Appendix S3. Derivation of Eq. 15

In Eq. 10, we defined $l_i$ as

$$l_i = r_i \ln \lambda_i(\alpha) - \lambda_i(\alpha) + r_i \ln T - \frac{\ln ((r_i T)!)}{T}.$$ 

The first and the second derivatives of $l_i$ are respectively given as follows:

$$\frac{\partial l_i}{\partial \alpha} = r_i \frac{\lambda_i'(\alpha)}{\lambda_i(\alpha)} - \lambda_i'(\alpha),$$

$$\frac{\partial^2 l_i}{\partial \alpha^2} = r_i \frac{\lambda_i''(\alpha)}{\lambda_i(\alpha)} - r_i \frac{(\lambda_i'(\alpha))^2}{(\lambda_i(\alpha))^2} - \lambda_i''(\alpha).$$

Substituting it into Eq. 14 yields the local Fisher information:

$$J_{\text{Local}}(\alpha; \phi_i) \equiv E \left[ -r_i \frac{\lambda_i''(\alpha)}{\lambda_i(\alpha)} + r_i \frac{(\lambda_i'(\alpha))^2}{(\lambda_i(\alpha))^2} + \lambda_i''(\alpha) \right] |_{\alpha}.$$ 

$$= -\frac{\lambda_i'(\alpha)^2}{\lambda_i(\alpha)} + \frac{\lambda_i(\alpha)(\lambda_i'(\alpha))^2}{(\lambda_i(\alpha))^2} + \frac{\lambda_i''(\alpha)}{\lambda_i(\alpha)}.$$ 

$$= \frac{(\lambda_i'(\alpha))^2}{\lambda_i(\alpha)}.$$