Supplementary S1 Material to:

“Thalassemia in the United Arab Emirates: Why It Can Be Prevented but Not Eradicated”

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I. Mathematical Model

The variables and the parameters are given as follows:

Variables

(v1) \( G_M \) and \( G_F \) are boys and girls (children) under the age of twenty;

(v2) \( S_M \) and \( S_F \) are the single male and female classes, and \( C_M \) and \( C_F \) are the single male and female carrier classes. These classes are populations at or over the age of twenty who can be called young adults;

(v3) \( S_A^M \) and \( S_A^F \) are the marriageable single male and female classes, \( C_A^M \) and \( C_A^F \) are the marriageable single carrier male and female classes;

(v4) \( U \) is the married (or united) class, and \( T_M \) and \( T_F \) are the male and female thalassemia major classes;

(v5) \( S_K \) and \( S_K^{AE} \) are uneducated/educated \( K \) populations, respectively for \( K = M \) or \( F \);

(v6) \( S_M^{AE}, S_F^{AE}, C_M^{AE}, \) and \( C_F^{AE} \) are educated marriageable male and female singles and carrier populations;

(v7) \( N_M = S_M + S_M^E + S_M^{AE} + C_M^E + C_M^{AE} \) and \( N_F = S_F + S_F^E + S_F^{AE} + C_F^E + C_F^{AE} \) are denoted as the total adult male and female populations, respectively;

Parameters

(p1) \( \alpha_M \) and \( \alpha_F \) are marriage rates of male and female;
(p2) $\alpha_s^M$ and $\alpha_s^F$ are the premarital screening rates of single male and female;

(p3) $\eta_T^M$ and $\eta_T^F$ are the rates of being diagnosed as thalassemia major of male and female, respectively;

(p4) $\eta_{C}^M$ and $\eta_{C}^F$ are the rates of being identified as thalassemia carrier of male and female, respectively;

(p5) $d_M$ and $d_F$ are the natural death rates of male and female, and $d_T$ is thalassemia induced death rate;

(p6) $b_M$ and $b_F$ are the birth rates of boys and girls, respectively;

(p7) $\gamma_M$ and $\gamma_F$ are the proportions of children becoming young adults;

(p8) $\nu_M$ and $\nu_F$ are the marriage reconsideration rates of uneducated male and female carrier populations;

(p9) $\varepsilon$ is a proportion of educating marriageable single populations;

(p10) $\tilde{\nu}_M$ and $\tilde{\nu}_F$ are marriage reconsideration rates of educated male and female carrier populations;

(p11) $d_M^G$ and $d_F^G$ are child mortality rates.

Then, the full scope of the mathematical model for the thalassemia dynamics with the premarital and education factor is given by
\[ \frac{dG_M}{dt} = b_M U - \zeta_T G_M - (1 - \xi_T) \gamma_M G_M - d_M G_M \]  
\[ \frac{dG_F}{dt} = b_F U - \zeta_T G_F - (1 - \xi_T) \gamma_F G_F - d_F G_F \]  
\[ \frac{dT_M}{dt} = \zeta_T G_M - \left( \frac{\varepsilon (1 - \xi_T) \gamma_M G_M}{\gamma_M G_M + \gamma_F G_F} \right) T_M - d_T T_M \]  
\[ \frac{dT_F}{dt} = \zeta_T G_F - \left( \frac{\varepsilon (1 - \xi_T) \gamma_F G_F}{\gamma_M G_M + \gamma_F G_F} \right) T_F - d_T T_F \]  
\[ \frac{dS_M}{dt} = (1 - \varepsilon)(1 - \xi_T) \gamma_M G_M - \alpha_s S_M - d_M S_M \]  
\[ \frac{dS_E}{dt} = \varepsilon (1 - \xi_T) \gamma_M G_M - \alpha_s S_E - d_M S_E \]  
\[ \frac{dS_{AE}}{dt} = (1 - \eta_C) \alpha_s S_{AE} - \alpha_M S_{AE} \left( \frac{S_{AE} + C_{AE} + S_A + C_A}{N_A} \right) - d_M S_{AE} \]  
\[ \frac{dC_M}{dt} = \eta_C \alpha_s S_M - \alpha_M C_M \left( \frac{S_{AE} + C_{AE} + S_A + C_A}{N_A} \right) \]  
\[ + \tilde{\nu}_M \alpha_M C_M \left( \frac{C_A + C_{AE}}{N_A} \right) - d_M C_M \]  
\[ \frac{dS_A}{dt} = (1 - \eta_C) \alpha_s S_M - \alpha_M S_A \left( \frac{S_{AE} + C_{AE} + S_A + C_A}{N_A} \right) - d_M S_A \]  
\[ \frac{dC_A}{dt} = \eta_C \alpha_s S_M - \alpha_M C_A \left( \frac{S_{AE} + C_{AE} + S_A + C_A}{N_A} \right) \]  
\[ + \nu_M \alpha_M C_A \left( \frac{C_A + C_{AE}}{N_A} \right) - d_M C_A \]
\[
\frac{dS_F}{dt} = (1 - \varepsilon)(1 - \zeta \eta_F^E)\gamma_F G_F - \alpha_s^F S_F - d_FS_F
\] (13)

\[
\frac{dS_E}{dt} = \varepsilon(1 - \zeta \eta_F^E)\gamma_F G_F - \alpha_s^F S_F - d_FS_E
\] (14)

\[
\frac{dS^AE_F}{dt} = (1 - \eta_C^F)\alpha_s^F S_F - \alpha_F S^AE_F \left(\frac{S^AE_M + C^AE_M + S^A_M + C^A_M}{N^A}\right) - d_FS^AE_F
\] (15)

\[
\frac{dC^AE_F}{dt} = \eta_C^F \alpha_s^F S_F - \alpha_F C^AE_F \left(\frac{S^AE_M + C^AE_M + S^A_M + C^A_M}{N^A}\right) + \nu_F \alpha_F C^AE_F \left(\frac{C^A_M + C^AE_M}{N^A}\right) - d_F C^AE_F
\] (16)

\[
\frac{dS^A_F}{dt} = (1 - \eta_C^F)\alpha_s^F S_F - \alpha_F S^A_F \left(\frac{S^AE_M + C^AE_M + S^A_M + C^A_M}{N^A}\right) - d_F S^A_F
\] (18)

\[
\frac{dC^A_F}{dt} = \eta_C^F \alpha_s^F S_F - \alpha_F C^A_F \left(\frac{S^AE_M + C^AE_M + S^A_M + C^A_M}{N^A}\right) + \nu_F \alpha_F C^A_F \left(\frac{C^A_M + C^AE_M}{N^A}\right) - d_F C^A_F
\] (19)

\[
\frac{dU}{dt} = \alpha_M(S^AE_M + C^AE_M + S^A_M + C^A_M) \left(\frac{S^AE_F + C^AE_F + S^A_F + C^A_F}{N^A}\right) + \alpha_F(S^AE_F + C^AE_F + S^A_F + C^A_F) \left(\frac{S^AE_M + C^AE_M + S^A_M + C^A_M}{N^A}\right)
\] (21)

\[
-\nu_M \alpha_M C^AB_F \left(\frac{C^A_F + C^AE_F}{N^A}\right) - \nu_M \alpha_M C^A_M \left(\frac{C^A_F + C^AE_F}{N^A}\right) - \frac{1}{2}(d_M + d_F)U,
\] (22)

where

\[
\zeta = \frac{1}{U} \left(\alpha_M(1 - \nu_M)C^AE_M \left(\frac{C^AE_F + C^A_F}{N^A}\right) + \alpha_M(1 - \nu_M)C^A_M \left(\frac{C^AE_F + C^A_F}{N^A}\right) + \alpha_F(1 - \nu_F)C^AE_F \left(\frac{C^AE_M + C^A_M}{N^A}\right) + \alpha_F(1 - \nu_F)C^A_F \left(\frac{C^AE_M + C^A_M}{N^A}\right)\right)
\] (23)

is the proportion of carrier-carrier marriages without marriage reconsideration even with the education and premarital screening.
We provide the details of the model equations in (1) to (24) as follows: Note that all female population will have the similar features as the male population presented here, hence we omit the explanation about the female classes.

- **Children Classes \((G_M \text{ and } G_F)\)**

  In Eqs. (1) and (2), \(b_M U \text{ and } b_F U\) are birth of boys and girls, \(\zeta M_T G_M\) and \(\zeta F_T G_F\) are diagnosed thalassemia populations proportional to the carrier-carrier marriages. \((1 - \zeta M_T) \gamma M_G M\) and \((1 - \zeta F_T) \gamma F_G F\) are non thalassemia children who become young adults at the age of twenty years old, and \(d_M G_M\) and \(d_F G_F\) are death of the children populations.

- **Thalassemia Classes \((T_M \text{ and } T_F)\)**

  In Eqs. (3) and (4), \(\zeta M_T G_M\) and \(\zeta F_T G_F\) are diagnosed thalassemia populations and \(d_T T_M\) is the death of thalassemia population due to the illness. \(- \left( \varepsilon (1 - \zeta M_T) \gamma M_G M \gamma M_G M + \gamma F_G F \right) T_M\) and \(- \left( \varepsilon (1 - \zeta F_T) \gamma F_G F \right) T_F\) are the reduction of thalassemia populations due to the education of non-thalassemia young adults on thalassemia.

- **Single non-educated and educated male classes \((S_M \text{ and } S^E_M)\)**

  In Eqs. (5) and (6), \((1 - \varepsilon) (1 - \zeta M_T) \gamma M_G M\) are non-educated single males who are non-thalassemia and young adults, and \(\varepsilon (1 - \zeta M_T) \gamma M_G M\) are educated single males who are non-thalassemia young adults. \(\alpha M_S M\) and \(\alpha M_S^E M\) are populations who take the premarital screening when they are about to marry. \(d_M S_M\) and \(d_M S EM\) are death of the two classes, respectively.

- **Single educated marriageable male class \((S^AE_M)\)**

  In Eq. (7), \((1 - \eta M C) \alpha M S M\) is the normal and non-carrier population screened from the educated single male population, and \(\alpha M S^AE M \left( S^AE + C^AE + S^A + C^A \right) N^A\) is the marrying population who will be flushed to the married class \(U\). \(d_M S^AE\) is the death of this class.

- **Carrier (single) educated marriageable male class \((C^AE_M)\)**

  In Eqs. (8) and (9), \(\eta C_M \alpha M S^E M\) is the carrier population screened from the educated single male population, and \(\alpha M C^AE M \left( C^AE + C^AE + S^A + C^A \right) N^A\) is the marrying carrier population who will be flushed to the married class \(U\). \(\tilde{\nu} M \alpha M C^AE M \left( C^A + C^AE \right) N^A\) is the
proportion of the educated carrier males who **decide not to marry carrier females**. This is the proportion of reconsideration of marriage of carrier male population who have been educated on thalassemia. \(d_M C_M^A\) is the death of this class.

- **Single and carrier (single) marriageable who are NOT educated on thalassemia** \((S_M^A\) and \(C_M^A\))

In Eqs. (10) and (11), \((1 - \eta_M^M)\alpha_M^MS_M^A\) and \(\eta_C^M\alpha_M^MS_M^A\) are normal and carrier male populations screened from the non educated single male and carrier male populations. The marriages of these populations are \(\alpha_M S_M^A \left(\frac{S_M^{AE} + C_M^{AE} + S_M^A + C_M^A}{N_A}\right)\) and \(\alpha_M C_M^A \left(\frac{S_M^{AE} + C_M^{AE} + S_M^A + C_M^A}{N_A}\right)\), respectively. \(d_M S_M^A\) and \(d_M C_M^A\) are the deaths of these classes, respectively. In Eq. (12) \(\nu_M \alpha_M C_M^A \left(\frac{C_A^A + C_M^{AE}}{N_A}\right)\) is the proportion of the uneducated carrier males who **decide not to marry carrier females**. This is the proportion of reconsideration of marriage of carrier male population who have been screened and knows the consequence of the carrier-carrier marriage.

- **Married (united) class** \((U)\)

In Eqs. (21) and (22), \(\alpha_M \left(\frac{S_M^{AE} + C_M^{AE} + S_M^A + C_M^A}{N_A}\right)\) and \(\alpha_F \left(\frac{S_F^{AE} + C_F^{AE} + S_F^A + C_F^A}{N_A}\right)\) are married males and females, respectively. In Eqs. (23) and (24), \(\tilde{\nu}_M \alpha_M C_M^{AE} \left(\frac{C_A^A + C_M^{AE}}{N_A}\right)\) and \(\tilde{\nu}_F \alpha_F C_F^{AE} \left(\frac{C_A^A + C_M^{AE}}{N_A}\right)\) are the proportion of carrier educated males and females who reconsider their carrier-carrier marriage, and \(\nu_M \alpha_M C_M^A \left(\frac{C_A^A + C_M^{AE}}{N_A}\right)\) and \(\nu_F \alpha_F C_F^A \left(\frac{C_A^A + C_M^{AE}}{N_A}\right)\) are that of carrier non-educated males and females who reconsider their carrier-carrier marriage. Finally, \(\frac{(d_M + d_F)U}{2}\) is the death of this class.

### II. Analysis of Model

Note that all variables remain nonnegative, i.e. \(\frac{dH}{dt} > 0\) if \(H = 0\), where

\[
\]

Also,

\[
\frac{dN}{dt} \leq (b_M + b_F)U - dN \Rightarrow N(t) \leq \frac{b_M + b_F U}{d}.
\]
where \( N = \sum_{K=M,F}\left(G_K + S_{K}^{E} + S_{K}^{AE} + C_{K}^{AE} + S_{K}^{A} + C_{K}^{A}\right) + U \), \( \bar{U} \) is the maximum number of married couples over a considered time span, and \( \bar{d} \) is the minimum of all death rates.

Then, we consider two ideal situations that are represented by two equilibrium points, namely,

(i) Type I: Thalassemia free equilibrium point, i.e., \( T_M = T_F = 0 \), \( C_M = C_M^A = C_M^{AE} = 0 \), and \( C_F = C_F^A = C_F^{AE} = 0 \);

(ii) Type II: Thalassemia major free only equilibrium point, i.e., \( T_M = T_F = 0 \) only \( C_M \), \( C_M^A \), \( C_M^{AE} \), \( C_F \), \( C_F^A \), \( C_F^{AE} \) are not necessarily zero

as time evolves in a long term. To investigate if (i) and (ii) are achievable via pre-marital screening and education factor, we will calculate two Jacobian matrices which are obtained by differentiating each equation in eqs. (1) to (24) with seventeen variables in order \( (G_M, G_F, T_M, T_F, S_M, S_{M}^{E}, S_{M}^{AE}, C_{M}^{AE}, S_{M}^{A}, C_{M}^{A}, S_F, S_{F}^{E}, S_{F}^{AE}, C_{F}^{AE}, S_{F}^{A}, C_{F}^{A}, U) \) and substituting the two equilibrium points in them.

**Type I. Thalassemia free equilibrium point**

\( (G_M^*, G_F^*, 0, 0, S_M^*, S_{M}^{AE*}, 0, S_{M}^{A*}, 0, S_F^*, S_{F}^{AE*}, 0, S_{F}^{A*}, 0, U^*) \)

For this equilibrium point, we obtain the following Jacobian matrix \( J_1 \):
\[
\begin{bmatrix}
-a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_M \\
0 & -b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_F \\
0 & 0 & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -d_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
e_1 & 0 & 0 & 0 & -e_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_i & 0 & 0 & 0 & 0 & -f_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & g_6 & -B & D & D & D & 0 & 0 & -D & -D & -D & -D & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & h_6 & 0 & -C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -l_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H & -H & -H & -H & 0 & m_{12} & -F & H & H & H & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n_{12} & 0 & -G & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_{11} & 0 & 0 & 0 & 0 & -G & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_{17} & \end{bmatrix}
\]

where

- \(a_1 = d_M^G + \gamma_M, b_2 = d_F^G + \gamma_F, c_3 = d_T + \frac{\varepsilon \gamma_M G_M^*}{\gamma_M G_M^* + \gamma_F G_F^*}, \)
- \(d_4 = d_T + \frac{\varepsilon \gamma_F G_F^*}{\gamma_M G_M^* + \gamma_F G_F^*}, \)
- \(e_1 = \gamma_M (1 - \varepsilon) \) and \(e_5 = \alpha_s^M + d_M, \) and \(f_1 = \varepsilon \gamma_M \) and \(f_6 = \alpha_s^M + d_M, \)
- \(g_6 = \alpha_s^M (1 - \eta_M^C), B = d_M + \alpha_M N_A^* \frac{(N_A^* - S_M^{AE^*})}{(N_A^*)^2} \) and \(D = \frac{\alpha_M N_A^* - S_M^{AE^*}}{(N_A^*)^2}, \) where \(N_A^* = N_{AM}^* + N_{AF}^*, \) \(N_{AM}^* = S_M^{AE^*} + S_M^{A^*} \) and \(N_{AF}^* = S_F^{AE^*} + S_F^{A^*}; \)
- \(h_6 = \alpha_s^M \eta_M^C \) and \(C = d_M + \frac{\alpha_M N_{AF}^*}{N_A^*}, \)
- \(i_5 = \alpha_s^M, \tilde{B} = d_M + \alpha_M N_{AF}^* \frac{(N_A^* - S_M^{A^*})}{(N_A^*)^2}, \) and \(E = \frac{\alpha_M N_{AF}^* S_M^{A^*}}{(N_A^*)^2}; \)
- \(k_2 = \gamma_F (1 - \varepsilon) \) and \(k_{11} = \alpha_s^F + d_F, l_2 = \varepsilon \gamma_F \) and \(l_{12} = \alpha_s^F + d_F, \)
- \(m_{12} = \alpha_s^F (1 - \eta_F), F = d_F + \alpha_F N_{AM}^* \frac{(N_A^* - S_M^{AE^*})}{(N_A^*)^2} \) and \(H = \frac{\alpha_F N_{AM}^* S_M^{AE^*}}{(N_A^*)^2}; \)
\[ n_{12} = \alpha_s \eta_F^C \text{ and } G = d_F + \frac{\alpha_FN_{AM}^*}{N_A^*}; \]

\[ p_{11} = \alpha_s^C (1 - \eta_F^C), \quad \bar{F} = d_F + \alpha_FN_{AM}^* \left( \frac{N_A^* - S_F^*}{(N_A^*)^2} \right), \text{ and } I = \frac{\alpha_FN_{AM}^* S_F^*}{(N_A^*)^2}; \]

\[ q_{11} = \alpha_s \eta_F^C, \quad r_{17} = -\frac{d_M + d_F}{2}, \quad J = \frac{(\alpha_M + \alpha_F)(N_{AP}^*)^2}{(N_A^*)^2}, \text{ and } \bar{J} = \frac{(\alpha_M + \alpha_F)(N_{AM}^*)^2}{(N_A^*)^2}. \]

**Type II.** Thalassemia major free only equilibrium point


For this equilibrium point we obtain the following Jacobian matrix \( J_2 \):

\[
\begin{bmatrix}
-a_1 & 0 & 0 & 0 & 0 & 0 & A & A_t & A & C_t & 0 & 0 & A & A'_t & A & C'_t & a_{17} \\
0 & -b_2 & 0 & 0 & 0 & 0 & \bar{A} & \bar{A}_t & \bar{A} & \bar{C}_t & 0 & 0 & \bar{A} & \bar{A}'_t & \bar{A} & C''_t & b_{17} \\
c_1 & 0 & -c_3 & 0 & 0 & 0 & -B & -B_t & -B & -D_t & 0 & 0 & -B & -B'_t & -B & -D'_t & c_{17} \\
0 & d_2 & 0 & -d_4 & 0 & 0 & -B & -B_t & -B & -D_t & 0 & 0 & -B & -B'_t & -B & -D'_t & d_{17} \\
e_1 & 0 & 0 & 0 & -e_5 & 0 & E & E_t & E & F_t & 0 & 0 & E & E'_t & E & F'_t & -e_{17} \\
f_1 & 0 & 0 & 0 & 0 & -f_6 & \bar{E} & \bar{E}_t & \bar{E} & \bar{F}_t & 0 & 0 & \bar{E} & \bar{E}'_t & \bar{E} & \bar{F}'_t & -f_{17} \\
0 & 0 & 0 & 0 & 0 & g_6 & -g_7 & G & G & G & 0 & 0 & -\bar{M} & -\bar{M} & -\bar{M} & -\bar{M} & 0 \\
0 & 0 & 0 & 0 & 0 & h_6 & H & -h_8 & H & H & 0 & 0 & -\bar{N} & -\bar{P} & -\bar{N} & -\bar{P} & 0 \\
0 & 0 & 0 & 0 & i_5 & 0 & I & I & i & I & 0 & 0 & -\bar{Q} & -\bar{Q} & -\bar{Q} & -\bar{Q} & 0 \\
0 & 0 & 0 & 0 & j_5 & 0 & J & J & J & J & 0 & 0 & -\bar{R} & -\bar{S} & -\bar{R} & -\bar{S} & 0 \\
0 & k_2 & 0 & 0 & 0 & 0 & K & L_t & K & L & 0 & 0 & K & L'_t & K & L' & -k_{17} \\
0 & l_2 & 0 & 0 & 0 & 0 & \bar{K} & \bar{L}_t & \bar{K} & \bar{L} & 0 & 0 & \bar{K} & \bar{L}'_t & \bar{K} & \bar{L}' & -l_{17} \\
0 & 0 & 0 & 0 & 0 & 0 & -M & -M & -M & -M & 0 & 0 & -M & -M & -M & -M & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -N & -P & -N & -P & 0 & 0 & -N & -N & -N & -N & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -Q & -Q & -Q & -Q & p_{11} & 0 & \bar{I} & \bar{I} & \bar{I} & p_{15} & \bar{I} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -R & -S & -R & -S & q_{11} & 0 & \bar{J} & \bar{J} & \bar{J} & \bar{J} & q_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X & Y & X & Y & 0 & 0 & X & Y & X & Y & -r_{17}
\end{bmatrix}
\]

(26)

where

(i) \( C_{Fadd} = (C_F^* + C_F^{AE^*}) \) and \( C_{Madd} = (C_M^* + C_M^{AE^*}) \)

\[
C_{FadduM} = \alpha_M (C_M^* + C_M^{AE^*}) \{ (1 - \bar{\nu}_M)C_M^{AE^*} + (1 - \nu_M)C_M^* \}
\]

\[
C_{MadduF} = \alpha_F (C_M^* + C_M^{AE^*}) \{ (1 - \bar{\nu}_F)C_F^{AE^*} + (1 - \nu_F)C_F^* \}
\]

\[ N_{AP}^* = S_M^{AE^*} + S_M^* + C_M^{AE^*} + C_M^* \text{ and } N_{AM}^* = S_F^{AE^*} + S_F^* + C_F^{AE^*} + C_F^* \]

\[ N_A^* = N_{AP}^* + N_{AM}^*; \]
(ii) \[ Z_1 = \frac{C_{Fadd}(1 + C_{Madd})}{N_A^*} \]

(iii) \[ \tilde{Z}_2 = \frac{\alpha_F((1 - \nu_F)C_A^* + (1 - \tilde{\nu}_F)C^{AE}) + \alpha_M(1 - \tilde{\nu}_M)C_{Fadd}}{N_A^*} \]

(iv) \[ Z_2 = \frac{\alpha_F((1 - \nu_F)C_A^* + (1 - \tilde{\nu}_F)C^{AE}) + \alpha_M(1 - \nu_M)C_{Fadd}}{N_A^*} \]

(v) \[ \tilde{Z}_3 = \frac{\alpha_M((1 - \nu_M)C_A^* + (1 - \tilde{\nu}_M)C^{AE}) + \alpha_F(1 - \tilde{\nu}_F)C_{Madd}}{N_A^*} \]

(vi) \[ Z_3 = \frac{\alpha_M((1 - \nu_M)C_A^* + (1 - \tilde{\nu}_M)C^{AE}) + \alpha_F(1 - \nu_F)C_{Madd}}{N_A^*} \]

- \( a_1 = d_M^* + \gamma_M + \frac{(1 - \gamma_M)\eta_{T}Z_1}{U^*} \), \( c_1 = \frac{\eta_{T}Z_1}{U^*} \), \( e_1 = \gamma_M(1 - \varepsilon)(1 - \eta_{T}^*) \frac{Z_1}{U^*} \) and 
  \( f_1 = \gamma_M\varepsilon(1 - \eta_{T}^*) \frac{Z_1}{U^*} \);

- \( b_2 = d_F^* + \gamma_F + \frac{(1 - \gamma_F)\eta_{T}Z_1}{U^*} \), \( d_2 = \frac{\eta_{T}Z_1}{U^*} \), \( k_2 = \gamma_F(1 - \varepsilon)(1 - \eta_{T}^*) \frac{Z_1}{U^*} \) and 
  \( l_2 = \gamma_F\varepsilon(1 - \eta_{T}^*) \frac{Z_1}{U^*} \);

- \( c_3 = d_T + \frac{\varepsilon\gamma_M(1 - \eta_{T}^*)G_MZ_1}{(\gamma_FG_F + \gamma_MG_M)U^*} \) and \( d_4 = d_T + \frac{\varepsilon\gamma_F(1 - \eta_{T}^*)G_FZ_1}{(\gamma_FG_F + \gamma_MG_M)U^*} \);

- \( e_5 = f_6 = d_M + \alpha_s^M \), \( i_5 = g_6 = \alpha_s^M(1 - \eta_{T}^*) \), and \( j_5 = h_6 = \alpha_s^M\eta_{T}^* \);

- \( g_7 = d_M + \frac{\alpha_MN_A^*(N_A^* - S^{AE})}{(N_A^*)^2} \) and \( h_8 = d_M + \frac{\alpha_M(N_A^* - C^{AE})(N_A^* - \tilde{\nu}_MC_{Fadd})}{(N_A^*)^2} \);

- \( i_9 = d_M + \frac{\alpha_MN_A^*(N_A^* - S^{AE})}{(N_A^*)^2} \) and \( j_10 = d_M + \frac{\alpha_M(N_A^* - C^{AE})(N_A^* - \nu_MC_{Fadd})}{(N_A^*)^2} \);

- \( k_{11} = l_{12} = d_F + \alpha_s^F \), \( p_{11} = m_{12} = \alpha_s^F(1 - \eta_{T}^*) \), and \( q_{11} = n_{12} = \alpha_s^F\eta_{T}^* \);

- \( m_{13} = d_F + \frac{\alpha_FN_A^*(N_A^* - S^{AE})}{(N_A^*)^2} \) and \( n_{14} = d_F + \frac{\alpha_F(N_A^* - C^{AE})(N_A^* - \tilde{\nu}_FC_{Madd})}{(N_A^*)^2} \);

- \( p_{15} = d_F + \frac{\alpha_FN_A^*(N_A^* - S^{AE})}{(N_A^*)^2} \) and \( q_{16} = d_F + \frac{\alpha_F(N_A^* - C^{AE})(N_A^* - \nu_FC_{Madd})}{(N_A^*)^2} \);

- \( a_{17} = b_M - \frac{(1 - \gamma_M)\eta_{T}G_M^*Z_1}{(U^*)^2} \) and \( b_{17} = b_F - \frac{(1 - \gamma_F)\eta_{T}G_F^*Z_1}{(U^*)^2} \);

- \( c_{17} = \frac{\eta_{T}G_M^*Z_1}{(U^*)^2} \) and \( d_{17} = \frac{\eta_{T}G_F^*Z_1}{(U^*)^2} \);

- \( e_{17} = \frac{(1 - \varepsilon)\gamma_M\eta_{T}G_M^*Z_1}{(U^*)^2} \) and \( f_{17} = \frac{\varepsilon\eta_{T}\gamma_MG_M^*Z_1}{(U^*)^2} \).
\[ k_{17} = \frac{(1 - \varepsilon) \eta_F^T \gamma_F G_F^* Z_1}{(U^*)^2} \quad \text{and} \quad l_{17} = \frac{\varepsilon \eta_F^T \gamma_F G_F^* Z_1}{(U^*)^2}, \]
\[ r_{17} = \frac{d_M + d_F}{2}; \]
\[ A = \frac{(1 - \gamma_M) \eta_M^T G_M Z_1}{N_A^* U^*} \quad \text{and} \quad \tilde{A} = \frac{(1 - \gamma_F) \eta_F^T G_F Z_1}{N_A^* U^*}, \]
\[ A_t = \frac{(1 - \gamma_M) \eta_M^T G_M (Z_1 - \tilde{Z}_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{A}_t = \frac{(1 - \gamma_F) \eta_F^T G_F (Z_1 - \tilde{Z}_2)}{N_A^* U^*}, \]
\[ A'_t = \frac{(1 - \gamma_M) \eta_M^T G_M (Z_1 - \tilde{Z}_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{A}'_t = \frac{(1 - \gamma_F) \eta_F^T G_F (Z_1 - \tilde{Z}_3)}{N_A^* U^*}, \]
\[ C_t = \frac{(1 - \gamma_M) \eta_M^T G_M (Z_1 - Z_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{C}_t = \frac{(1 - \gamma_F) \eta_F^T G_F (Z_1 - Z_2)}{N_A^* U^*}, \]
\[ C'_t = \frac{(1 - \gamma_M) \eta_M^T G_M (Z_1 - Z_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{C}'_t = \frac{(1 - \gamma_F) \eta_F^T G_F (Z_1 - Z_3)}{N_A^* U^*}; \]
\[ B = \frac{\eta_M^T G_M Z_1}{N_A^* U^*} \quad \text{and} \quad \tilde{B} = \frac{\eta_F^T G_F Z_1}{N_A^* U^*}, \]
\[ B_t = \frac{\eta_M^T G_M (Z_1 - \tilde{Z}_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{B}_t = \frac{\eta_F^T G_F (Z_1 - \tilde{Z}_2)}{N_A^* U^*}, \]
\[ B'_t = \frac{\eta_M^T G_M (Z_1 - \tilde{Z}_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{B}'_t = \frac{\eta_F^T G_F (Z_1 - \tilde{Z}_3)}{N_A^* U^*}, \]
\[ D_t = \frac{\eta_M^T G_M (Z_1 - Z_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{D}_t = \frac{\eta_F^T G_F (Z_1 - Z_2)}{N_A^* U^*}, \]
\[ D'_t = \frac{\eta_M^T G_M (Z_1 - Z_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{D}'_t = \frac{\eta_F^T G_F (Z_1 - Z_3)}{N_A^* U^*}; \]
\[ E = \frac{(1 - \varepsilon) \eta_M^T G_M Z_1}{N_A^* U^*} \quad \text{and} \quad \tilde{E} = \frac{\varepsilon \eta_M^T G_M Z_1}{N_A^* U^*}, \]
\[ E_t = \frac{(1 - \varepsilon) \eta_M^T G_M (Z_1 - \tilde{Z}_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{E}_t = \frac{\varepsilon \eta_M^T G_M (Z_1 - \tilde{Z}_2)}{N_A^* U^*}, \]
\[ E'_t = \frac{(1 - \varepsilon) \eta_M^T G_M (Z_1 - \tilde{Z}_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{E}'_t = \frac{\varepsilon \eta_M^T G_M (Z_1 - \tilde{Z}_3)}{N_A^* U^*}, \]
\[ F_t = \frac{(1 - \varepsilon) \eta_M^T G_M (Z_1 - Z_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{F}_t = \frac{\varepsilon \eta_M^T G_M (Z_1 - Z_2)}{N_A^* U^*}, \]
\[ F'_t = \frac{(1 - \varepsilon) \eta_M^T G_M (Z_1 - Z_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{F}'_t = \frac{\varepsilon \eta_M^T G_M (Z_1 - Z_3)}{N_A^* U^*}; \]
\[ K = \frac{(1 - \varepsilon) \eta_F^T \gamma_F G_F Z_1}{N_A^* U^*} \quad \text{and} \quad \tilde{K} = \frac{\varepsilon \eta_F^T \gamma_F G_F Z_1}{N_A^* U^*}, \]
\[ L_t = \frac{(1 - \varepsilon) \eta_F^T \gamma_F G_F (Z_1 - \tilde{Z}_2)}{N_A^* U^*} \quad \text{and} \quad \tilde{L}_t = \frac{\varepsilon \eta_F^T \gamma_F G_F (Z_1 - \tilde{Z}_2)}{N_A^* U^*}, \]
\[ L'_t = \frac{(1 - \varepsilon) \eta_F^T \gamma_F G_F (Z_1 - \tilde{Z}_3)}{N_A^* U^*} \quad \text{and} \quad \tilde{L}'_t = \frac{\varepsilon \eta_F^T \gamma_F G_F (Z_1 - \tilde{Z}_3)}{N_A^* U^*}; \]
\[ L = \frac{(1 - \varepsilon)\eta_F^2 G_F}{N_A^* U^*}(Z_1 - Z_2) \quad \text{and} \quad \tilde{L} = \frac{\varepsilon \eta_F^2 G_F}{N_A^* U^*}(Z_1 - Z_2), \]
\[ L' = \frac{(1 - \varepsilon)\eta_F^2 G_F}{N_A^* U^*}(Z_1 - Z_3) \quad \text{and} \quad \tilde{L}' = \frac{\varepsilon \eta_F^2 G_F}{N_A^* U^*}(Z_1 - Z_3), \]

- \( G = \frac{\alpha_M S_M^{AE^*} N_A^*}{(N_A^*)^2} \) and \( \tilde{G} = \frac{\alpha_F S_F^{AE^*} N_A^*}{(N_A^*)^2} \)
- \( H = \frac{\alpha_M C_M^{AE^*} (N_A^* - \tilde{\nu}_M C_{Fadd})}{(N_A^*)^2} \) and \( \tilde{H} = \frac{\alpha_F C_F^{AE^*} (N_A^* - \tilde{\nu}_F C_{Madd})}{(N_A^*)^2} \)
- \( J = \frac{\alpha_M C_M^{A^*} (N_A^* - \tilde{\nu}_M C_{Fadd})}{(N_A^*)^2} \) and \( \tilde{J} = \frac{\alpha_F C_F^{A^*} (N_A^* - \tilde{\nu}_F C_{Madd})}{(N_A^*)^2} \)
- \( P = \frac{\alpha_F C_F^{AE^*} (N_A^* - \tilde{\nu}_F (N_A^* - C_{Madd}))}{(N_A^*)^2} \) and \( \tilde{P} = \frac{\alpha_M C_M^{AE^*} (N_A^* - \tilde{\nu}_M (N_A^* - C_{Fadd}))}{(N_A^*)^2} \)
- \( S = \frac{\alpha_F C_F^{A^*} (N_A^* - \tilde{\nu}_F (N_A^* - C_{Madd}))}{(N_A^*)^2} \) and \( \tilde{S} = \frac{\alpha_M C_M^{A^*} (N_A^* - \tilde{\nu}_M (N_A^* - C_{Fadd}))}{(N_A^*)^2} \)
- \( M = \frac{\alpha_F S_F^{AE^*} N_A^*}{(N_A^*)^2} \) and \( \tilde{M} = \frac{\alpha_M S_M^{AE^*} N_A^*}{(N_A^*)^2} \)
- \( N = \frac{\alpha_F C_F^{AE^*} (N_A^* + \tilde{\nu}_M C_{Madd})}{(N_A^*)^2} \) and \( \tilde{N} = \frac{\alpha_M C_M^{AE^*} (N_A^* + \tilde{\nu}_M C_{Fadd})}{(N_A^*)^2} \)
- \( R = \frac{\alpha_F C_F^{A^*} (N_A^* + \tilde{\nu}_M C_{Madd})}{(N_A^*)^2} \) and \( \tilde{R} = \frac{\alpha_M C_M^{A^*} (N_A^* + \tilde{\nu}_M C_{Fadd})}{(N_A^*)^2} \)
- \( X = \frac{(\alpha_M + \alpha_F)(N_A^*)^2}{(N_A^*)^2} + \frac{\alpha_F C_{Madd}(\tilde{\nu}_F C_F^{A^*} + \tilde{\nu}_F C_F^{AE^*}) + \alpha_M C_{Fadd}(\tilde{\nu}_M C_M^{A^*} + \tilde{\nu}_M C_M^{AE^*})}{(N_A^*)^2}, \)
- \( Y = X - \frac{\alpha_F(\tilde{\nu}_F C_F^{A^*} + \tilde{\nu}_F C_F^{AE^*}) + \alpha_M \tilde{\nu}_M C_{Fadd}}{N_A^*}, \)
- \( \tilde{Y} = X - \frac{\alpha_M(\tilde{\nu}_M C_M^{A^*} + \tilde{\nu}_M C_M^{AE^*}) + \alpha_F \tilde{\nu}_F C_{Madd}}{N_A^*}. \)

Proposition 1 [1] [Chapter 6 Corollary 6.1.3] The eigenvalues of \( A = [m_{ij}]_{n \times n} \), an \( n \) by \( n \) matrix, are in the union of \( n \) discs

\[ \bigcup_{j=1}^{n} \{z \in \mathbb{C} : |z - a_{jj}| \leq C'_j(A)\}, \tag{27} \]

where \( \mathbb{C} \) is the set of complex numbers, \( a_{jj} \) are the diagonal entries in \( A \),
\[ \{z \in \mathbb{C} : |z - a_{jj}| \leq C'_j(A)\}, \]

is a disc in \( \mathbb{C} \) centered at \( a_{jj} \) with the radius \( C'_j(A) \), and

\[ C'_j(A) = \sum_{i \neq j} |a_{ij}|, \quad j = 1, \ldots, n, \tag{28} \]
which is the absolute column sum without the the diagonal entry in the jth column.

The above proposition is known as Geršgorin disc theorem. By using Proposition 1, we have the following result:

**Theorem 2** Type I equilibrium point, the thalassemia free equilibrium point

\[
(G^*_M, G^*_F, 0, 0, S^*_M, S^*_{AE}^*, 0, S^*_A, 0, S^*_F, S^*_{AE}^*, 0, S^*_A, 0, U^*)
\]

and Type II equilibrium point, the thalassemia major free only equilibrium point

\[
\]

are unstable.

**Proof** From the jacobian matrix \(J_1\), we can calculate \(C'_j(J_1) = \sum_{i \neq j} |a_{ij}|, \ j = 1, \cdots, n.\). Then, we have

\[
\bigcup_{j=1}^{17} \{z \in \mathbb{C} : |z - a_{jj}| \leq C'_j(J_1)\}, \tag{29}
\]

where the discs from \(j = 1, \cdots, 6\), and \(j = 10, 11\) locate in the left half plane of \(\mathbb{C}\) since \(|a_{jj}| > C'_j(J_1)\) for such \(j\). However, the rest of discs may cross the origin and the right half plane of \(\mathbb{C}\). Note that the 17th disc is given by

\[
\{z \in \mathbb{C} : |z - d_M + d_F| \leq b_M + b_F\} \tag{30}
\]

which is the disc centered at \(-\frac{d_M + d_F}{2}\) with the radius \(C'_{17}(J_1) = b_M + b_F\). Since the UAE population is in the increasing trend, i.e. the birth rate is greater than the averaged death rate, we conclude

\[
\frac{d_M + d_F}{2} < b_M + b_F. \tag{31}
\]

Thus, the 17th disc in (30) surely crosses the origin and the right half plane of \(\mathbb{C}\). Thus, Type I equilibrium point is unstable. By the similar argument, from the jacobian matrix \(J_2\) we can calculate \(C'_j(J_2) = \sum_{i \neq j} |a_{ij}|, \ j = 1, \cdots, n.\). Then, we have

\[
\bigcup_{j=1}^{17} \{z \in \mathbb{C} : |z - a_{jj}| \leq C'_j(J_2)\}, \tag{32}
\]
where the discs from $j = 1, \cdots, 6$, and $j = 10, 11$ locate in the left half plane of $\mathbb{C}$ since $|a_{jj}| > C'_j(J_2)$ for such $j$. However, the rest of discs may cross the origin and the right half plane of $\mathbb{C}$. In particular, the 17th disc is given by

$$\{z \in \mathbb{C} : |z - \left(-\frac{d_M + d_F}{2}\right)| \leq b_M + b_F + \frac{(\eta_T^T \gamma_M G_M^* + \eta_F^T \gamma_F G_F^*) Z_1}{(U^*)^2}\}$$

(33)

which is the disc centered at $-\frac{d_M + d_F}{2}$ with the radius $C'_{17}(J_2) = b_M + b_F + \frac{(\eta_M^T \gamma_M G_M^* + \eta_F^T \gamma_F G_F^*) Z_1}{(U^*)^2}$, where $Z_1 = \frac{C_{FaddnuM} + C_{MaddnuF}}{N_A}$.

Thus, the 17th disc surely crosses the origin and hence the right half plane of $\mathbb{C}$. Thus, Type II equilibrium point is unstable. This completes the proof. 

**Remark** In the result in Theorem 2 we observed the 17th disc crosses through the origin from the left half plane to the right half plane of $\mathbb{C}$ from the relation between the birth and death rates of the whole population. Thus, Types I and II equilibrium points are unstable regardless of the premarital screening and education factor. Hence, Types I and II equilibrium points that are our current goal in thalassemia control will not be achievable with the premarital and education factor in a long term.

### III. Parameter Estimation

We obtain and estimate the parameter values for the simulations from the 2015 data of the UAE bureau of statistics [2], 2013 Abu Dhabi Statistics Yearbook [3] and 2013 Health Authority of Abh Dhabi [4]. The part of the UAE national population data is summarized in Table 1.

Then, the parameter values are estimated as follows:

- Birth rates: $b_M = \frac{17,279}{476,712} = 0.0362$ and $b_F = \frac{16,761}{476,712} = 0.0352$. 


### Table 1: The UAE National Census Data

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<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>F</td>
<td>Total</td>
</tr>
<tr>
<td>Population</td>
<td>231,383</td>
<td>245,329</td>
<td>476,712</td>
</tr>
</tbody>
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Death rates: \( d_M = \frac{1432}{476,712} = 0.0030 \) and \( d_F = \frac{910}{476,712} = 0.0019 \)

- **Marriage rates:**
  \( \alpha_M = \frac{\text{Male Marriage Contracts}}{\text{Total Population}} = \frac{9560}{476712} = 0.02 \)
  \( \alpha_F = \frac{\text{Female Marriage Contracts}}{\text{Total population}} = \frac{8239}{476712} = 0.0172 \)

- **Screening rates:**
  \( \alpha_M^s = \frac{\text{Male Marriage Contracts}}{\text{Male total population}} = \frac{9560}{231383} = 0.0413 \)
  \( \alpha_F^s = \frac{\text{Female Marriage Contracts}}{\text{Female total Population}} = \frac{8239}{245329} = 0.0335 \)

- Thalassemia diagnosis adjusting factor: \( \eta_M^T = 0.015 \) and \( \eta_F^T = 0.144 \) are chosen such that the proportion of thalassemia major male is less than 0.00025 and the proportion of thalassemia major female is than 0.00024.

### References


