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| Weibull 4b  Kind: asymptotic sigmoidal  References: [2,15,27,28] | | | | | | | | | |
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| Extreme Value 2  Kind: asymptotic attenuating  References: [2,7,8] | | | | |
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| Gompertz 2  Kind: asymptotic attenuating  References: [4,9–11] | | | | |
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