

Supplementary material

1 Formulas

The overall likelihood can be expressed as:

$$P(Z|R, E, \Theta) = \prod_{i=1}^N \prod_{j=1}^M \left[\left(\frac{1}{\sqrt{2\pi}\sigma_{1j}} e^{-\frac{z_{ij}^2}{2\sigma_{1j}^2}} \right)^{r_i \cdot e_j} \cdot \left(\frac{1}{\sqrt{2\pi}\sigma_{0j}} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j} \right]$$

$$p(\sigma_{1j}^2) \sim \text{Inverse Gamma}(\alpha_{1j}, \beta_{1j})$$

$$p(\sigma_{0j}^2) \sim \text{Inverse Gamma}(\alpha_{0j}, \beta_{0j})$$

$$p(\mu_{0j}|\sigma_{0j}^2) \sim N(\tau_{0j}, \sigma_{0j}^2)$$

$$p(r_i) \sim \text{Bernoulli}(\pi_{r_i})$$

$$p(e_j) \sim \text{Bernoulli}(\pi_{e_j})$$

$$i = 1, \dots, N \quad j = 1, \dots, M$$

$$\begin{aligned} p(R, E, \Theta|Z) &\propto p(R, E, \Theta, Z) \\ &= p(Z|R, E, \Theta) \prod_{i=1}^N p(r_i) \prod_{j=1}^M [p(\mu_{0j}|\sigma_{0j}^2)p(\sigma_{0j}^2)p(\sigma_{1j}^2)p(e_j)] \end{aligned}$$

The marginal posterior distribution is:

$$\begin{aligned} p(R, E|Z) &= \int p(R, E, \Theta|Z) d\Theta \\ &\propto \int p(Z|R, E, \Theta) \prod_{i=1}^N p(r_i) \prod_{j=1}^M [p(\mu_{0j}|\sigma_{0j}^2)p(\sigma_{0j}^2)p(\sigma_{1j}^2)p(e_j)] d\Theta \\ &\propto \int \prod_{i=1}^N \prod_{j=1}^M \left[\left(\sigma_{1j}^{-1} e^{-\frac{z_{ij}^2}{2\sigma_{1j}^2}} \right)^{r_i \cdot e_j} \cdot \left(\sigma_{0j}^{-1} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j} \right] \prod_{i=1}^N \pi_{r_i}^{r_i} (1 - \pi_{r_i})^{1-r_i} \\ &\quad \prod_{j=1}^M \left[\sigma_{0j}^{-1} e^{-\frac{(\mu_{0j}-\tau_{0j})^2}{2\sigma_{0j}^2}} \cdot (\sigma_{0j}^2)^{-(\alpha_{0j}+1)} e^{-\frac{\beta_{0j}}{\sigma_{0j}^2}} (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} \pi_{e_j}^{e_j} (1 - \pi_{e_j})^{1-e_j} \right] d\Theta \end{aligned}$$

Divide the integral into two part: I_1 and I_2 :

$$\begin{aligned}
I_1 &= \int \prod_{i=1}^N \prod_{j=1}^M \left(\sigma_{1j}^{-1} \exp \left\{ -\frac{z_{ij}^2}{2\sigma_{1j}^2} \right\} \right)^{r_i \cdot e_j} \cdot \prod_{j=1}^M (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int \prod_{i=1}^N \left(\sigma_{1j}^{-1} \exp \left\{ -\frac{z_{ij}^2}{2\sigma_{1j}^2} \right\} \right)^{r_i \cdot e_j} \cdot (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int \sigma_{1j}^{-e_j \sum_{i=1}^N r_i} \exp \left\{ -\frac{e_j}{2\sigma_{1j}^2} \sum_{i=1}^N z_{ij}^2 r_i \right\} \cdot (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int (\sigma_{1j}^2)^{-(\alpha_{1j}+1+\frac{e_j}{2} \sum_{i=1}^N r_i)} \exp \left\{ -\frac{1}{\sigma_{1j}^2} \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i \right) \right\} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \frac{\Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i \right)}{\left[\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i \right]^{\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i}}
\end{aligned}$$

Let $f_{ij} = 1 - r_i e_j$, $A_j = \sum_{i=1}^N z_{ij}^2 f_{ij}$, $B_j = \sum_{i=1}^N z_{ij} f_{ij}$, $C_j = \sum_{i=1}^N f_{ij}$.

$$\begin{aligned}
I_2 &= \int \int \prod_{i=1}^N \prod_{j=1}^M \left(\sigma_{0j}^{-1} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j} \sigma_{0j}^{-1} e^{-\frac{(\mu_{0j}-\tau_{0j})^2}{2\sigma_{0j}^2}} (\sigma_{0j}^2)^{-(\alpha_{0j}+1)} e^{-\frac{\beta_{0j}}{\sigma_{0j}^2}} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \int (\sigma_{0j}^2)^{-\frac{C_j+3}{2}-\alpha_{0j}} \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left(\sum_{i=1}^N (z_{ij}-\mu_{0j})^2 f_{ij} + (\mu_{0j}-\tau_{0j})^2 + 2\beta_{0j} \right) \right\} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \int (\sigma_{0j}^2)^{-\frac{C_j+3}{2}-\alpha_{0j}} \\
&\quad \exp \left\{ -\frac{1}{2\sigma_{0j}^2} [A_j + \tau_{0j}^2 + 2\beta_{0j} - 2\mu_{0j}(B_j + \tau_{0j}) + \mu_{0j}^2(1+C_j)] \right\} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \left\{ \int (\sigma_{0j}^2)^{-\frac{C_j+3}{2}-\alpha_{0j}} \exp \left[-\frac{1+C_j}{2\sigma_{0j}^2} \left(\mu_{0j} - \frac{B_j + \tau_{0j}}{1+C_j} \right)^2 \right] d\mu_{0j} \right\} \\
&\quad \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left[A_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(B_j + \tau_{0j})^2}{1+C_j} \right] \right\} d\sigma_{0j}^2 \\
&\propto \prod_{j=1}^M (1+C_j)^{-1/2} \int (\sigma_{0j}^2)^{-\frac{C_j+2}{2}-\alpha_{0j}} \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left[A_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(B_j + \tau_{0j})^2}{1+C_j} \right] \right\} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \frac{(1+C_j)^{-1/2} \Gamma(\alpha_{0j} + C_j/2) 2^{\alpha_{0j} + C_j/2}}{\left[A_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(B_j + \tau_{0j})^2}{1+C_j} \right]^{\alpha_{0j} + C_j/2}}
\end{aligned}$$

Then the log transformed marginal posterior distribution is:

$$\begin{aligned}
\log p(R, E|Z) &= \log I_1 + \log I_2 + \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} + \text{constant} \\
&= \sum_{j=1}^M \log \Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i \right) - \sum_{j=1}^M \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i \right) \log \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i \right) \\
&\quad - \sum_{j=1}^M \frac{1}{2} \log(1 + C_j) + \sum_{j=1}^M \log \Gamma(\alpha_{0j} + C_j/2) \\
&\quad - \sum_{j=1}^M (\alpha_{0j} + C_j/2) \log \left[A_j/2 + \tau_{0j}^2/2 + \beta_{0j} - \frac{(B_j + \tau_{0j})^2}{2(1 + C_j)} \right] \\
&\quad + \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} + \text{constant} \\
&= \sum_{j=1}^M \log \Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i \right) - \sum_{j=1}^M \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i \right) \log \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i \right) \\
&\quad - \sum_{j=1}^M \frac{1}{2} \log \left[1 + \sum_{i=1}^N (1 - r_i e_j) \right] + \sum_{j=1}^M \log \Gamma \left(\alpha_{0j} + \sum_{i=1}^N \frac{1 - r_i e_j}{2} \right) \\
&\quad - \sum_{j=1}^M \left(\alpha_{0j} + \sum_{i=1}^N \frac{1 - r_i e_j}{2} \right) \\
&\quad \times \log \left\{ \frac{\tau_{0j}^2}{2} + \beta_{0j} + \sum_{i=1}^N \frac{z_{ij}^2 (1 - r_i e_j)}{2} - \frac{\left(\sum_{i=1}^N z_{ij} (1 - r_i e_j) + \tau_{0j} \right)^2}{2 \left[1 + \sum_{i=1}^N (1 - r_i e_j) \right]} \right\} \\
&\quad + \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} + \text{constant} \tag{S1}
\end{aligned}$$

Allowing cell level noise, the overall likelihood is modified as follows:

$$\begin{aligned}
P(Z|R, E, C, \Theta) &= \prod_{i=1}^N \prod_{j=1}^M \left[\left(\frac{1}{\sqrt{2\pi}\sigma_{1j}} e^{-\frac{z_{ij}^2}{2\sigma_{1j}^2}} \right)^{r_i \cdot e_j \cdot c_{ij}} \cdot \left(\frac{1}{\sqrt{2\pi}\sigma_{0j}} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j \cdot c_{ij}} \right] \\
p(\sigma_{1j}^2) &\sim \text{Inverse Gamma}(\alpha_{1j}, \beta_{1j}) \\
p(\sigma_{0j}^2) &\sim \text{Inverse Gamma}(\alpha_{0j}, \beta_{0j}) \\
p(\mu_{0j}|\sigma_{0j}^2) &\sim N(\tau_{0j}, \sigma_{0j}^2) \\
p(r_i) &\sim \text{Bernoulli}(\pi_{r_i}) \\
p(e_j) &\sim \text{Bernoulli}(\pi_{e_j}) \\
p(c_{ij}) &\sim \text{Bernoulli}(\pi_{c_{ij}}) \\
i &= 1, \dots, N \quad j = 1, \dots, M
\end{aligned}$$

$$\begin{aligned}
p(R, E, C, \Theta|Z) &\propto p(R, E, C, \Theta, Z) \\
&= p(Z|R, E, C, \Theta) \prod_{i=1}^N p(r_i) \prod_{j=1}^M [p(\mu_{0j}|\sigma_{0j}^2)p(\sigma_{0j}^2)p(\sigma_{1j}^2)p(e_j)] \prod_{i=1}^N \prod_{j=1}^M p(c_{ij})
\end{aligned}$$

The marginal posterior distribution is:

$$\begin{aligned}
p(R, E, C|Z) &= \int p(R, E, C, \Theta|Z) d\Theta \\
&\propto \int p(Z|R, E, C, \Theta) \prod_{j=1}^M p(\mu_{0j}|\sigma_{0j}^2) \prod_{j=1}^M p(\sigma_{0j}^2) \prod_{j=1}^M p(\sigma_{1j}^2) \\
&\quad \cdot \prod_{i=1}^N p(r_i) \prod_{j=1}^M p(e_j) \prod_{i=1}^N \prod_{j=1}^M p(c_{ij}) d\Theta \\
&\propto \int \prod_{i=1}^N \prod_{j=1}^M \left[\left(\sigma_{1j}^{-1} e^{-\frac{z_{ij}^2}{2\sigma_{1j}^2}} \right)^{r_i \cdot e_j \cdot c_{ij}} \cdot \left(\sigma_{0j}^{-1} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j \cdot c_{ij}} \right] \\
&\quad \cdot \prod_{j=1}^M \sigma_{0j}^{-1} e^{-\frac{(\mu_{0j}-\tau_{0j})^2}{2\sigma_{0j}^2}} \cdot \prod_{j=1}^M (\sigma_{0j}^2)^{-(\alpha_{0j}+1)} e^{-\frac{\beta_{0j}}{\sigma_{0j}^2}} \cdot \prod_{j=1}^M (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} \\
&\quad \prod_{i=1}^N \pi_{r_i}^{r_i} (1-\pi_{r_i})^{1-r_i} \prod_{j=1}^M \pi_{e_j}^{e_j} (1-\pi_{e_j})^{1-e_j} \prod_{i=1}^N \prod_{j=1}^M \pi_{c_{ij}}^{c_{ij}} (1-\pi_{c_{ij}})^{1-c_{ij}} d\Theta
\end{aligned}$$

Divide the integral into two part: I_1 and I_2 :

$$\begin{aligned}
I_1 &= \int \prod_{i=1}^N \prod_{j=1}^M \left(\sigma_{1j}^{-1} \exp \left\{ -\frac{z_{ij}^2}{2\sigma_{1j}^2} \right\} \right)^{r_i \cdot e_j \cdot c_{ij}} \cdot \prod_{j=1}^M (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int \prod_{i=1}^N \left(\sigma_{1j}^{-1} \exp \left\{ -\frac{z_{ij}^2}{2\sigma_{1j}^2} \right\} \right)^{r_i \cdot e_j \cdot c_{ij}} \cdot (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int \sigma_{1j}^{-e_j \sum_{i=1}^N r_i c_{ij}} \exp \left\{ -\frac{e_j}{2\sigma_{1j}^2} \sum_{i=1}^N z_{ij}^2 r_i c_{ij} \right\} \cdot (\sigma_{1j}^2)^{-(\alpha_{1j}+1)} e^{-\frac{\beta_{1j}}{\sigma_{1j}^2}} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \int (\sigma_{1j}^2)^{-(\alpha_{1j}+1+\frac{e_j}{2} \sum_{i=1}^N r_i c_{ij})} \exp \left\{ -\frac{1}{\sigma_{1j}^2} \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i c_{ij} \right) \right\} d\sigma_{1j}^2 \\
&= \prod_{j=1}^M \frac{\Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij} \right)}{\left[\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i c_{ij} \right]^{\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij}}}
\end{aligned}$$

Let $g_{ij} = 1 - r_i e_j c_{ij}$, $D_j = \sum_{i=1}^N z_{ij}^2 g_{ij}$, $E_j = \sum_{i=1}^N z_{ij} g_{ij}$, $F_j = \sum_{i=1}^N g_{ij}$.

$$\begin{aligned}
I_2 &= \int \int \prod_{i=1}^N \prod_{j=1}^M \left(\sigma_{0j}^{-1} e^{-\frac{(z_{ij}-\mu_{0j})^2}{2\sigma_{0j}^2}} \right)^{1-r_i \cdot e_j \cdot c_{ij}} \sigma_{0j}^{-1} e^{-\frac{(\mu_{0j}-\tau_{0j})^2}{2\sigma_{0j}^2}} (\sigma_{0j}^2)^{-(\alpha_{0j}+1)} e^{-\frac{\beta_{0j}}{\sigma_{0j}^2}} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \int (\sigma_{0j}^2)^{-\frac{F_j+3}{2}-\alpha_{0j}} \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left(\sum_{i=1}^N (z_{ij} - \mu_{0j})^2 g_{ij} + (\mu_{0j} - \tau_{0j})^2 + 2\beta_{0j} \right) \right\} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \int (\sigma_{0j}^2)^{-\frac{F_j+3}{2}-\alpha_{0j}} \\
&\quad \exp \left\{ -\frac{1}{2\sigma_{0j}^2} [D_j + \tau_{0j}^2 + 2\beta_{0j} - 2\mu_{0j}(E_j + \tau_{0j}) + \mu_{0j}^2(1 + F_j)] \right\} d\mu_{0j} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \int \left\{ \int (\sigma_{0j}^2)^{-\frac{F_j+3}{2}-\alpha_{0j}} \exp \left[-\frac{1+F_j}{2\sigma_{0j}^2} \left(\mu_{0j} - \frac{E_j + \tau_{0j}}{1+F_j} \right)^2 \right] d\mu_{0j} \right\} \\
&\quad \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left[D_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(E_j + \tau_{0j})^2}{1+F_j} \right] \right\} d\sigma_{0j}^2 \\
&\propto \prod_{j=1}^M (1+F_j)^{-1/2} \int (\sigma_{0j}^2)^{-\frac{F_j+2}{2}-\alpha_{0j}} \exp \left\{ -\frac{1}{2\sigma_{0j}^2} \left[D_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(E_j + \tau_{0j})^2}{1+F_j} \right] \right\} d\sigma_{0j}^2 \\
&= \prod_{j=1}^M \frac{(1+F_j)^{-1/2} \Gamma(\alpha_{0j} + F_j/2) 2^{\alpha_{0j} + F_j/2}}{\left[D_j + \tau_{0j}^2 + 2\beta_{0j} - \frac{(E_j + \tau_{0j})^2}{1+F_j} \right]^{\alpha_{0j} + F_j/2}}
\end{aligned}$$

Then the log transformed marginal posterior distribution is:

$$\begin{aligned}
\log p(R, E, C|Z) &= \log I_1 + \log I_2 + \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} \\
&+ \sum_{i=1}^N \sum_{j=1}^M c_{ij} \log \frac{\pi_{c_{ij}}}{1 - \pi_{c_{ij}}} + \text{constant} \\
&= \sum_{j=1}^M \log \Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij} \right) \\
&- \sum_{j=1}^M \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij} \right) \log \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i c_{ij} \right) \\
&- \sum_{j=1}^M \frac{1}{2} \log(1 + F_j) + \sum_{j=1}^M \log \Gamma(\alpha_{0j} + F_j/2) \\
&- \sum_{j=1}^M (\alpha_{0j} + F_j/2) \log \left[D_j/2 + \tau_{0j}^2/2 + \beta_{0j} - \frac{(E_j + \tau_{0j})^2}{2(1 + F_j)} \right] \\
&+ \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} + \sum_{i=1}^N \sum_{j=1}^M c_{ij} \log \frac{\pi_{c_{ij}}}{1 - \pi_{c_{ij}}} + \text{constant} \\
&= \sum_{j=1}^M \log \Gamma \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij} \right) \\
&- \sum_{j=1}^M \left(\alpha_{1j} + \frac{e_j}{2} \sum_{i=1}^N r_i c_{ij} \right) \log \left(\beta_{1j} + \frac{e_j}{2} \sum_{i=1}^N z_{ij}^2 r_i c_{ij} \right) \\
&- \sum_{j=1}^M \frac{1}{2} \log \left[1 + \sum_{i=1}^N (1 - r_i e_j c_{ij}) \right] + \sum_{j=1}^M \log \Gamma \left(\alpha_{0j} + \sum_{i=1}^N \frac{1 - r_i e_j c_{ij}}{2} \right) \\
&- \sum_{j=1}^M \left(\alpha_{0j} + \sum_{i=1}^N \frac{1 - r_i e_j c_{ij}}{2} \right) \\
&\times \log \left\{ \frac{\tau_{0j}^2}{2} + \beta_{0j} + \sum_{i=1}^N \frac{z_{ij}^2 (1 - r_i e_j c_{ij})}{2} - \frac{\left(\sum_{i=1}^N z_{ij} (1 - r_i e_j c_{ij}) + \tau_{0j} \right)^2}{2 \left[1 + \sum_{i=1}^N (1 - r_i e_j c_{ij}) \right]} \right\} \\
&+ \sum_{i=1}^N r_i \log \frac{\pi_{r_i}}{1 - \pi_{r_i}} + \sum_{j=1}^M e_j \log \frac{\pi_{e_j}}{1 - \pi_{e_j}} + \sum_{i=1}^N \sum_{j=1}^M c_{ij} \log \frac{\pi_{c_{ij}}}{1 - \pi_{c_{ij}}} \\
&+ \text{constant} \tag{S2}
\end{aligned}$$

2 Prior Specification

We adopt standard conjugate priors for the model parameters. The prior parameters τ_{0j} , α_{0j} , β_{0j} , α_{1j} and β_{1j} are specified as follows:

First we estimate the linear transformation factor \hat{a}_i using least square without intercept. To keep our model simple and avoid over-fitting, the linear transformation factor \hat{a}_i is restricted to be significantly different from 0 and 1. The difference z_{ij} is defined as $y_{ij} - \hat{a}_i \cdot x_j$.

We calculate the sample mean $\hat{\mu}_{0j}$ and sample variance \hat{v}_{0j}^2 of the difference z_{ij} in each column, and set $\tau_{0j} = \hat{\mu}_{0j}$. Next we calculate the sample variance $\hat{\sigma}_0^2$ of column variance \hat{v}_{0j}^2 , and choose α_{0j} , β_{0j} , such that the prior distribution Inverse Gamma(α_{0j}, β_{0j}) has mean \hat{v}_{0j} and variance $\hat{\sigma}_0^2$.

To choose α_{1j} , β_{1j} in each column, we select z_{ij} with absolute value less than $\hat{\sigma}_{0j}/2$ in that column and calculate their sample variance \hat{v}_{1j}^2 . Let $\hat{\sigma}_1^2$ be the variance of all \hat{v}_{1j}^2 s, and choose α_{1j} and β_{1j} such that the prior distribution Inverse Gamma(α_{1j}, β_{1j}) has mean \hat{v}_{1j} and variance $\hat{\sigma}_1^2$.

We use non-informative priors for row indicator r_i , column indicator e_j and cell-level indicator c_{ij} , i.e., $\pi_{r_i} = \pi_{e_j} = \pi_{c_{ij}} = 1/2$.