Text S1. Calculating steady states of system (6) and their stability

Steady states of system (6) can be found by solving the following set of equations:

\[
\begin{align*}
0 &= X(r_x + b_{xy}Y)(1 - \frac{\beta X}{K_x}), \\
0 &= Y(r_y + b_{yx}X)(1 - \frac{Y}{K_y}).
\end{align*}
\]  

(A1)

for \(X\) and \(Y\). It is easy to see that there are four steady states \((X_1^*, Y_1^*)=(0, 0),\) \((X_2^*, Y_2^*)=(K_x/\beta, 0),\) \((X_3^*, Y_3^*)=(0, K_y)\) and \((X_4^*, Y_4^*)=(K_x/\beta, K_y)\). The Jacobian matrix of (A1) takes the form

\[
J(X,Y) = \begin{pmatrix}
(r_x + b_{xy}Y)(1 - \frac{2\beta X}{K_x}) & b_{yx}X(1 - \frac{\beta X}{K_x}) \\
-(r_y + b_{yx}X)(1 - \frac{2Y}{K_y}) & (r_y + b_{yx}X)(1 - \frac{\beta Y}{K_y})
\end{pmatrix}
\]

evaluated at \((X,Y)=(X_i^*, Y_i^*)\) where \(i=1..4\). For the trivial steady state we have

\[
J(0,0) = \begin{pmatrix}
r_x & 0 \\
0 & r_y
\end{pmatrix},
\]

hence the trivial steady state is unstable. Similarly for the semi-trivial steady states we have

\[
J(K_x/\beta, 0) = \begin{pmatrix}
-r_y & 0 \\
0 & r_y + b_{yx}K_x/\beta
\end{pmatrix}
\]

and \(J(0,K_y) = \begin{pmatrix}
r_x + b_{yx}K_y & 0 \\
0 & -r_y
\end{pmatrix},\)

from which we conclude that both semi-trivial steady states are also unstable. Finally for the non-trivial steady state we have

\[
J(K_x/\beta, K_y) = \begin{pmatrix}
-(r_x + b_{yx}K_y) & 0 \\
0 & -(r_y + b_{yx}K_y/\beta)
\end{pmatrix},
\]

and therefore the non-trivial steady state is stable.