

S1 Supplement.

The steady state solution where all living pools are extinct:

$$H_C = 0, \quad D_N = 0, \quad D_C = 0, \quad P_C = 0, \quad N = \frac{N_{ext}}{K_{NL}}.$$

The steady state solution where the herbivore is extinct:

$$H_C = 0, \quad D_N = \frac{N_{ext}}{K_{DL}} - \frac{K_{NL}d_P}{K_{DL}r_P}, \quad D_C = D_N CNP \left(1 + \frac{K_{DL}}{r_{min}} \right), \quad P_C = \frac{D_C r_{min}}{d_P}, \quad N = \frac{d_P}{r_P}.$$

The steady state solution where all pools coexist:

$$\begin{aligned} P_C &= \frac{d_H + r_{2H}}{e_1 r_{1H}}, \quad N = \frac{r_{1H} H_C + d_P}{r_P}, \\ H_C &= \frac{e_1(r_{min} + k_{DL})(r_P N_{ext} - k_{NL}d_P) - \frac{1}{CNP} \frac{r_P}{r_{1H}} d_P k_{DL}(d_H + r_{2H})}{e_1 r_{1H} k_{NL}(r_{min} + k_{DL}) + r_P k_{DL} \left(\frac{(1-e_1)}{CNP} (d_H + r_{2H}) + \frac{e_1}{CNH} d_H \right)}, \\ D_N &= \frac{\frac{d_P(d_H + r_{2H})}{CNP \cdot e_1 \cdot r_{1H}} + \left(\frac{(1-e_1)(d_H + r_{2H})}{e_1 \cdot CNP} + \frac{d_H}{CNH} \right) H_C}{r_{min} + k_{DL}}, \\ D_C &= \left(\frac{d_H}{e_1 r_{min}} + \left(\frac{1}{e_1} - 1 \right) \frac{r_{2H}}{r_{min}} \right) H_C + \frac{d_P(d_H + r_{2H})}{e_1 r_{1H} r_{min}} \end{aligned}$$