**S2 Equation:**

**Dynamic transmission model**

These are difference equations of the dynamic model of the pneumococcal transmission between serogroups, vaccination efficacy parameters against colonisation of vaccine serogroups and a potential parameter ($α\_{i}$) to describe the rapid increase in NVT IPD cases since 2014/15.

**Unvaccinated**:

$$S\_{i,1}\left(t\right)=\left(1-λ\_{1,i}\left(t-1\right)-λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)S\_{i-1,1}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,1}\left(t-1\right)+VT2\_{i-1,1}\left(t-1\right)+NVT\_{i-1,1}\left(t-1\right)\right),$$

$$VT1\_{i,1}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)VT1\_{i-1,1}\left(t-1\right)+λ\_{1,i}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,1}\left(t-1\right)+VT1NVT\_{i-1,1}\left(t-1\right)\right),$$

$$VT2\_{i,1}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)VT2\_{i-1,1}\left(t-1\right)+λ\_{2,i}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,1}\left(t-1\right)+VT2NVT\_{i-1,1}\left(t-1\right)\right),$$

$$NVT\_{i,1}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}λ\_{1,i}\left(t-1\right)-π\_{6,i}λ\_{2,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)NVT\_{i-1,1}\left(t-1\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,1}\left(t-1\right)+VT2NVT\_{i-1,1}\left(t-1\right)\right),$$

$$VT1VT2\_{i,1}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)VT1VT2\_{i-1,1}\left(t-1\right)+π\_{1,i}λ\_{2,i}VT1\_{i-1,1}\left(t-1\right)+π\_{4,i}λ\_{1,i}VT2\_{i-1,1}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,1}\left(t-1\right),$$

$$VT1NVT\_{i,1}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}λ\_{2,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)VT1NVT\_{i-1,1}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,1}\left(t-1\right)+π\_{5,i}λ\_{1,i}\left(t-1\right)NVT\_{i-1,1}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,1}\left(t-1\right),$$

$$VT2NVT\_{i,1}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}λ\_{1,i}\left(t-1\right)-φ\_{0,i}(t-1)\right)VT2NVT\_{i-1,1}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,1}\left(t-1\right)+π\_{6,i}λ\_{2,i}NVT\_{i-1,1}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,1}\left(t-1\right),$$

$$ALL\_{i,1}\left(t\right)=\left(1-3ρ\_{i-1}-φ\_{0,i}(t-1)\right)ALL\_{i-1,1}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,1}\left(t-1\right) +π\_{8,i}λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,1}\left(t-1\right)+πλ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,1}\left(t-1\right),$$

**PCV7 partially protected**:

$$S\_{i,2}\left(t\right)=\left(1-\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)S\_{i-1,2}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,2}\left(t-1\right)+VT2\_{i-1,2}\left(t-1\right)+NVT\_{i-1,2}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+ωS\_{i-1,3}\left(t-1\right),$$

$$VT1\_{i,2}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1\_{i-1,2}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT1NVT\_{i-1,2}\left(t-1\right)\right)+\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1\_{i-1,1}\left(t-1\right)+ωVT1\_{i-1,3}\left(t-1\right),$$

$$VT2\_{i,2}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)VT2\_{i-1,2}\left(t-1\right)+λ\_{2,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT2NVT\_{i-1,2}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT2\_{i-1,1}\left(t-1\right)+ωVT2\_{i-1,3}\left(t-1\right),$$

$$NVT\_{i,2}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-π\_{6,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)NVT\_{i-1,2}\left(t-1\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,2}\left(t-1\right)+VT2NVT\_{i-1,2}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)NVT\_{i-1,1}\left(t-1\right)+ωNVT\_{i-1,3}\left(t-1\right),$$

$$VT1VT2\_{i,2}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1VT2\_{i-1,2}\left(t-1\right)+π\_{1,i}λ\_{2,i}VT1\_{i-1,2}\left(t-1\right)+π\_{4,i}\left(1-d\_{7}/2\right)λ\_{1,i}VT2\_{i-1,2}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1VT2\_{i-1,1}\left(t-1\right)+ωVT1VT2\_{i-1,3}\left(t-1\right),$$

$$VT1NVT\_{i,2}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1NVT\_{i-1,2}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,2}\left(t-1\right)+π\_{5,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)NVT\_{i-1,2}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1NVT\_{i-1,1}\left(t-1\right)+ωVT1NVT\_{i-1,3}\left(t-1\right),$$

$$VT2NVT\_{i,2}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT2NVT\_{i-1,2}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,2}\left(t-1\right)+π\_{6,i}λ\_{2,i}NVT\_{i-1,2}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT2NVT\_{i-1,1}\left(t-1\right)+ωVT2NVT\_{i-1,3}\left(t-1\right),$$

$$ALL\_{i,2}\left(t\right)=\left(1-3ρ\_{i-1}-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)ALL\_{i-1,2}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,2}\left(t-1\right) +π\_{8,i}λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,2}\left(t-1\right)+π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)ALL\_{i-1,1}\left(t-1\right)+ωALL\_{i-1,3}\left(t-1\right),$$

**PCV7 fully protected**:

$$S\_{i,3}\left(t\right)=\left(1-\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-ω\right)S\_{i-1,3}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,3}\left(t-1\right)+VT2\_{i-1,3}\left(t-1\right)+NVT\_{i-1,3}\left(t-1\right)\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+φ\_{1,i}(t-1)\left(S\_{i-1,2}\left(t-1\right)+S\_{i-1,4}\left(t-1\right)\right),$$

$$VT1\_{i,3}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-ω\right)VT1\_{i-1,3}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,3}\left(t-1\right)+VT1NVT\_{i-1,3}\left(t-1\right)\right)+\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT1\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1\_{i-1,2}\left(t-1\right)+VT1\_{i-1,4}\left(t-1\right)\right),$$

$$VT2\_{i,3}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-ω\right)VT2\_{i-1,3}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,3}\left(t-1\right)+VT2NVT\_{i-1,3}\left(t-1\right)\right)+λ\_{2,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT2\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT2\_{i-1,2}\left(t-1\right)+VT2\_{i-1,4}\left(t-1\right)\right),$$

$$NVT\_{i,3}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-π\_{6,i}λ\_{2,i}\left(t-1\right)-ω\right)NVT\_{i-1,3}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,3}\left(t-1\right)+VT2NVT\_{i-1,3}\left(t-1\right)\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,2}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(NVT\_{i-1,2}\left(t-1\right)+NVT\_{i-1,4}\left(t-1\right)\right),$$

$$VT1VT2\_{i,3}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-ω\right)VT1VT2\_{i-1,3}\left(t-1\right)+π\_{1,i}λ\_{2,i}VT1\_{i-1,3}\left(t-1\right)+π\_{4,i}\left(1-d\_{7}\right)λ\_{1,i}VT2\_{i-1,3}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,3}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT1VT2\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT1VT2\_{i-1,4}\left(t-1\right)\right),$$

$$VT1NVT\_{i,3}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}λ\_{2,i}\left(t-1\right)-ω\right)VT1NVT\_{i-1,3}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,3}\left(t-1\right)+π\_{5,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)NVT\_{i-1,3}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,3}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT1VT2\_{i-1,4}\left(t-1\right)\right),$$

$$VT2NVT\_{i,3}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-ω\right)VT2NVT\_{i-1,3}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,3}\left(t-1\right)+π\_{6,i}λ\_{2,i}NVT\_{i-1,3}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,3}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT2NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT2NVT\_{i-1,2}\left(t-1\right)+VT2NVT\_{i-1,4}\left(t-1\right)\right),$$

$$ALL\_{i,3}\left(t\right)=\left(1-3ρ\_{i-1}-ω\right)ALL\_{i-1,3}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,3}\left(t-1\right) +π\_{8,i}λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,3}\left(t-1\right)+π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,3}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)ALL\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(ALL\_{i-1,2}\left(t-1\right)+ALL\_{i-1,4}\left(t-1\right)\right),$$

**PCV7 waned**:

$$S\_{i,4}\left(t\right)=\left(1-λ\_{1,i}\left(t-1\right)-λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)S\_{i-1,4}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,4}\left(t-1\right)+VT2\_{i-1,4}\left(t-1\right)+NVT\_{i-1,4}\left(t-1\right)\right)+ωS\_{i-1,2}\left(t-1\right),$$

$$VT1\_{i,4}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1\_{i-1,4}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,4}\left(t-1\right)+VT1NVT\_{i-1,4}\left(t-1\right)\right)+λ\_{1,i}\left(t-1\right)S\_{i-1,4}\left(t-1\right)+ωVT1\_{i-1,2}\left(t-1\right),$$

$$VT2\_{i,4}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)VT2\_{i-1,4}\left(t-1\right)+λ\_{2,i}\left(t-1\right)S\_{i-1,4}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,4}\left(t-1\right)+VT2NVT\_{i-1,4}\left(t-1\right)\right)+ωVT2\_{i-1,2}\left(t-1\right),$$

$$NVT\_{i,4}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}λ\_{1,i}\left(t-1\right)-π\_{6,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)NVT\_{i-1,4}\left(t-1\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,4}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,4}\left(t-1\right)+VT2NVT\_{i-1,4}\left(t-1\right)\right)+ωNVT\_{i-1,2}\left(t-1\right),$$

$$VT1VT2\_{i,4}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1VT2\_{i-1,4}\left(t-1\right)+π\_{1,i}λ\_{2,i}VT1\_{i-1,4}\left(t-1\right)+π\_{4,i}λ\_{1,i}VT2\_{i-1,4}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,4}\left(t-1\right)+ωVT1VT2\_{i-1,2}\left(t-1\right),$$

$$VT1NVT\_{i,4}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1NVT\_{i-1,4}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,4}\left(t-1\right)+π\_{5,i}λ\_{1,i}\left(t-1\right)NVT\_{i-1,4}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,4}\left(t-1\right)+ωVT1NVT\_{i-1,2}\left(t-1\right),$$

$$VT2NVT\_{i,4}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}λ\_{1,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT2NVT\_{i-1,4}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,4}\left(t-1\right)+π\_{6,i}λ\_{2,i}NVT\_{i-1,4}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,4}\left(t-1\right)+ωVT2NVT\_{i-1,2}\left(t-1\right),$$

$$ALL\_{i,4}\left(t\right)=\left(1-3ρ\_{i-1}-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)ALL\_{i-1,4}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,4}\left(t-1\right) +π\_{8,i}λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,4}\left(t-1\right)+π\_{9,i}λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,4}\left(t-1\right)+ωALL\_{i-1,2}\left(t-1\right),$$

**PCV13 partially protected**:

$$S\_{i,5}\left(t\right)=\left(1-\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)S\_{i-1,5}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,5}\left(t-1\right)+VT2\_{i-1,5}\left(t-1\right)+NVT\_{i-1,5}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+ωS\_{i-1,6}\left(t-1\right),$$

$$VT1\_{i,5}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1\_{i-1,5}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,5}\left(t-1\right)+VT1NVT\_{i-1,5}\left(t-1\right)\right)+\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)S\_{i-1,5}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1\_{i-1,1}\left(t-1\right)+ωVT1\_{i-1,6}\left(t-1\right),$$

$$VT2\_{i,5}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)VT2\_{i-1,5}\left(t-1\right)+\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)S\_{i-1,5}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,5}\left(t-1\right)+VT2NVT\_{i-1,5}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT2\_{i-1,1}\left(t-1\right)+ωVT2\_{i-1,6}\left(t-1\right),$$

$$NVT\_{i,5}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-π\_{6,i}\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)NVT\_{i-1,5}\left(t-1\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,5}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,5}\left(t-1\right)+VT2NVT\_{i-1,5}\left(t-1\right)\right)+φ\_{0,i(if age<1)}\left(t-1\right)NVT\_{i-1,1}\left(t-1\right)+ωNVT\_{i-1,6}\left(t-1\right),$$

$$VT1VT2\_{i,5}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1VT2\_{i-1,5}\left(t-1\right)+π\_{1,i}\left(1-d\_{13}/2\right)λ\_{2,i}VT1\_{i-1,5}\left(t-1\right)+π\_{4,i}\left(1-d\_{7}/2\right)λ\_{1,i}VT2\_{i-1,5}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,5}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1VT2\_{i-1,1}\left(t-1\right)+ωVT1VT2\_{i-1,6}\left(t-1\right),$$

$$VT1NVT\_{i,5}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1NVT\_{i-1,5}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,5}\left(t-1\right)+π\_{5,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)NVT\_{i-1,5}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,5}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1NVT\_{i-1,1}\left(t-1\right)+ωVT1NVT\_{i-1,6}\left(t-1\right),$$

$$VT2NVT\_{i,5}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT2NVT\_{i-1,5}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,5}\left(t-1\right)+π\_{6,i}\left(1-d\_{13}/2\right)λ\_{2,i}NVT\_{i-1,5}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,5}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT2NVT\_{i-1,1}\left(t-1\right)+ωVT2NVT\_{i-1,6}\left(t-1\right),$$

$$ALL\_{i,5}\left(t\right)=\left(1-3ρ\_{i-1}-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)ALL\_{i-1,5}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,5}\left(t-1\right) +π\_{8,i}\left(1-d\_{13}/2\right)λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,5}\left(t-1\right)+π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,5}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)ALL\_{i-1,1}\left(t-1\right)+ωALL\_{i-1,6}\left(t-1\right),$$

**PCV13 fully protected:**

$$S\_{i,6}\left(t\right)=\left(1-\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-ω\right)S\_{i-1,6}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,6}\left(t-1\right)+VT2\_{i-1,6}\left(t-1\right)+NVT\_{i-1,6}\left(t-1\right)\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)S\_{i-1,1}\left(t-1\right)+φ\_{1,i}(t-1)\left(S\_{i-1,2}\left(t-1\right)+S\_{i-1,4}\left(t-1\right)+S\_{i-1,5}\left(t-1\right)+S\_{i-1,7}\left(t-1\right)\right),$$

$$VT1\_{i,6}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-ω\right)VT1\_{i-1,6}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,6}\left(t-1\right)+VT1NVT\_{i-1,6}\left(t-1\right)\right)+\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)S\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT1\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1\_{i-1,2}\left(t-1\right)+VT1\_{i-1,4}\left(t-1\right)+VT1\_{i-1,5}\left(t-1\right)+VT1\_{i-1,7}\left(t-1\right)\right),$$

$$VT2\_{i,6}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-ω\right)VT2\_{i-1,6}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,6}\left(t-1\right)+VT2NVT\_{i-1,6}\left(t-1\right)\right)+\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)S\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT2\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT2\_{i-1,2}\left(t-1\right)+VT2\_{i-1,4}\left(t-1\right)+VT2\_{i-1,5}\left(t-1\right)+VT2\_{i-1,7}\left(t-1\right)\right),$$

$$NVT\_{i,6}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)-π\_{6,i}\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)-ω\right)NVT\_{i-1,6}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,6}\left(t-1\right)+VT2NVT\_{i-1,6}\left(t-1\right)\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(NVT\_{i-1,2}\left(t-1\right)+NVT\_{i-1,4}\left(t-1\right)+NVT\_{i-1,5}\left(t-1\right)+NVT\_{i-1,7}\left(t-1\right)\right),$$

$$VT1VT2\_{i,6}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-ω\right)VT1VT2\_{i-1,6}\left(t-1\right)+π\_{1,i}\left(1-d\_{13}\right)λ\_{2,i}VT1\_{i-1,6}\left(t-1\right)+π\_{4,i}\left(1-d\_{7}\right)λ\_{1,i}VT2\_{i-1,6}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT1VT2\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT1VT2\_{i-1,4}\left(t-1\right)+VT1VT2\_{i-1,5}\left(t-1\right)+VT1VT2\_{i-1,7}\left(t-1\right)\right),$$

$$VT1NVT\_{i,6}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)-ω\right)VT1NVT\_{i-1,6}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,6}\left(t-1\right)+π\_{5,i}\left(1-d\_{7}\right)λ\_{1,i}\left(t-1\right)NVT\_{i-1,6}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age<1)}\left(t-1\right)VT1NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT1VT2\_{i-1,2}\left(t-1\right)+VT1VT2\_{i-1,4}\left(t-1\right)+VT1VT2\_{i-1,5}\left(t-1\right)+VT1VT2\_{i-1,7}\left(t-1\right)\right),$$

$$VT2NVT\_{i,6}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)-ω\right)VT2NVT\_{i-1,6}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,6}\left(t-1\right)+π\_{6,i}\left(1-d\_{13}\right)λ\_{2,i}NVT\_{i-1,6}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)VT2NVT\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(VT2NVT\_{i-1,2}\left(t-1\right)+VT2NVT\_{i-1,4}\left(t-1\right)+VT2NVT\_{i-1,5}\left(t-1\right)+VT2NVT\_{i-1,7}\left(t-1\right)\right),$$

$$ALL\_{i,6}\left(t\right)=\left(1-3ρ\_{i-1}-ω\right)ALL\_{i-1,3}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,6}\left(t-1\right) +π\_{8,i}\left(1-d\_{13}\right)λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,6}\left(t-1\right)+π\_{9,i}\left(1-d\_{7}/2\right)λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,6}\left(t-1\right)+φ\_{0,i(if age\geq 1)}\left(t-1\right)ALL\_{i-1,1}\left(t-1\right)+φ\_{1,i}\left(t-1\right)\left(ALL\_{i-1,2}\left(t-1\right)+ALL\_{i-1,4}\left(t-1\right)+ALL\_{i-1,5}\left(t-1\right)+ALL\_{i-1,7}\left(t-1\right)\right),$$

**PCV13 waned:**

$$S\_{i,7}\left(t\right)=\left(1-λ\_{1,i}\left(t-1\right)-λ\_{2,i}\left(t-1\right)-λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)S\_{i-1,7}\left(t-1\right)+ρ\_{i-1}\left(VT1\_{i-1,7}\left(t-1\right)+VT2\_{i-1,7}\left(t-1\right)+NVT\_{i-1,7}\left(t-1\right)\right)+ωS\_{i-1,5}\left(t-1\right),$$

$$VT1\_{i,7}\left(t\right)=\left(1-ρ\_{i-1}-π\_{1,i}λ\_{2,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1\_{i-1,7}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,7}\left(t-1\right)+VT1NVT\_{i-1,7}\left(t-1\right)\right)+λ\_{1,i}\left(t-1\right)S\_{i-1,7}\left(t-1\right)+ωVT1\_{i-1,5}\left(t-1\right),$$

$$VT2\_{i,7}\left(t\right)=\left(1-ρ\_{i-1}-π\_{4,i}λ\_{1,i}\left(t-1\right)-π\_{2,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)VT2\_{i-1,7}\left(t-1\right)+λ\_{2,i}\left(t-1\right)S\_{i-1,7}\left(t-1\right)+ρ\_{i-1}\left(VT1VT2\_{i-1,7}\left(t-1\right)+VT2NVT\_{i-1,7}\left(t-1\right)\right)+ωVT2\_{i-1,5}\left(t-1\right),$$

$$NVT\_{i,7}\left(t\right)=\left(1-ρ\_{i-1}-π\_{5,i}λ\_{1,i}\left(t-1\right)-π\_{6,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}\left(t-1\right)\right)NVT\_{i-1,7}\left(t-1\right)+λ\_{3,i}\left(t-1\right)S\_{i-1,7}\left(t-1\right)+ρ\_{i-1}\left(VT1NVT\_{i-1,7}\left(t-1\right)+VT2NVT\_{i-1,7}\left(t-1\right)\right)+ωNVT\_{i-1,5}\left(t-1\right),$$

$$VT1VT2\_{i,7}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{7,i}λ\_{3,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1VT2\_{i-1,7}\left(t-1\right)+π\_{1,i}λ\_{2,i}VT1\_{i-1,7}\left(t-1\right)+π\_{4,i}λ\_{1,i}VT2\_{i-1,7}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,7}\left(t-1\right)+ωVT1VT2\_{i-1,5}\left(t-1\right),$$

$$VT1NVT\_{i,7}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{8,i}λ\_{2,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT1NVT\_{i-1,7}\left(t-1\right)+π\_{2,i}λ\_{3,i}\left(t-1\right)VT1\_{i-1,7}\left(t-1\right)+π\_{5,i}λ\_{1,i}\left(t-1\right)NVT\_{i-1,7}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,7}\left(t-1\right)+ωVT1NVT\_{i-1,5}\left(t-1\right),$$

$$VT2NVT\_{i,7}\left(t\right)=\left(1-2ρ\_{i-1}-π\_{9,i}λ\_{1,i}\left(t-1\right)-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)VT2NVT\_{i-1,7}\left(t-1\right)+π\_{3,i}λ\_{3,i}VT1\_{i-1,7}\left(t-1\right)+π\_{6,i}λ\_{2,i}NVT\_{i-1,7}\left(t-1\right)+ρ\_{i-1}ALL\_{i-1,7}\left(t-1\right)+ωVT2NVT\_{i-1,5}\left(t-1\right),$$

$$ALL\_{i,7}\left(t\right)=\left(1-3ρ\_{i-1}-φ\_{1,i}(t-1)-φ\_{2,i}(t-1)\right)ALL\_{i-1,7}\left(t-1\right)+π\_{7,i}λ\_{3,i}\left(t-1\right)VT1VT2\_{i-1,7}\left(t-1\right) +π\_{8,i}λ\_{2,i}\left(t-1\right)VT1NVT\_{i-1,7}\left(t-1\right)+π\_{9,i}λ\_{1,i}\left(t-1\right)VT2NVT\_{i-1,7}\left(t-1\right)+ωALL\_{i-1,5}\left(t-1\right),$$

for $i=2,…,4800$ age cohorts (48 cohorts for each annual age cohort comprising 100 year cohorts between 0y and 99y) , where the initial values of Unvaccinated group is obtained from the pre-vaccination equilibrium from the static model result. The movement between vaccine protected groups depends on the monthly vaccine uptake and vaccine protection waning according to the duration of vaccine protection. $π$ is a reduction parameter on the FOI, $λ,$ which is 1- Competition parameter between serogroups, $φ$s are monthly vaccination rates for each dose (two primary and booster doses), d7 and d13 are reduction in FOIs due to PCV7 and PCV13 against acquisition of VT1 and VT2, $ω$ is a waning parameter (1/duration of vaccine protection), and $ρ$ is a clearance rate, 1/ duration of colonisation.

The Nelder-Mead method finds the set of model parameters with the maximum Poisson likelihood:

$LogLikelihood (Model|data)= \sum\_{Y=2005}^{2015}\sum\_{Sero=1}^{3}\sum\_{Age=1}^{6}\left(IPDData\_{Y,Sero,Age} log⁡(IPDModel\_{Y,Sero,Age})-IPDModel\_{Y,Sero,Age}\right)$

for three serogroups and six age groups between 2005/06 and 2015/16.