Description of the confidence interval calculations for the slope of the mean squared displacement.

Let \( Y_{il} = ||X_l(t_i)||^2 \) denote the squared displacement of the \( l \)th trajectory at the \( i \)th time point for \( l = 1, \ldots, N \) and \( i = 1, \ldots, m \). The sample mean squared displacement \( \bar{Y}_i \) at the \( i \)th time point is computed by

\[
\bar{Y}_i = \frac{1}{N} \sum_{l=1}^{N} Y_{il},
\]

which provides an estimate of the expected value \( E(Y_{il}) \). We also denote by \( C_{ij} \) the sample covariance between the square displacements at times \( t_i \) and \( t_j \), where \( C_{ij} \) are given by

\[
C_{ij} = \frac{1}{N} \sum_{l=1}^{N} (Y_{il} - \bar{Y}_i)(Y_{jl} - \bar{Y}_j) = \left( \frac{1}{N} \sum_{l=1}^{N} Y_{il}Y_{jl} \right) - \bar{Y}_i\bar{Y}_j,
\]

and provide estimates of \( \text{Cov}(Y_{il}, Y_{jl}) \) the covariance between \( Y_{il} \) and \( Y_{jl} \).

The slope \( s \) of the least squares line through the origin is given by

\[
s = \sum_{i=1}^{m} \alpha_i \bar{Y}_i,
\]

where

\[
\alpha_i = \frac{t_i}{\sum_{j=1}^{m} t_j^2}.
\]

In order to calculate the 95% confidence intervals for the estimated slope \( s \) we use the formula for the variance \( \text{Var}(s) \) of \( s \) given by

\[
\text{Var}(s) = \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i\alpha_j \text{Cov}(\bar{Y}_i, \bar{Y}_j),
\]

where \( \text{Cov}(\bar{Y}_i, \bar{Y}_j) \) is the covariance between \( \bar{Y}_i \) and \( \bar{Y}_j \). Since

\[
\text{Cov}(\bar{Y}_i, \bar{Y}_j) = \frac{1}{N} \text{Cov}(Y_{il}, Y_{jl}),
\]

we estimate \( \text{Cov}(\bar{Y}_i, \bar{Y}_j) \) by \( C_{ij}/N \). This gives the formula

\[
V = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i\alpha_j C_{ij},
\]

for the estimated variance \( V \) of \( s \). Finally, the 95% confidence interval for the slope is computed by

\[
CI = (s - 1.96\sqrt{V}, s + 1.96\sqrt{V}).
\]