**S1 Appendix. Detailed description of the simulation design**

**Baseline simulation design**

We modeled a multi-input, multi-output production function as shown in equation (1). We assumed that the production technology can be represented by the transformation of three discretionary inputs, and , into a total productive capacity , according to the following linear production function which satisfied constant returns to scale (CRS):

(1)

where is the facility-index. We also assumed that inefficient behavior exists and can be modeled with uniformly distributed efficiency. The efficiency score scaled down the total productive capacity, (), . Three inputs were drawn for a sample of 200 facilities from the following uniform distributions:, , and Total productive capacity was used to produce up to three outputs. In reality, however, it is unlikely that every decision-making unit (DMU) produces all possible outputs. We accounted for such scenarios with a model specification that assumed all facilities produce output, while only a subset of facilities produces output and/or output. Production of these two outputs was defined by a random parameter. Each facility was assigned two random numbers, one for each output of choice. The production rule was defined in equation (2):

(2)

A DMU produced output only if it was assigned a random number smaller than 0.4 (). Similarly, DMU produced output only if it was assigned a random number smaller than 0.6 (). For each positive output, we determined how much of the total output capacity was devoted to its production (output shares). Output shares (were defined exogenously to reflect that demand was given, and that the shares were drawn from a uniform distribution when ,

To ensure that the sum of output shares equaled one, output-specific productive capacity () was calculated as shown in equation (3):

(3)

Last, we assumed that each output’s volume () was dependent on the resources required to produce the output. If a particular output was more resource-intensive to produce, given a DMU’s productive capacity, fewer outputs could be produced by the DMU. The final volume of outputs produced by a DMU was defined in equation (4), where we assumed that output was the most resource-intensive output to produce, followed by and .

(4)

.

**Functional form**

We replicated a Cobb-Douglas and piecewise Cobb-Douglas multiple-output production function assuming that all inputs were drawn from a uniform distribution between 1 and 15, (Table A). Multiple-output productions functions assume that the transformation function is separable, such that outputs are separable from inputs. In this design we modeled the output aggregate as Cobb-Douglas, while the input aggregate was modeled as a Cobb-Douglas and piecewise Cobb-Douglas. To model the output aggregate, we followed the approach used by Collier and Ruggiero [1], which ensured that all outputs followed a uniform distribution. This approach was preferred for computing a Cobb-Douglas multiple-output production function, as other methods have led to high skewedness for one of the outputs [2].

Inputs were drawn from a uniform distribution, after which an input aggregate was constructed based on the form,, with , and . Further, inefficient behavior was modeled as where

To generate outputs, we used three normally distributed random variables,. We defined each output as where was chosen to satisfy the equation. The values of output exponents were specified analogously to the inputs, with, and .

**Table A. Variations in the functional form of the multiple-output production function.**

|  |  |  |
| --- | --- | --- |
| **Functional form** | **Multiple-output production function specification** | **Input coefficients parameterization** |
| Cobb-Douglas |  | , , |
| Piecewise Cobb-Douglas |  | For ≤ 5, ≤ 5, ≤ 5  = 0.2, = 0.5, = 0.3    For 5 < ≤ 10, 5 <≤ 10, 5 <≤ 10  = 0.15, = 0.45, = 0.25  For 10 < ≤ 15, 10 <≤ 15, 10 <≤ 15  = 0.1, = 0.4, = 0.2 |

**Varied measurement error (simulation scenario *g*)**

Tables B and C provide additional detail regarding variations implemented as part of the simulation scenario *g*.

**Table B. Measurement error types.**

|  |  |
| --- | --- |
| **Additive measurement error** | **Multiplicative error** |
| For :  For :  with | For all :  with |

**Table C. Measurement error scenarios.**

|  |  |  |
| --- | --- | --- |
| **Low measurement error** | **High measurement error** | **Mixed measurement error** |
| = 0.02 | = 0.08 | For : = 0.08  For: = 0.02 |

**References**

1. Collier T, Johnson AL, Ruggiero J. Technical efficiency estimation with multiple inputs and multiple outputs using regression analysis. Eur J Oper Res. 2011;208: 153–160. doi:10.1016/j.ejor.2010.08.024

2. Bifulco R, Bretschneider S. Estimating school efficiency: A comparison of methods using simulated data. Econ Educ Rev. 2001;20: 417–429. doi:10.1016/S0272-7757(00)00025-X