## Text S5: Correlated random walks in one dimension

Suppose animals perform correlated random walks in one dimension (*i.e.,* $\Omega = \mathbf{R}$) and let $P_m(x,t)$ be the probability density of an animal being at location $x \in \Omega$ at time $t \geq 0$ (we restrict our attention to one dimension for analytical tractability). The initial value problem that determines the evolution of $P_m$ is given by a telegraph equation [1]:

$$\frac{\partial^2 P_m}{\partial t^2} + \frac{2}{\tau}\frac{\partial P_m}{\partial t} = u^2 \frac{\partial^2 P_m}{\partial x^2}, \qquad P_m(x,0) = \delta(x) \quad \text{and} \quad \frac{\partial P_m}{\partial t}(x,0) = 0 \tag{1}$$

Here, $\tau$ is the characteristic time an animal moves before changing directions and $u$ is its speed. As correlation time ($\tau$) approaches zero, Eq (1) reduces to a diffusion equation in which $D$ equals the limit of $\frac{1}{2}u^2\tau$ as $\tau \to 0$ and $u \to \infty$. The solution of (1) is [2]:

$$P_m(x,t) = \begin{cases} e^{-\frac{t}{\tau}}\{\delta(x-ut) + \delta(x+ut) + \frac{1}{2u\tau}[I_0(z) + \frac{t}{\tau z}I_1(z)]\}, & |x| < ut \\ 0, & |x| > ut \end{cases}$$

where $z = \frac{1}{\tau}\sqrt{t^2 - \frac{x^2}{v^2}}$ and $I_0$ and $I_1$ are the modified Bessel functions of the first kind (there is no known closed form solution to the telegraph equation in two dimensions). Below, we compute the summary statistics of $P_s$.

**Mean of $P_s$:** Multiply Eq (1) by $x$ and integrate over $\Omega$ to get

$$\mu_m''(t) + \frac{2}{\tau}\mu_m'(t) = 0, \quad \mu_m(0) = \mu_m'(0) = 0$$

Here, we have used the fact that $P_m$ and $\partial P_m/\partial x$ both approach zero as $x \to \pm\infty$. It follows from the uniqueness of solutions to differential equations that $\mu_m = 0$, and hence from Eq (1) of Text S1 that $\mu_s = 0$.

**Scale of $P_s$:** To determine $\sigma_s^2$ we first determine $\mu_m^2(t)$ (the second moment of $P_m$, not the square of its first moment). Repeating the same procedure as above but multiplying Eq (1) by $x^2$ instead of $x$ yields

$$(\mu_m^2)''(t) + \frac{2}{\tau}(\mu_m^2)'(t) = 2u^2, \quad \mu_m^{12}(0) = (\mu_m^2)'(0) = 0$$

whose solution is

$$\mu_m^2(t) = \frac{u^2\tau^2}{2}\Big( -1 + \frac{2t}{\tau} + e^{-\frac{2t}{\tau}}\Big) \tag{2}$$

Straightforward calculations involving Eqs (1) and (4) of Text S1, (2), and (3*) lead to

$$\sigma_s^2 = \mu_s^2 = \int_0^\infty \mu_m^2(t)P_r(t)\,dt = \frac{u^2\tau^2}{2}\left(-1 + \frac{2ab}{\tau} + \left(1 + \frac{2b}{\tau}\right)^{-a}\right)$$

where $a$ and $b$ are parameters of gamma distributed retention times ($P_r$) of Eq (3*). In comparing this result with the corresponding one for random motion, and to identify the effects of correlation, it will be convenient to introduce the dimensionless quantities $\omega = \frac{\mu_r}{\tau} = \frac{ab}{\tau}$ and $\xi^2 = \frac{\sigma_r^2}{\mu_r^2} = \frac{1}{a}$. In so doing, we fix $D$ to be a constant and choose $u$ and $\tau$ such that $\frac{1}{2}u^2\tau = D$. With these substitutions,

$$\sigma_s^2(D\tau, \omega, \xi^2) = D\tau\left(-1 + 2\omega + (1 + 2\omega\xi^2)^{-1/\xi^2}\right) \tag{3}$$

**Shape of $P_s$:** To determine $\kappa_s$ we first determine $\mu_m^4(t)$ (the fourth moment of $P_m$). Multiplying Eq (1) by $x^4$ and then integrating over $\Omega$ produces

$$(\mu_m^4)''(t) + \frac{2}{\tau}(\mu_m^4)'(t) = 12u^2\mu_m^2, \quad \mu_m^4(0) = (\mu_m^4)'(0) = 0$$

The solution of this differential equation is

$$\mu_m^4(t) = \frac{3u^4\tau^4}{2}\left(3 + \frac{2t}{\tau}\left(\frac{t}{\tau} - 2\right) - \left(3 + \frac{2t}{\tau}\right)e^{-\frac{2t}{\tau}}\right) \tag{4}$$

Straightforward calculations involving Eq (4) of Text S1, (4), and (3*) lead to

$$\mu_s^4 = \frac{3u^4\tau^4}{2}\left(3 - \frac{4ab}{\tau} + \frac{2a(a+1)b^2}{\tau^2} - \left(3 + \frac{2(a+3)b}{\tau}\right)\left(1 + \frac{2b}{\tau}\right)^{-1-a}\right)$$

Utilizing the dimensionless parameters,

$$\mu_s^4 = 6(D\tau)^2\left(3 - 4\omega + 2\omega^2(1+\xi^2) - \left(3 + 2\omega(1+3\xi^2)\right)(1+2\omega\xi^2)^{-1-\frac{1}{\xi^2}}\right) \tag{5}$$

Eq (1) of Text S1, (3), and (5) and the relation $\mu_s = 0$ together imply that

$$\kappa_s(\omega,\xi^2) = \frac{\mu_s^4}{\sigma_s^4} - 3 = 6\left\{\frac{3 - 4\omega + 2\omega^2(1+\xi^2) - \left(3 + 2\omega(1+3\xi^2)\right)(1+2\omega\xi^2)^{-1-\frac{1}{\xi^2}}}{\left(-1 + 2\omega + (1+2\omega\xi^2)^{-1/\xi^2}\right)^2}\right\} - 3$$

Although the shape ($\kappa_s$) of $P_s$ does not depend directly on $D = \frac{1}{2}u^2\tau$ (as was also the case with random motion), it does depend on correlation time $\tau$ via $\omega$. See the panel in Fig S(1)). Also note that, unlike in previous random walk models where summary statistics of $P_s$ were general with respect to $P_r$, the expressions for the scale and shape above depend on explicit form of $P_r$ to be a gamma distribution.

**Form of $P_s$:** Although we are unable to obtain a closed form for the seed dispersal kernel ($P_s$), we can find it using numerical integration. See Fig S(2).

# References


[1] Othmer HG, Dunbar SR, Alt W (1988) Models of dispersal in biological systems. J Math Biol 26: 263–298.

[2] Morse P, Feshbach H, Hill E (1954) In: Methods of theoretical physics, volume 22. p. 410.

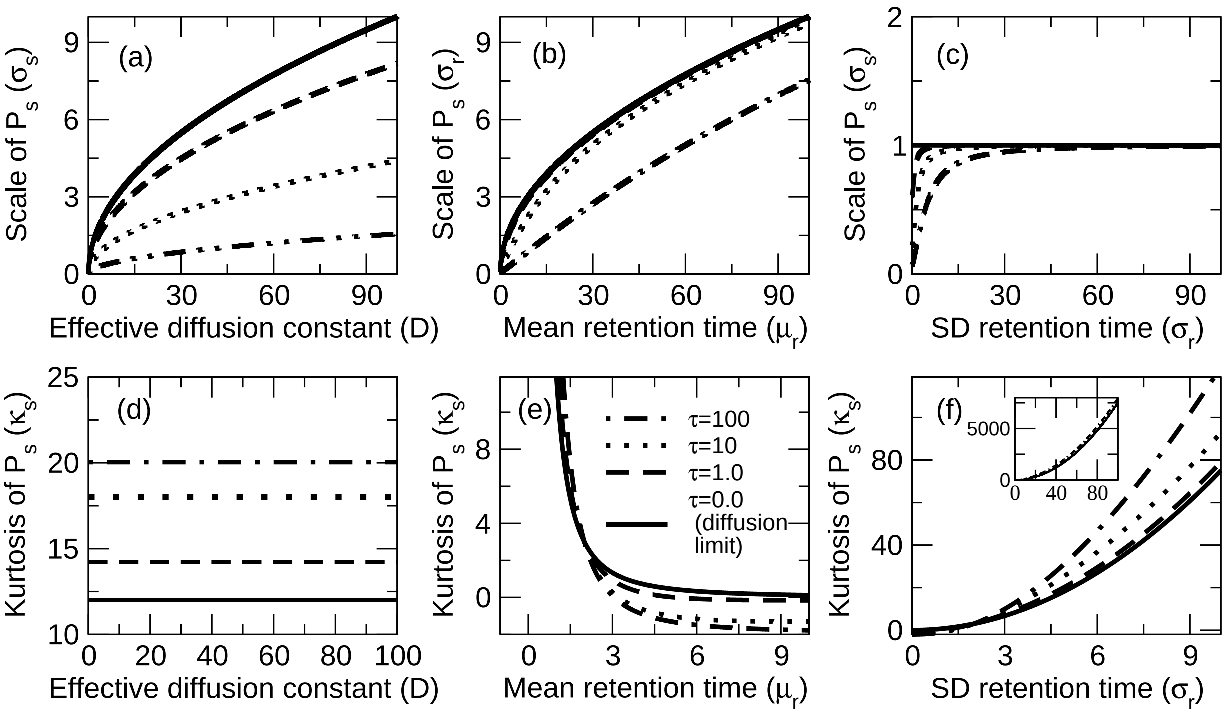


**Figure S1**: Scale ($\sigma_s$) and kurtosis ($\kappa_s$) of the seed dispersal kernel for animals that move according to correlated random walks (CRW). Scale as a function of: (a) The effective diffusion constant, $D = \frac{1}{2}u^2\tau$; Parameters: $\mu_r = 0.5$ and $\sigma_r = 1.0$. (b) Mean seed retention time, $\mu_r$; Parameters: $D = 0.5$ and $\sigma_r = 1.0$. (c) Standard deviation (SD) of seed retention time, $\sigma_r$; Parameters: $D = 1.0$ and $\mu_r = 1.0$. Excess kurtosis as a function of: (d) The effective diffusion constant, $D$; Parameters: $\mu_r = 1.0$ and $\sigma_r = 2.0$. (e) Mean seed retention time, $\mu_r$; Parameters: $D = 1.0$ and $\sigma_r = 2.0$. (f) Standard deviation (SD) of seed retention time, $\sigma_r$; Parameters: $D = 1.0$ and $\mu_r = 2.0$. Inset in (f) shows that the trend of $\kappa_s$ over large scales of $\sigma_r$ is qualitatively unaffected by the choice of the correlation time scale ($\tau$).

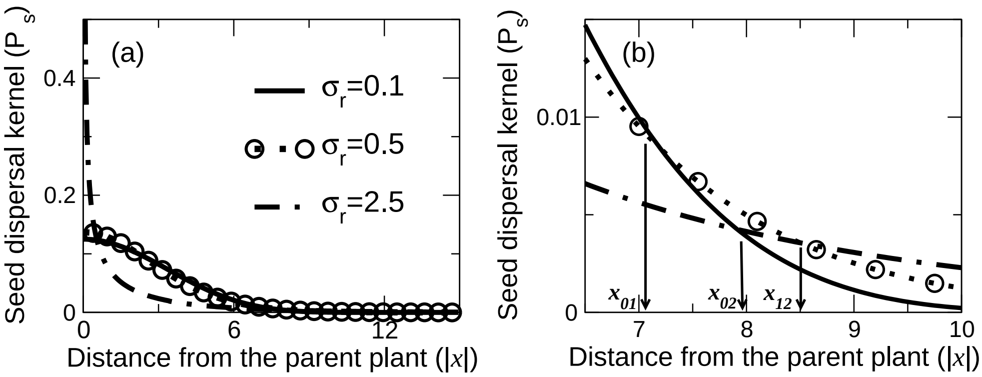


**Figure S2**: Correlated random walk in one dimension. (a) The seed dispersal kernel as a function of distance from the source tree ($|x|$) and standard deviation in seed retention time ($\sigma_r$). (b) The seed dispersal kernel at larger distances. The larger the $\sigma_r$, the more frequent the LDD events. Note that $x_{01} < x_{02} < x_{12}$ ($x_{01} \approx 7.1, x_{02} \approx 8.0, x_{12} \approx 8.5$). Parameters: $\tau = 1.0$ and $\mu_r = 10.0$.