Appendix A. The Proof of Proposition 1.

Proposition 1. For states $p \in Q$, $q \in Q$, if there exists a symbol $a$ that maintains $\delta(p, a) = s$ and $\delta(q, a) = t$ and furthermore, $s$ and $t$ are distinguishable, then $p$ and $q$ are distinguishable.

Proof. Suppose $\delta(p, a) = s$, $\delta(q, a) = t(s \neq t)$, and $p \equiv q$.

Because $s \neq t$, there must be a word $w$ that satisfies ($\tilde{\delta}(s, w) \in F$, $\tilde{\delta}(t, w) \notin F$) or ($\tilde{\delta}(s, w) \notin F$, $\tilde{\delta}(t, w) \in F$).

Therefore, $\tilde{\delta}(p, aw) \in F$ and $\tilde{\delta}(q, aw) \notin F$, or $\tilde{\delta}(p, aw) \notin F$ and $\tilde{\delta}(q, aw) \in F$.

This means that $p \neq q$, which contradicts the supposition. Hence, proposition 1 is proved.

Appendix B. The Proof of Proposition 2.

Proposition 2. If the backward depths of two states $p$ and $q$ for any accepted state $t$ are different, $p$ and $q$ must be distinguishable. Formally, if $BD(p, t) \neq BD(q, t)$, then $p \neq q$.

Proof. Because $BD(p, t) \neq BD(q, t)$, there exist words $w_i$ and $w_j$ that maintain $\tilde{\delta}(p, w_i) = t$ and $\tilde{\delta}(q, w_j) = t$, respectively, where $|w_i| \neq |w_j|$.

If $|w_i| < |w_j|$, then $\tilde{\delta}(p, w_i) = t$ and $\tilde{\delta}(q, w_j) \neq t$.

Thus, $p$ and $q$ are distinguishable $(p \neq q)$.

\qed