Appendix S2

Derivation of $\Delta r$

According to Eq. (11) in the main body of the paper, the herding degree of bull markets ($r(t) > 0$) and bear markets ($r(t) < 0$) are defined as

$$\begin{align*}
\text{bull} & \iff d_{\text{bull}}(r(t)) = \sum_{t, r(t) > 0} [V(t) \cdot r(t)] / \sum_{t, r(t) > 0} V(t), \\
\text{bear} & \iff d_{\text{bear}}(r(t)) = \sum_{t, r(t) < 0} [V(t) \cdot |r(t)|] / \sum_{t, r(t) < 0} V(t).
\end{align*}$$

We introduce a shifting of $r(t)$, denoted by $\Delta r$, such that $d_{\text{bull}}(r'(t)) = d_{\text{bear}}(r'(t))$ with $r'(t) = r(t) + \Delta r$. With $r(t)$ replaced by $r'(t)$ in the above equation, we have

$$\begin{align*}
&\begin{cases}
\text{bull} & \iff d_{\text{bull}}(r'(t)) = \sum_{t, r'(t) > 0} [V(t) \cdot [r(t) + \Delta r]] / \sum_{t, r'(t) > 0} V(t), \\
\text{bear} & \iff d_{\text{bear}}(r'(t)) = \sum_{t, r'(t) < 0} [V(t) \cdot |r(t) + \Delta r|] / \sum_{t, r'(t) < 0} V(t).
\end{cases}
\end{align*}$$

$\Delta r$ is first assumed to be small, and this is verified from the practical calculation. Hence, $r'(t) > 0$ and $r'(t) < 0$ are approximately $r(t) > 0$ and $r(t) < 0$, respectively. Therefore,

$$\begin{align*}
&\begin{cases}
\text{bull} & \iff d_{\text{bull}}(r'(t)) = \sum_{t, r(t) > 0} [V(t) \cdot [r(t) + \Delta r]] / \sum_{t, r(t) > 0} V(t), \\
\text{bear} & \iff d_{\text{bear}}(r'(t)) = \sum_{t, r(t) < 0} [V(t) \cdot |r(t) + \Delta r|] / \sum_{t, r(t) < 0} V(t).
\end{cases}
\end{align*}$$

Thus, we have

$$d_{\text{bull}}(r'(t)) = \sum_{t, r(t) > 0} [V(t) \cdot r(t)] / \sum_{t, r(t) > 0} V(t) + \Delta r,$$

and

$$d_{\text{bear}}(r'(t)) = -\sum_{t, r(t) < 0} [V(t) \cdot r(t)] / \sum_{t, r(t) < 0} V(t) - \Delta r$$

$$= \sum_{t, r(t) < 0} [V(t) \cdot |r(t)|] / \sum_{t, r(t) < 0} V(t) - \Delta r.$$

Inserting the above two equations into $d_{\text{bull}}(r'(t)) = d_{\text{bear}}(r'(t))$, we have

$$\sum_{t, r(t) > 0} [V(t) \cdot r(t)] / \sum_{t, r(t) > 0} V(t) - \sum_{t, r(t) < 0} [V(t) \cdot |r(t)|] / \sum_{t, r(t) < 0} V(t) + 2\Delta r = 0.$$

Therefore,

$$\Delta r = \frac{1}{2} \left\{ \frac{\sum_{t, r(t) < 0} [V(t) \cdot |r(t)|]}{\sum_{t, r(t) < 0} V(t)} - \frac{\sum_{t, r(t) > 0} [V(t) \cdot r(t)]}{\sum_{t, r(t) > 0} V(t)} \right\}$$

$$= \frac{1}{2} [d_{\text{bear}}(r(t)) - d_{\text{bull}}(r(t))].$$