Supporting Information 1

Example of a symmetric matrix A with negative values and multiple largest eigenvalues so that the first principal component is not unique. Note that A is positive semidefinite and symmetric (as are covariance matrices). The columns in matrix E are the eigenvectors of A with eigenvalues $\lambda_1 = 1.2, \ \lambda_2 = 1.2, \ \lambda_3 = 0.6$.

$$
A = \begin{pmatrix} 1.000 & 0.200 & 0.200 \\ 0.200 & 1.000 & -0.200 \\ 0.200 & -0.200 & 1.000 \end{pmatrix}, \quad E = \begin{pmatrix} 0.816 & 0.000 & 0.577 \\ 0.408 & -0.707 & -0.577 \\ 0.408 & 0.707 & -0.577 \end{pmatrix}
$$