Supporting Information of

Excess Relative Risk as an Effect Measure in Case-Control Studies of Rare Diseases

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S7 Exhibit. A proof that a constant excess relative risk (ERR) and a constant risk ratio (RR) models cannot be reconciled except for a weak exposure or when disease risks vary little across strata.

Under a constant ERR model, we have

\[
RR_s = \frac{\text{Risk}_{s,2} + \text{ERR}}{\text{Risk}_{s,2}} = 1 + \frac{\text{Risk}_{E=2} \times \text{ERR}}{\text{Risk}_{s,2}}.
\]

Therefore,

\[
\text{Var}(RR_s) = \text{Var} \left( \frac{\text{Risk}_{E=2} \times \text{ERR}}{\text{Risk}_{s,2}} \right)
= \frac{\text{Risk}_{E=2}^2 \times \text{ERR}^2}{\text{Risk}_{E=2}^4} \times \text{Var}(\text{Risk}_{s,2})
= \text{ERR}^2 \times \text{CV}^2(\text{Risk}_{s,2}),
\]

where \(\text{CV}(\text{Risk}_{s,2})\) is the coefficient of variation of the disease risks of the unexposed population across the strata. Under a constant RR model, we have

\[
\text{ERR}_s = \frac{\text{RR} \times \text{Risk}_{s,2} - \text{Risk}_{s,2}}{\text{Risk}_{E=2}} = \frac{(\text{RR} - 1) \times \text{Risk}_{s,2}}{\text{Risk}_{E=2}}.
\]

Therefore,

\[
\text{Var}(\text{ERR}_s) = \text{Var} \left( \frac{(\text{RR} - 1) \times \text{Risk}_{s,2}}{\text{Risk}_{E=2}} \right)
= \frac{(\text{RR} - 1)^2}{\text{Risk}_{E=2}^2} \times \text{Var}(\text{Risk}_{s,2})
= (\text{RR} - 1)^2 \times \text{CV}^2(\text{Risk}_{s,2}).
\]