Supplement S1 Bifurcation analysis of the stress response $F(t)$

\[ \frac{dF}{dt} = 0 \]

$F_1: F_1 = 0$

\[ F_2: T \left(1 - \frac{F(t)}{K}\right) = \frac{F(t)}{1+4F(t)^2} \]  \( \text{(S1.1)} \)

When is $T \left(1 - \frac{F(t)}{K}\right)$ at a tangent to $\frac{F(t)}{1+4F(t)^2}$?

\[ \frac{d}{dF} \left(T \left(1 - \frac{F(t)}{K}\right)\right) = \frac{d}{dF} \frac{F(t)}{1+4F(t)^2} \]

\[ - \frac{T}{K} = 1 - \frac{F(t)^2}{(1+4F(t)^2)^2} \]  \( \text{(S1.2)} \)

Substituting (S1.2) into (S1.1) yields

\[ T - \frac{F(t)^2 - 1}{(1+4F(t)^2)^2} F(t) = \frac{F(t)}{1+4F(t)^2} \]

\[ T = \frac{F(t)^3 - F(t) + F(t) + F(t)^3}{(1+4F(t)^2)^2} \]

\[ T = \frac{2F(t)^3}{(1+4F(t)^2)^2} \]  \( \text{(S1.3)} \)

Thus, a saddle-note bifurcation occurs if $T$ increases and becomes greater than (S1.3). The current stable fixpoint is annihilated by the instable one. Increasing $F$ further leads to a rapid change to the only stable fixpoint left, resembling bistable behavior.