Text S3  Average fixation probability

We wish to find the rate of change of the average probability of fixation of a focal allele,

$$\partial_t \bar{P} = \sum_X g(X) \partial_t P(X) + \sum_X \partial_t g(X) P(X).$$  \hspace{1cm} (21)

The first sum is given by the average of Eq. (2), weighted by genotype frequencies:

$$- \sum_X g(X) \partial_t P(X) = s \bar{P} + \sum_X g(X) S(X) P(X) + \sum_{X,Y} g(X) r(X,Y) (P(Y) - P(X) - \frac{1}{2} \sum_X g(X) P(X)^2.$$ \hspace{1cm} (22)

To calculate the second sum in Eq. (21), we require the rate of change of background frequencies:

$$\partial_t g(X) = S(X) g(X) + \frac{1}{2} \sum_Y (g(Y) R(X,Y) - g(X) R(X,Y)),$$ \hspace{1cm} (23)

where $R(X,Y)$ is the rate at which individuals with genotype $X$ recombine to form individuals with genotype $Y$. (The factor of 1/2 in Eq. (23) is necessary because each recombination event involves two parents recombining to form two offspring.) Note that unlike $r(X,Y)$ (the rate at which recombination moves the focal allele from background $X$ to background $Y$), the definition of $R(X,Y)$ does not involve the focal allele, and in general $r(X,Y) \neq \frac{1}{2} R(X,Y)$. However, it is true that $\sum_X g(X) r(X,Y) = \frac{1}{2} \sum_X g(X) R(X,Y)$ for all $Y$, since both sides are expressions for the total rate of recombination events producing offspring with genotype $Y$. Thus, when we substitute Eqs. (22) and (23) into Eq. (21), we find that the terms involving $S$ and $r$ cancel, leaving

$$- \frac{\partial \bar{P}}{\partial t} = s \bar{P} - \frac{1}{2} \sum_X g(X) P(X)^2.$$ \hspace{1cm} (24)

Rewriting the second term in terms of the mean and variance of $P(X)$, we obtain Eq. (3).