Appendix A: Different phases of growth and shrinkage

![Diagram](image)

Fig. S1. (a) Phase diagram of $N=1$ microtubule in the force ($f$)-concentration ($c$) plane. The curve of mean wall-velocity $\langle V \rangle = 0$ demarcates between two phases, namely the bounded and unbounded growth phases. (b) and (c): Typical time traces of the wall position in the bounded phase. The trajectory of (b) shows that the system length (wall position) $x(t)$ first shrinks rapidly with a negative velocity, but ultimately it fluctuates around a constant mean value — the later part is zoomed in (c), which shows catastrophes of the filament. (d) A typical trajectory of the system length in the unbounded growth phase, where $x(t)$ grows in time with a positive velocity. Parameters are specified in Table 1 and inside the figure panels.

Depending on the values of the applied force $f$ and concentration $c$, there are two dynamical phases of a $N$-filament system — (i) the bounded growth phase, and (ii) the unbounded growth phase [1, 2]. The phase diagram for $N = 1$ microtubule is shown in the $f$–$c$ plane, in Fig. S1a. The $\langle V \rangle = 0$ curve marks the phase-boundary (where, $\langle V \rangle$ is the mean wall velocity). It should be noted that, in the absence of force ($f = 0$), there exists a critical ATP/GTP concentration $c = c_{\text{crit}}$, at which $\langle V \rangle = 0$ i.e. the system is stalled (see Fig. S1a). In the presence of force ($f > 0$), and for a concentration $c > c_{\text{crit}}$, the system can only be stalled when we apply the “stall force”, $f = f_s^{(N)}$, at which the average wall velocity $\langle V \rangle = 0$ [3, 4].

For the parameter regime $c > c_{\text{crit}}$ with $f > f_s^{(N)}$, or for $c < c_{\text{crit}}$ with $f \geq 0$, the filament shrinks on an average with a negative velocity — see the trajectory of the wall position in Fig. S1b. During shrinkage, when the filament length becomes very short, the filament eventually encounters the immovable left wall, and then the length fluctuates around a constant mean value — see Fig. S1c. This is called the bounded phase. On the other hand, for the parameter regime $c > c_{\text{crit}}$ and $f < f_s^{(N)}$, the filament indefinitely grows on an average, with a positive velocity — this is the unbounded growth phase — see
Appendix B: Comparison of stall forces obtained from the collapse time measurements, and from the force-velocity relations

In this appendix, we discuss two possible measurement procedures for the stall force. In theories, the stall force is usually measured from force-velocity relations by finding the force \( f = f_s^{(N)} \), at which average wall velocity \( \langle V(f) \rangle = 0 \) [3,5,6]. But a fact is that \( \langle V \rangle \) becomes very small for \( f \to f_s^{(N)} \), with increasing \( N \). As a result, for \( N > 2 \), the task of experimentally measuring the precise \( f_s^{(N)} \) is challenging. In fact, from simulations we calculated that for \( N = 3 \) microtubules (or actin filaments), at a force \( f = 3f_s^{(1)} \) and concentration \( c = 100 \mu M \) (or \( c = 1 \mu M \)), the wall velocity is \( \sim 0.1 \) nm/s (or \( \sim 0.02 \) nm/s). Monitoring such slow motion, and finding the force for which the wall truly halts maybe difficult.

We have already shown in the main text that, if \( f \to f_s^{(N)} \) from above, the average collapse time \( T_{\text{coll}} \) tends to diverge. This behavior of \( T_{\text{coll}} \) may be used to precisely determine the collective stall force \( f_s^{(N)} \) of a system. This relies on approaching \( f_s^{(N)} \) from above, as opposed to from below as in the case of \( f \to \langle V \rangle \) measurements. Note that such a measurement of \( T_{\text{coll}} \) should be easy experimentally, as one is dealing with large values, while monitoring vanishingly small values of \( \langle V \rangle \) is more difficult. We present the values of \( f_s^{(N)} \) obtained numerically by noticing the limits \( \langle V \rangle \to 0 \), and \( T_{\text{coll}} \to \infty \), in Table S1 — they match quite well.

Table S1. Comparison of values of stall forces obtained numerically by monitoring the limits \( \langle V \rangle \to 0 \), and \( T_{\text{coll}} \to \infty \). ATP/GTP Concentrations are taken to be \( c = 1 \mu M \) for actin and \( c = 100 \mu M \) for microtubule (for other parameters see Table 1).

<table>
<thead>
<tr>
<th></th>
<th>( \langle V \rangle ) measurement</th>
<th>( f_s^{(1)} ) (pN)</th>
<th>( f_s^{(2)} ) (pN)</th>
<th>( f_s^{(3)} ) (pN)</th>
<th>( f_s^{(4)} ) (pN)</th>
</tr>
</thead>
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<td>Actin</td>
<td>( T_{\text{coll}} ) measurement</td>
<td>3.134</td>
<td>6.389</td>
<td>9.619</td>
<td>12.834</td>
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<tr>
<td>MT</td>
<td>( \langle V \rangle ) measurement</td>
<td>16.748</td>
<td>35.010</td>
<td>52.793</td>
<td>70.384</td>
</tr>
<tr>
<td></td>
<td>( T_{\text{coll}} ) measurement</td>
<td>16.741</td>
<td>35.017</td>
<td>52.814</td>
<td>70.473</td>
</tr>
</tbody>
</table>

Appendix C: Results for \( \delta = 0.8 \)

In the main text, we considered the force distribution factor \( \delta = 1 \) (i.e. only the polymerisation rate is force dependent), as estimated from the experiments on microtubules [7], and consistent with the earlier theories [2,3,5]. This estimation of \( \delta \) for microtubule has been debated [8]. For actin the value of \( \delta \) is not known. To check if our results hold for \( \delta \neq 1 \) (i.e. when both the polymerisation and depolymerisation rates are force dependent) we calculate different dynamical quantities with \( \delta = 0.8 \) for actin filaments — this is shown in Fig. S2. We see in Fig. S2a that the average cap sizes for filament number \( N > 1 \) are orders of magnitudes higher than \( N = 1 \) at large forces — this is similar to the \( \delta = 1 \) case (although the cap sizes for \( N > 1 \) do not saturate with force). In Fig. S2b we show the diffusion constant \( D \) for the length fluctuations versus force for \( \delta = 0.8 \) — this figure is quite similar to its counterpart for \( \delta = 1 \) (see Fig. 8a in main paper). The diffusion constant \( D \) does not decay with force for \( N > 1 \) unlike the \( N = 1 \) case. Finally in Fig. S2c, we show the diffusion constant \( D \) derived from the fluctuations of switching events between the capped and uncapped states as a function of force. The figure shows qualitatively similar behavior as the length diffusion constant \( D \). This again demonstrates our point (already shown
for $\delta = 1$) that the overall length fluctuations are closely related to the fluctuations of the switching events between the capped and uncapped states. Thus our conclusions seem to remain qualitatively unchanged even for $\delta \neq 1$.

Fig. S2. (a) Average cap size $\langle k \rangle$, (b) the diffusion constant $D$ for the system-length fluctuations, and (c) the diffusion constant $D_s$ for the fluctuations of switching events between capped and uncapped states — these are plotted against the scaled force $f/f_s(1)$. All data are for actin parameters (see Table 1) with a concentration $c = 0.2\mu M$ and for $\delta = 0.8$.

References


