Text S1: The Axiomatic Basis of the Shapley Value

Let player $i$ be a null player in $v$ if $\Delta_i(S) = 0$ for every coalition $S$ ($i \notin S$). Players $i$ and $j$ are interchangeable in $v$ if $\Delta_i(S) = \Delta_j(S)$ for every coalition $S$ that contains neither $i$ nor $j$. The Shapley value is the only efficient value that satisfies the three following axioms:

**Axiom 1 (Symmetry)** If $i$ and $j$ are interchangeable in game $v$ then $\gamma_i(v) = \gamma_j(v)$.

Intuitively, this axiom states that the value should not be affected by a mere change in the players’ “names”.

**Axiom 2 (Null player property)** If $i$ is a null player in game $v$ then $\gamma_i(v) = 0$.

This axiom sets the baseline of the value to be zero for a player whose marginal importance is always zero.

**Axiom 3 (Additivity)** For any two games $v$ and $w$ on a set $N$ of players, $\gamma_i(v + w) = \gamma_i(v) + \gamma_i(w)$ for all $i \in N$, where $v + w$ is the game defined by $(v + w)(S) = v(S) + w(S)$.

This last axiom constrains the value to be consistent in the space of all games.