**Supporting material: Results in a sparse network**

1. **Stimulus-enhanced theta/gamma oscillation in a sparse network**

The results in the main text are obtained using an all-to-all connected network. We have additionally investigated whether our results can also be generated using a more physiological sparsely connected network. Here the sparseness of the network is realized by randomly assigning the coupling between neurons and the probability that a pair of neurons are connected in either direction is p=0.8. The network size is still the same as in the all-to-all network i.e. NR=100, Nf=50, Ns=50. The maximum synaptic conductances are slightly modified as:

Figure S1 shows that for a sparse network, the theta-gamma dual oscillations can be generated by by utilizing a combination of fast and slow-type GABAA receptor interneurons. Similar to the case in an all-to-all network, these theta/gamma dual oscillations in a sparse arrangement also show stimulus-enhanced theta power with a weak gamma-band power. As for the cross frequency coupling (CFC) between theta phase and gamma amplitude, the result is also similar to the all-to-all network, i.e., the coherence of the CFC is increased during the stimulus compared with the pre-stimulus period. Figure S2 shows that when the network size increases to *N*EX =200, *N*INf =100, *N*INs =100, the coupling probability can be reduced to p=0.6 and all results still hold true. We anticipate that the probability of connectivity could be further reduced with an increased network size.

1. **Different synaptic mechanism for learning-related changes in the theta amplitude and the CFC**

In Figure S3 and Figure S4, we show the dependence of the theta and gamma amplitudes, the coherence of the CFC between theta phase and gamma amplitude, the variation of theta-band phase on different types of synaptic connections in a sparse network. Figure S3 shows the dependence on and conductances. Over a small range the effects of these two parameters on theta amplitude have a slight difference to results shown using an all-to-all network shown in Figure 4 in the main text. With the sparse network both theta amplitude and the coherence of CFC can be increased by moderately increasing the conductance , however further increases result in a decrease of theta amplitude and the CFC. Changing the conductance , does not have a significant effect in increasing theta amplitude, but can lead to an increase of the CFC strength. In Figure S4 effects of altering and conductances can be seen. Increasing and can lead to a significant and more robust increase of both theta amplitude and the coherence of CFC, and the theta-band phase becomes more synchronized among neurons. These results are consistent with those shown in Figure 5 in the main text for an all-to-all connected network.

In conclusion in a sparsened network: i). both theta amplitude and the CFC can also be increased by moderately increasing the EX-to-EX connection and the EX-to-INs connection mediated by NMDA receptors, but only in a narrow parameter range. ii). Co-ordinately increasing the EX-to-EX connection mediated by NMDA receptors and the INs-to- EX connection mediated by slow GABAA receptors can also significantly and robustly increase theta amplitude and the CFC as well as the synchronization of theta-band phase among neurons.