{matrix} \right]-\left[\begin{matrix}infiltration rate \\at \vec{x} at t\end{matrix}\right]\pm \left[overland flow\right]=R-αO\frac{P+k\_{2}W\_{0}}{P+k\_{2}}+D\_{O}∆O $$ | $$(1c)$$ |

**Table A. Parameters for the RDE model as adapted from Rietkerk (2002).**

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| Parameter | Description  |
| $$D\_{P}=0.1m^{2}/day$$ | Plant dispersal |
| $$D\_{W}=0.1m^{2}/day$$ | Diffusion coefficient for soil water |
| $$D\_{O}=100m^{2}/day$$ | Diffusion coefficient for surface water |
| *O* [*mm* ] | Surface water |
| *P* [$g/m^{2} $] | Plant density |
| $$R [mm/day ]$$ | Rainfall (ranged between 0 to 3) |
| *W* [*mm*] | Soil water |
| $$W\_{0}=0.2$$ | Water infiltration rate in absence of plants |
| $$c=10g/(mm⋅m)$$ | Conversion of water uptake by plants to plant growth |
| $$d=0.25\frac{1}{day}$$ | Specific loss of plant density due to mortality |
| $g\_{max}=0.05mm/(g⋅m\^2⋅day) $  | Maximum specific water uptake |
| $$k\_{1}=5mm$$ | Half-saturation constant of specific plant growth and water uptake |
| $$k\_{2}=5 g/m^{2}$$ | Saturation constant of water infiltration |
| $$r\_{w}=0.2\frac{1}{day} $$ | Specific soil water loss due to evaporation and drainage |
| $$α=0.2\frac{1}{day}$$ | Maximum infiltration rate |